GPDs and fitting procedures for DVCS

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Based mostly on:

K.K., D. Müller, M. Murray, to appear in Phys. Part. Nucl. (2014), `arXiv:1301.1230`
K.K., D. Müller, A. Schäfer, JHEP **07** (2011) 073, `arXiv:1106.2808`
K.K., D. Müller, Nucl. Phys. **B841** (2010) 1-58, `arXiv:0904.0458`

QCD Evolution Workshop,
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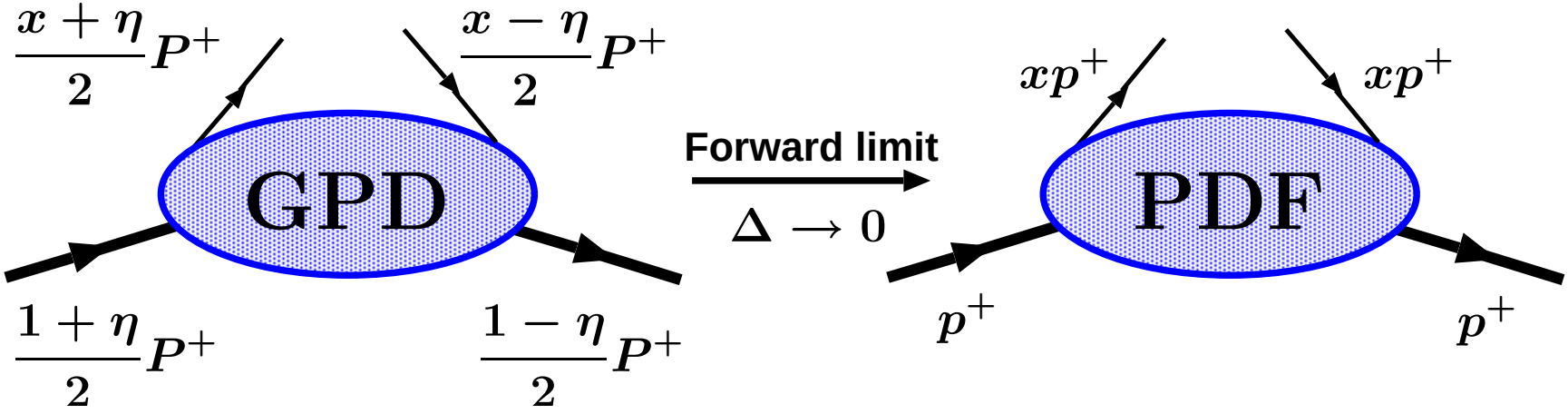
Outline

Attractiveness of GPDs (1/3)

- **1. Well-defined within the QCD**

  [Müller '92, et al. '94, Ji, Radyushkin '96]

$$F^q(x,\eta,t=\Delta^2) = \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P_2|\bar{q}(-z)\gamma^+ q(z)|P_1\rangle\Big|_{z^+=0,\,\mathbf{z}_\perp=\mathbf{0}}$$



$$P = P_1 + P_2\,; \qquad \Delta = P_2 - P_1\,; \qquad \eta = -\frac{\Delta^+}{P^+} \text{ (skewedness)}$$

Attractiveness of GPDs (2/3)

- Decomposition into nucleon helicity conserving and non-conserving parts:

$$F^a = \frac{\bar{u}(P_2)\gamma^+ u(P_1)}{P^+} H^a + \frac{\bar{u}(P_2) i\sigma^{+\nu} u(P_1)\Delta_\nu}{2MP^+} E^a \qquad a = q, g$$

$$H^q(x,0,0)\big|_{x\geq 0} = q(x) \qquad \int_{-1}^{1} dx\, H^q(x,\eta,t) = F_1^q(t)$$

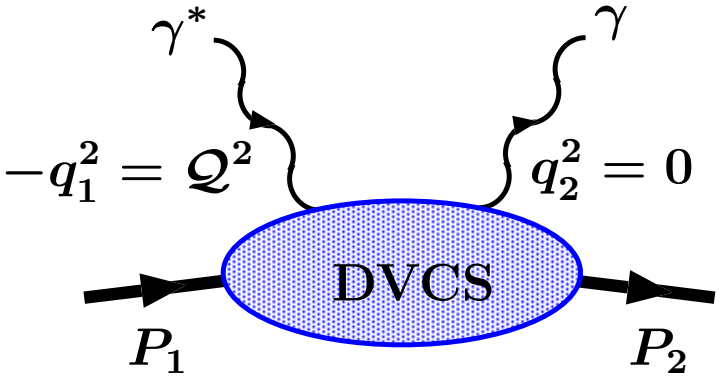
- **2. Close contact to 3D quark-gluon hadron structure**

$$\frac{1}{2}\int_{-1}^{1} dx\, x\Big[H^q(x,\eta,t) + E^q(x,\eta,t)\Big] = J^q(t) \qquad \text{[Ji '97]}$$

$$q(x, b_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2}\, e^{ib_\perp\cdot\Delta_\perp}\, H^q(x,0,-\Delta_\perp^2) \qquad \text{[Burkardt '00]}$$

Attractiveness of GPDs (3/3)

- **3. Accessible to experiments**
- Deeply virtual Compton scattering (DVCS)



$$P = P_1 + P_2 \,, \qquad t = (P_2 - P_1)^2$$

$$q = (q_1 + q_2)/2$$

Generalized Bjorken limit:

$$-q^2 \simeq \mathcal{Q}^2/2 \to \infty$$

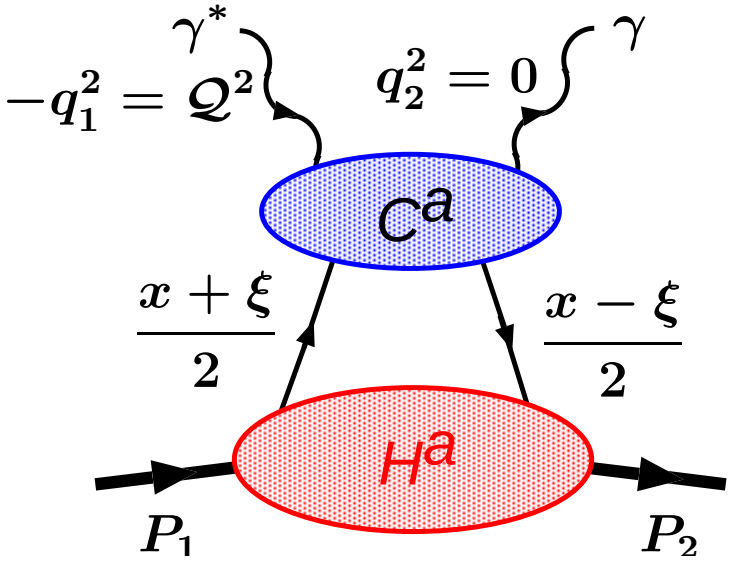
$$\xi = \frac{-q^2}{2P \cdot q} = \frac{x_B}{2 - x_B} \to \text{const}$$

- We work at leading order accuracy where cross-section can be expressed in terms of four Compton form factors (CFFs)

$$\mathcal{F} \in \{\mathcal{H}(\xi, t, \mathcal{Q}^2), \mathcal{E}(\xi, t, \mathcal{Q}^2), \tilde{\mathcal{H}}(\xi, t, \mathcal{Q}^2), \tilde{\mathcal{E}}(\xi, t, \mathcal{Q}^2)\}$$

Factorization of DVCS $\longrightarrow$ GPDs

- [Collins et al. '98]



- Compton form factor is a convolution:

$$^a\mathcal{H}(\xi, t, \mathcal{Q}^2) = \int \mathrm{d}x \; C^a(x, \xi, \mathcal{Q}^2/\mathcal{Q}_0^2) \; H^a(x, \eta = \xi, t, \mathcal{Q}_0^2)$$

a=NS,S(Σ,G)

Dispersion-relation access to GPDs at LO

[Teryaev '05; K.K., Müller and Passek-K. '07, '08; Diehl and Ivanov '07]

- LO perturbative prediction is "handbag" amplitude

$$\mathcal{H}(\xi, t, \mathcal{Q}^2) \overset{\text{LO}}{=} \int_{-1}^{1} dx \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H(x, \xi, t, \mathcal{Q}^2)$$

- giving access to GPD on the "cross-over" line $\eta = x$

$$\frac{1}{\pi} \mathfrak{Im}\, \mathcal{H}(\xi = x, t, \mathcal{Q}^2) \overset{\text{LO}}{=} H(x, x, t, \mathcal{Q}^2) - H(-x, x, t, \mathcal{Q}^2)$$

- while dispersion relation connects it to $\mathfrak{Re}\, \mathcal{H}$

$$\mathfrak{Re}\, \mathcal{H}(\xi, t, \mathcal{Q}^2) = \frac{1}{\pi} \mathrm{PV} \int_0^1 d\xi' \left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'} \right) \mathfrak{Im}\, \mathcal{H}(\xi', t, \mathcal{Q}^2) + \mathcal{C}_{\mathcal{H}}(t, \mathcal{Q}^2)$$

Curse of dimensionality

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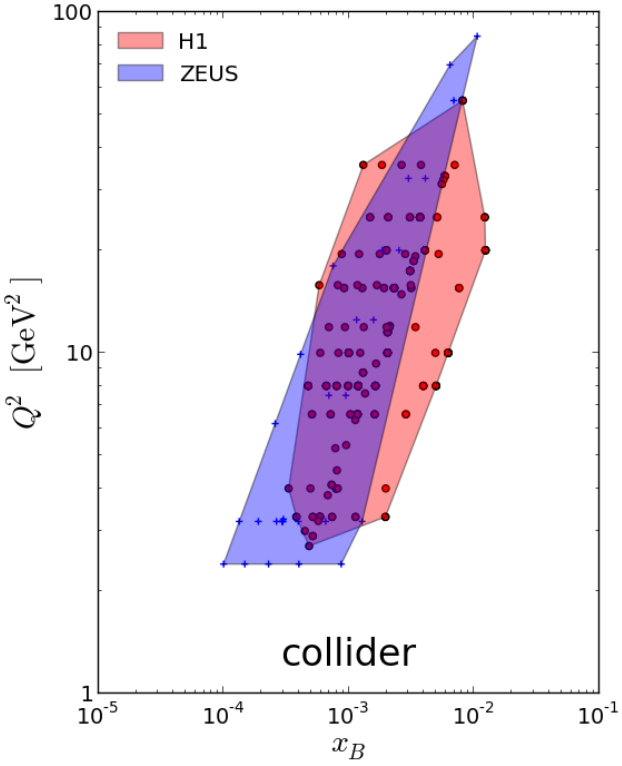
Curse of dimensionality

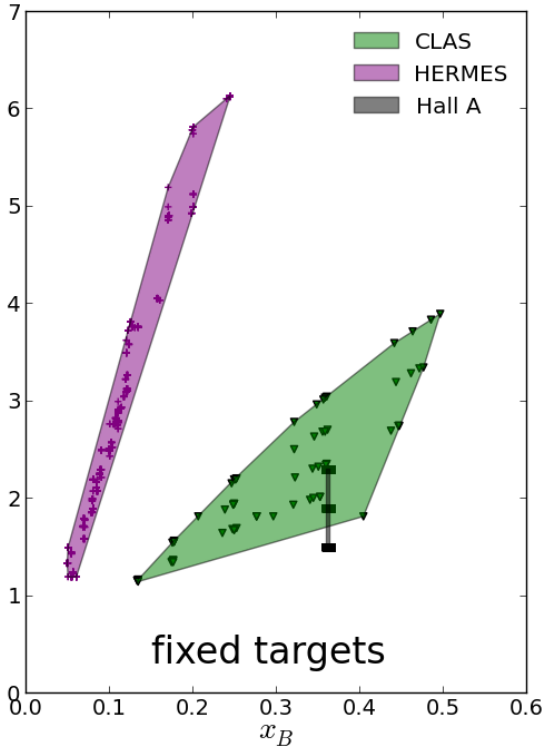
- *It is relatively easy to find a coin lying somewhere on* 100 *meter string. It is very difficult to find it on a football field.*
- When the dimensionality increases, the volume of the space increases so fast that the available data becomes sparse.
- Analogously, in contrast to $PDFs(x)$, it is very difficult to perform truly model independent extraction of $GPDs(x, \xi, t)$
- Known GPD constraints don't decrease the dimensionality of the GPD domain space.
- As an intermediate step, one can attempt extraction of $CFFs(\xi, t)$

- (Dependence on additional variable, photon virtuality $\mathcal{Q}^2$, is in principle known — given by evolution equations.)

Fit types

	Fit type	Data used	pQCD order	Target
1.	Local fits	fixed target	LO	CFFs
2.	Global fits	collider/fixed target	((N)N)LO	CFFs/GPDs
3.	Neural nets	fixed target	LO	CFFs





Local fits to HERMES data

- Most complete set of asymmetries (14) is measured in 12 bins:

bin no.	1	2	3	4	5	6	7	8	9	10	11	12
$-t$ [GeV^2]	0.03	0.1	0.2	0.42	0.1	0.1	0.13	0.2	0.08	0.1	0.13	0.19
x_{B}	0.08	0.1	0.11	0.12	0.05	0.08	0.12	0.2	0.06	0.08	0.11	0.17
$\mathcal{Q}^2$ [GeV^2]	1.9	2.5	2.9	3.5	1.5	2.2	3.1	5.0	1.2	1.9	2.8	4.9

$$A_C \equiv \frac{\mathrm{d}\sigma_{e^+} - \mathrm{d}\sigma_{e^-}}{\mathrm{d}\sigma_{e^+} + \mathrm{d}\sigma_{e^-}} = \frac{\mathcal{A}_{\mathrm{Interference}}(\mathcal{F})}{|\mathcal{A}_{\mathrm{DVCS}}|^2(\mathcal{F}^2) + |\mathcal{A}_{\mathrm{BH}}|^2}$$

$$A_{\mathrm{C}}^{\cos(1\phi)} \propto \left[F_1\,\mathfrak{Re}\,\mathcal{H} - \frac{t}{4M_p^2}F_2\,\mathfrak{Re}\,\mathcal{E} + \frac{x_{\mathrm{B}}}{2}(F_1+F_2)\,\mathfrak{Re}\,\widetilde{\mathcal{H}}\right]$$

- To express asymmetries in terms of CFFs we use formulas from [Belitsky, Müller, Kirchner '01, Belitsky, Müller '10].

Method of stepwise regression

- Constraints from data are too weak to constrain simultaneously all eight $\{\mathfrak{Im}\,\mathcal{F}, \mathfrak{Re}\,\mathcal{F}\}$ CFFs
- Let us take smaller number of CFFs, choosing only those which are reliably extracted. Stepwise regression algorithm:
  1. Perform single-CFF fit with each of 8 CFFs and see which one alone describes data best (it is $\mathfrak{Im}\,\mathcal{H}$, by far).
  2. Combine $\mathfrak{Im}\,\mathcal{H}$ with each of other seven CFFs and see which pair describes data best.
  3. Proceed until there is either no improvement in data description or new CFFs are not extracted with any statistical significance

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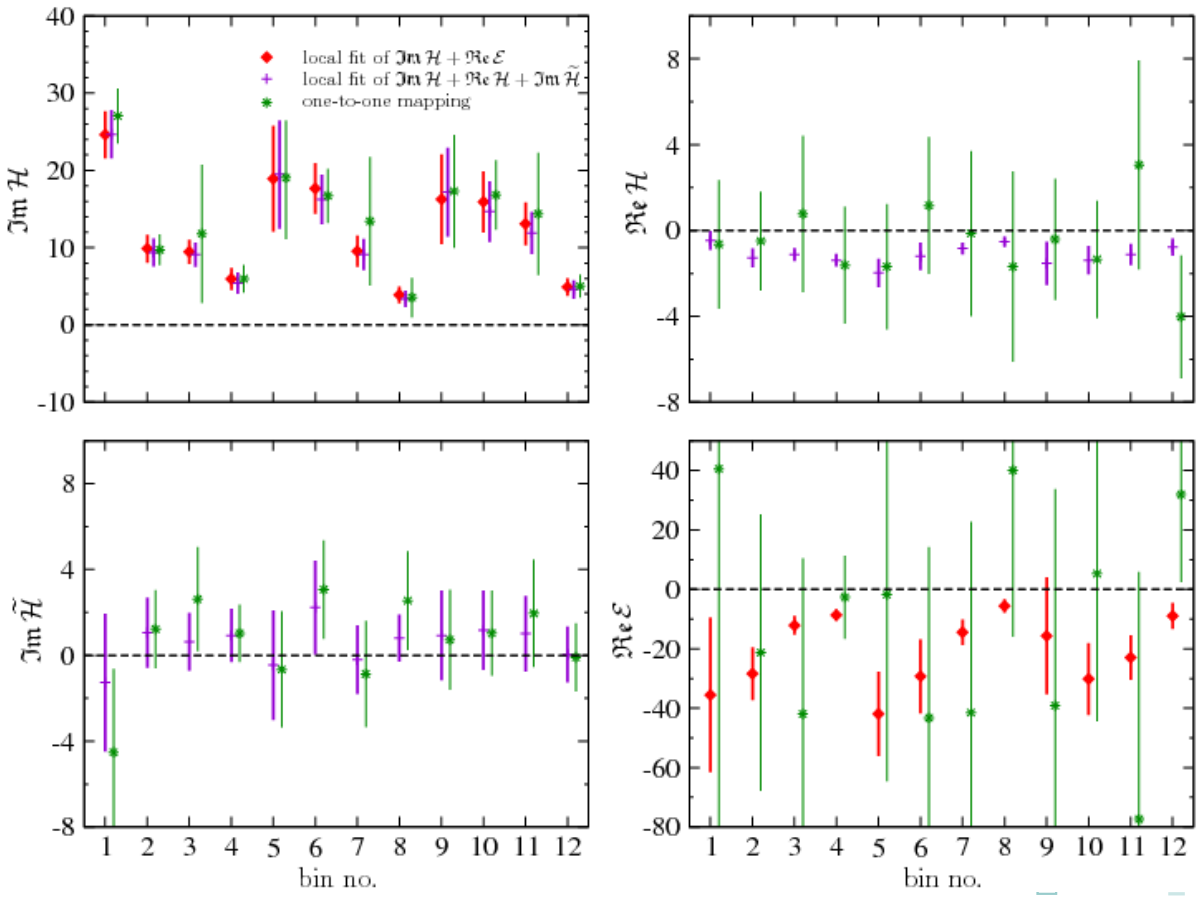
- It turns out that already 2nd step is the final one, and there are two equally good pairs of CFFs:
  1. $(\mathfrak{Im}\,\mathcal{H}, \mathfrak{Re}\,\mathcal{H})$ with $\chi^2/n_{\mathrm{d.o.f.}} = 102.3/120$, and
  2. $(\mathfrak{Im}\,\mathcal{H}, \mathfrak{Re}\,\mathcal{E})$ with $\chi^2/n_{\mathrm{d.o.f.}} = 103.0/120$.

Stepwise regression — results

- Scenario 1: Fit of $\mathfrak{Im}\,\mathcal{H}$, $\mathfrak{Re}\,\mathcal{H}$ and $\mathfrak{Im}\,\widetilde{\mathcal{H}}$. $\chi^2/n_{\mathrm{d.o.f.}} = 148.8/144$. (In good agreement with [Guidal '10])
- Scenario 2: Fit of $\mathfrak{Im}\,\mathcal{H}$ and $\mathfrak{Re}\,\mathcal{E}$. $\chi^2/n_{\mathrm{d.o.f.}} = 134.2/144$.



Modelling GPDs in moment space

- Instead of considering momentum fraction dependence $H(x, \ldots)$
- ...it is convenient to make a transform into complementary space of conformal moments j:

$$H_j^q(\eta, \ldots) \equiv \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} \mathrm{d}x \, \eta^j \, C_j^{3/2}(x/\eta) \, H^q(x, \eta, \ldots)$$

- They are analogous to Mellin moments in DIS: $x^j \to C_j^{3/2}(x)$
- $C_j^{3/2}(x)$ — Gegenbauer polynomials

Advantages of conformal moments

1. The evolution equations are most simple: There is **no mixing** among moments at LO, and in special ($\overline{CS}$) scheme not even at NLO
2. Stable and fast **computer code** for evolution and fitting
3. Moments are equal to matrix elements of **local** operators and are thus directly accessible on the **lattice**

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- To model η-dependence we use SO(3) partial wave expansion of crossed process $\gamma^*\gamma \to p\bar{p}$ where scattering angle θ corresponds to $-\frac{1}{\eta}$.

I-PW model — only leading SO(3) partial wave

- $$\mathbf{H}_j(\xi, t, \mu_0^2) = \begin{pmatrix} N'_\Sigma\, F_\Sigma(t)\, \mathrm{B}\big(1+j-\alpha_\Sigma(0), 8\big) \\ N'_\mathrm{G}\, F_\mathrm{G}(t)\, \mathrm{B}\big(1+j-\alpha_\mathrm{G}(0), 6\big) \end{pmatrix}$$

$$\alpha_a(t)=\alpha_a(0)+0.15t \qquad F_a(t) = \frac{j+1-\alpha(0)}{j+1-\alpha(t)} \left(1 - \frac{t}{M_0^{a2}}\right)^{-p_a}$$

. . . corresponding in forward case to PDFs of form

$$\Sigma(x) = N'_\Sigma\, x^{-\alpha_\Sigma(0)}\, (1-x)^7\ ; \qquad G(x) = N'_\mathrm{G}\, x^{-\alpha_\mathrm{G}(0)}\, (1-x)^5$$

- $M_0^G = \sqrt{0.7}\,\mathrm{GeV}$ is fixed by the J/ψ production data
- Free parameter (for DVCS): M_0^Σ

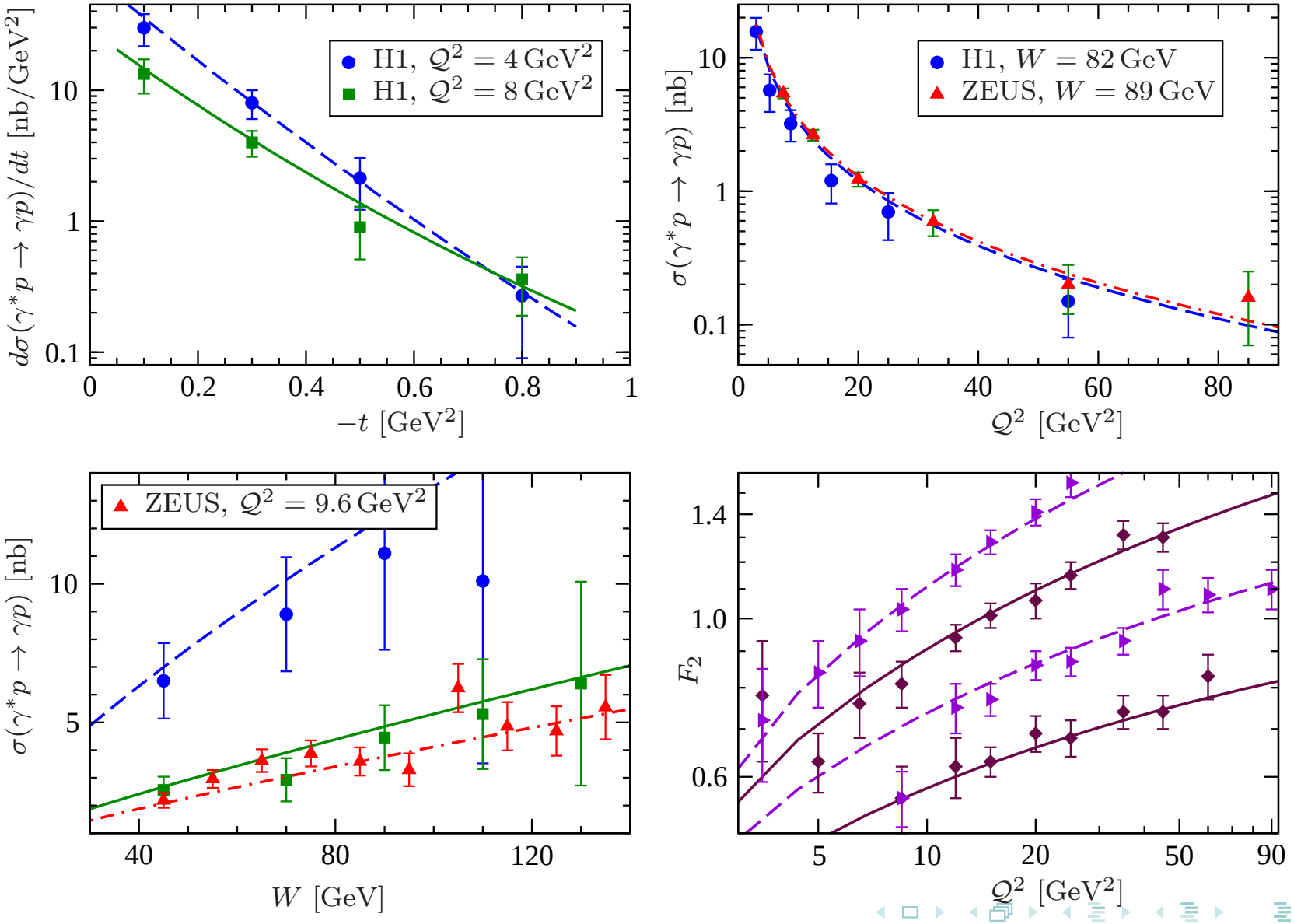
For small ξ (small x_{Bj}) valence quarks are less important $\Rightarrow \Sigma \approx$ sea

Inclusion of subleading PW — flexible models

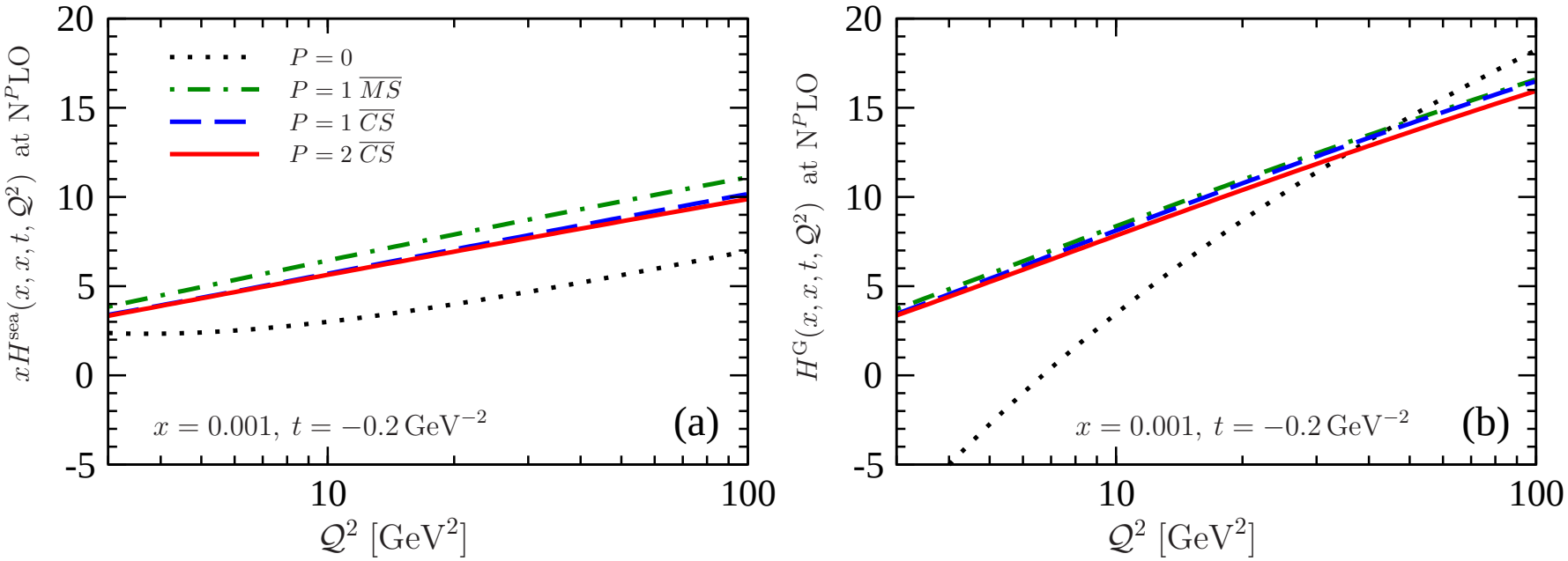
$$\mathbf{H}_j(\eta,t) = \underbrace{\begin{pmatrix} N'_{\rm sea}\, F_{\rm sea}(t)\, \mathrm{B}\big(1+j-\alpha_{\rm sea}(0),8\big) \\ N'_{\rm G}\, F_{\rm G}(t)\, \mathrm{B}\big(1+j-\alpha_{\rm G}(0),6\big) \end{pmatrix}}_{\text{skewness } r\approx 1.6 \text{ (too large)}} + \underbrace{\begin{pmatrix} s_{\rm sea} \\ s_{\rm G} \end{pmatrix} \begin{pmatrix} \text{subleading par-} \\ \text{tial waves, } \eta\text{-} \\ \text{dependence!} \end{pmatrix}}_{<0 \text{ negative skewness}}$$

- nl-PW — addition of second PW needed for good fits
- two new parameters: $s^{(2)}_{\rm sea}$ and $s^{(2)}_{\rm G}$

Example of fit result



Resulting small-x $H(x, x, t)$



- P=0: LO; P=1: NLO; P=2: NNLO
- The whole procedure is extended to meson production [Müller, Lautenschlager, Passek-Kumerički, Schäfer '13]

Extending global analysis to fixed target data

- **Hybrid models** at LO
- Sea quarks and gluons modelled like just described (conformal moments + SO(3) partial wave expansion + $\mathcal{Q}^2$ evolution).
- Valence quarks model (ignoring $\mathcal{Q}^2$ evolution):

$$\mathfrak{Im}\,\mathcal{H}(\xi,t) = \pi\left[\frac{4}{9}H^{u_{\rm val}}(\xi,\xi,t) + \frac{1}{9}H^{d_{\rm val}}(\xi,\xi,t) + \frac{2}{9}H^{\rm sea}(\xi,\xi,t)\right]$$

$$H(x,x,t) = n\,r\,2^{\alpha}\left(\frac{2x}{1+x}\right)^{-\alpha(t)}\left(\frac{1-x}{1+x}\right)^{b}\frac{1}{\left(1-\frac{1-x}{1+x}\frac{t}{M^2}\right)^{p}}\,.$$

- Fixed: n (from PDFs), $\alpha(t)$ (eff. Regge), p (counting rules)

$$\alpha^{\rm val}(t) = 0.43 + 0.85\,t/{\rm GeV}^2 \quad (\rho,\,\omega)$$

- $\mathfrak{Re}\,\mathcal{H}$ determined by dispersion relations

$$\mathfrak{Re}\,\mathcal{H}(\xi,t) = \frac{1}{\pi}\mathrm{PV}\int_0^1 d\xi'\left(\frac{1}{\xi-\xi'}-\frac{1}{\xi+\xi'}\right)\mathfrak{Im}\,\mathcal{H}(\xi',t) - \frac{C}{\left(1-\frac{t}{M_C{}^2}\right)^2}$$

- Typical set of free parameters:

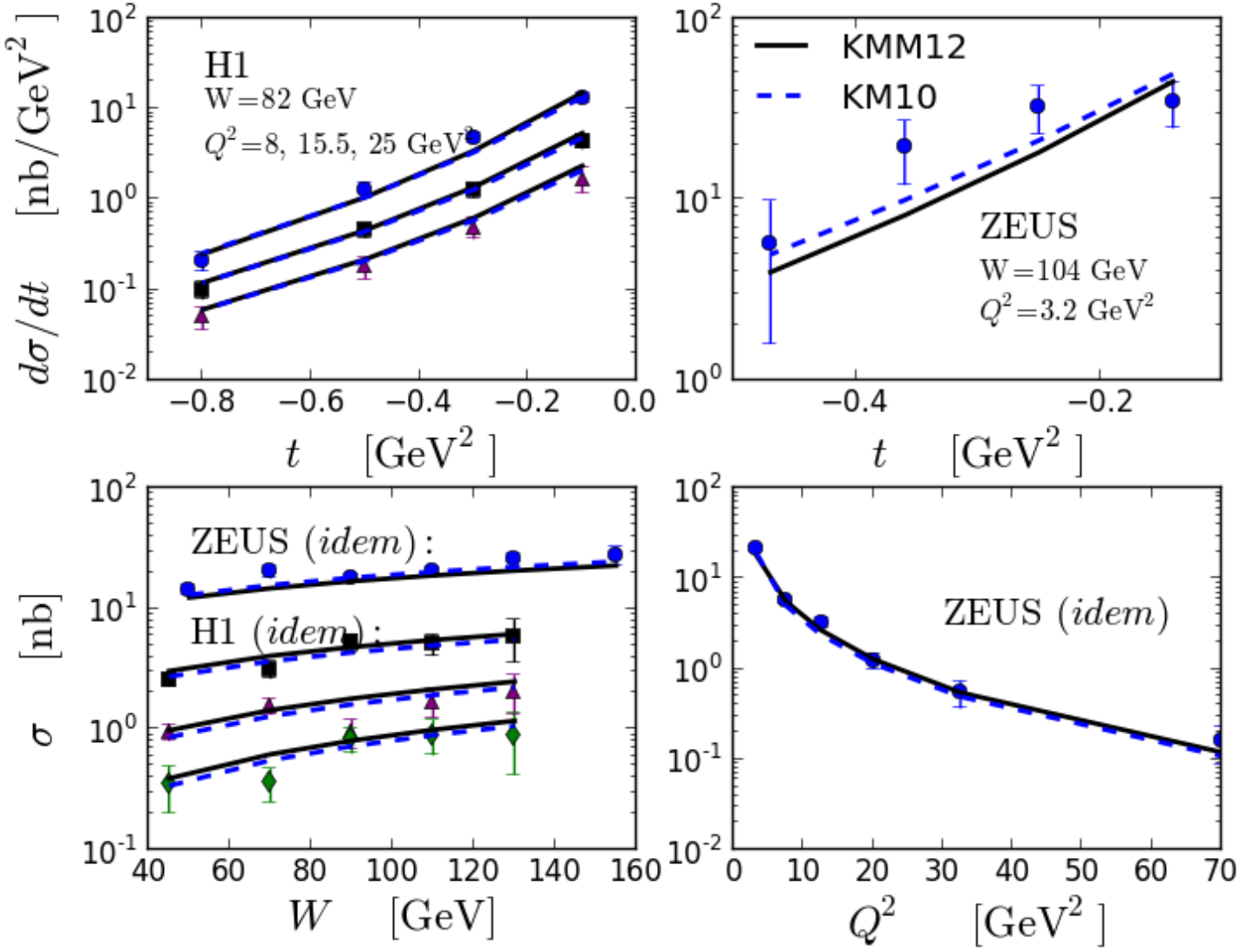
M_0^{sea}, $s_{\mathrm{sea}}^{(2,4)}$, $s_{\mathrm{G}}^{(2,4)}$	sea* quarks and gluons H
r^{val}, M^{val}, b^{val}	valence H
$\tilde{r}^{\mathrm{val}}$, $\tilde{M}^{\mathrm{val}}$, $\tilde{b}^{\mathrm{val}}$	valence $\widetilde{H}$
C, M_C	subtraction constant (H, E)
r_π, M_π	"pion pole" $\widetilde{E}$

- Global fit to 175 data points turns out fine:

$$\texttt{KM10 model: } \chi^2/d.o.f. = 135.9/160.$$

* $s_{\mathrm{sea,G}}$ = strengths of subleading partial waves. LO evolution is included.

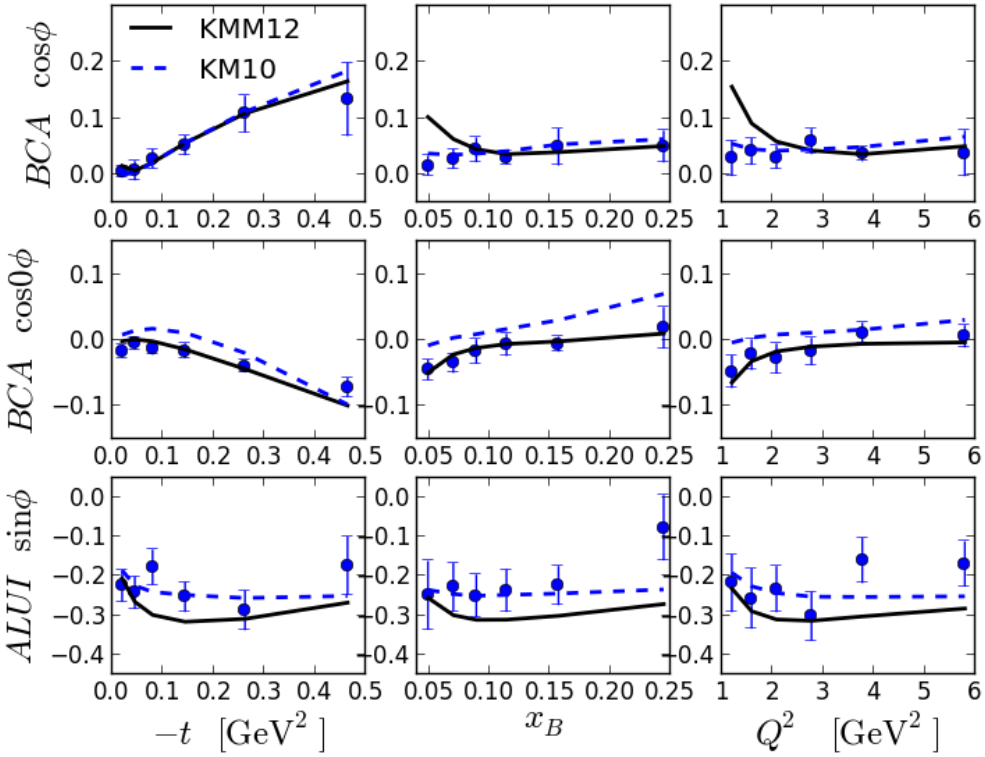
H1 (2007), ZEUS (2008)



HERMES (2008)

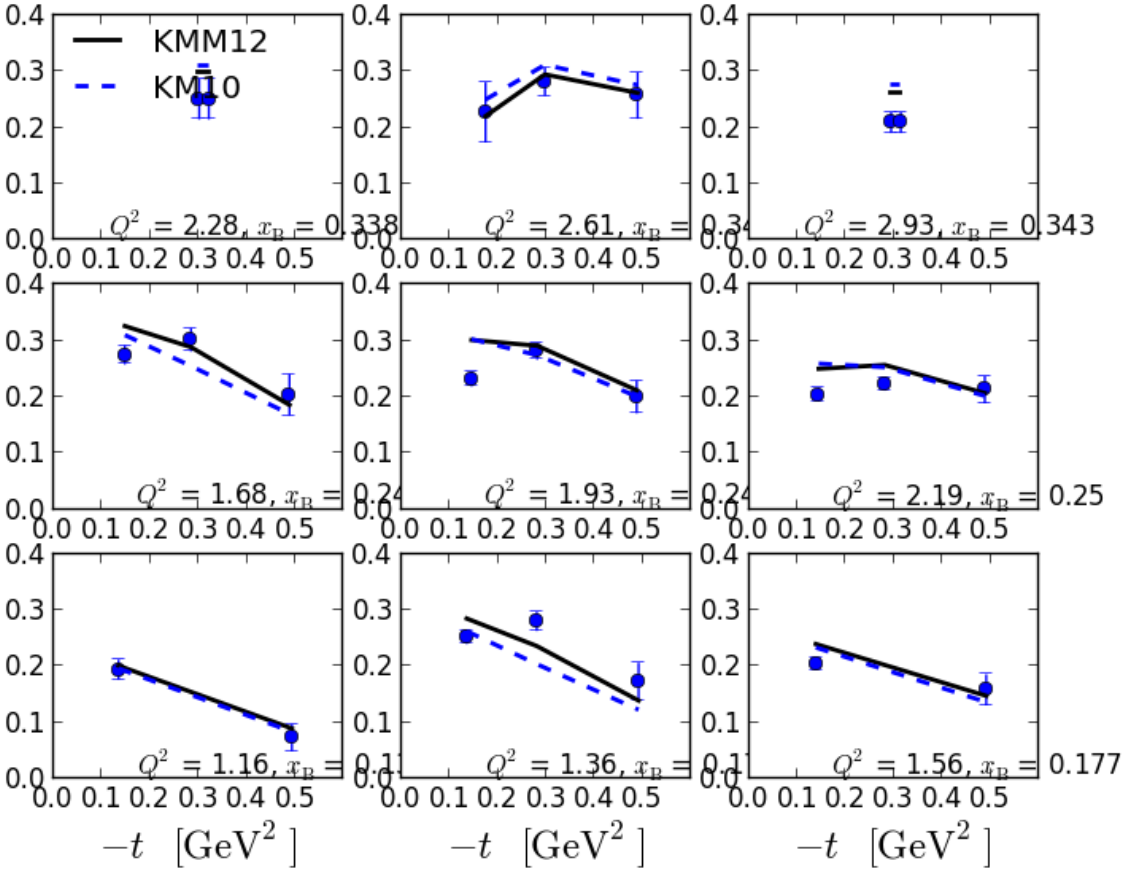
$$BCA \equiv \frac{d\sigma_{e^+} - d\sigma_{e^-}}{d\sigma_{e^+} + d\sigma_{e^-}} \sim A_C^{\cos 0\phi} + A_C^{\cos 1\phi} \cos\phi \sim \Re\mathrm{e}\,\mathcal{H}$$

$$BSA \equiv \frac{d\sigma_{e\uparrow} - d\sigma_{e\downarrow}}{d\sigma_{e\uparrow} + d\sigma_{e\downarrow}} \sim A_{LU}^{\sin 1\phi} \sin\phi \sim \Im\mathrm{m}\,\mathcal{H}$$



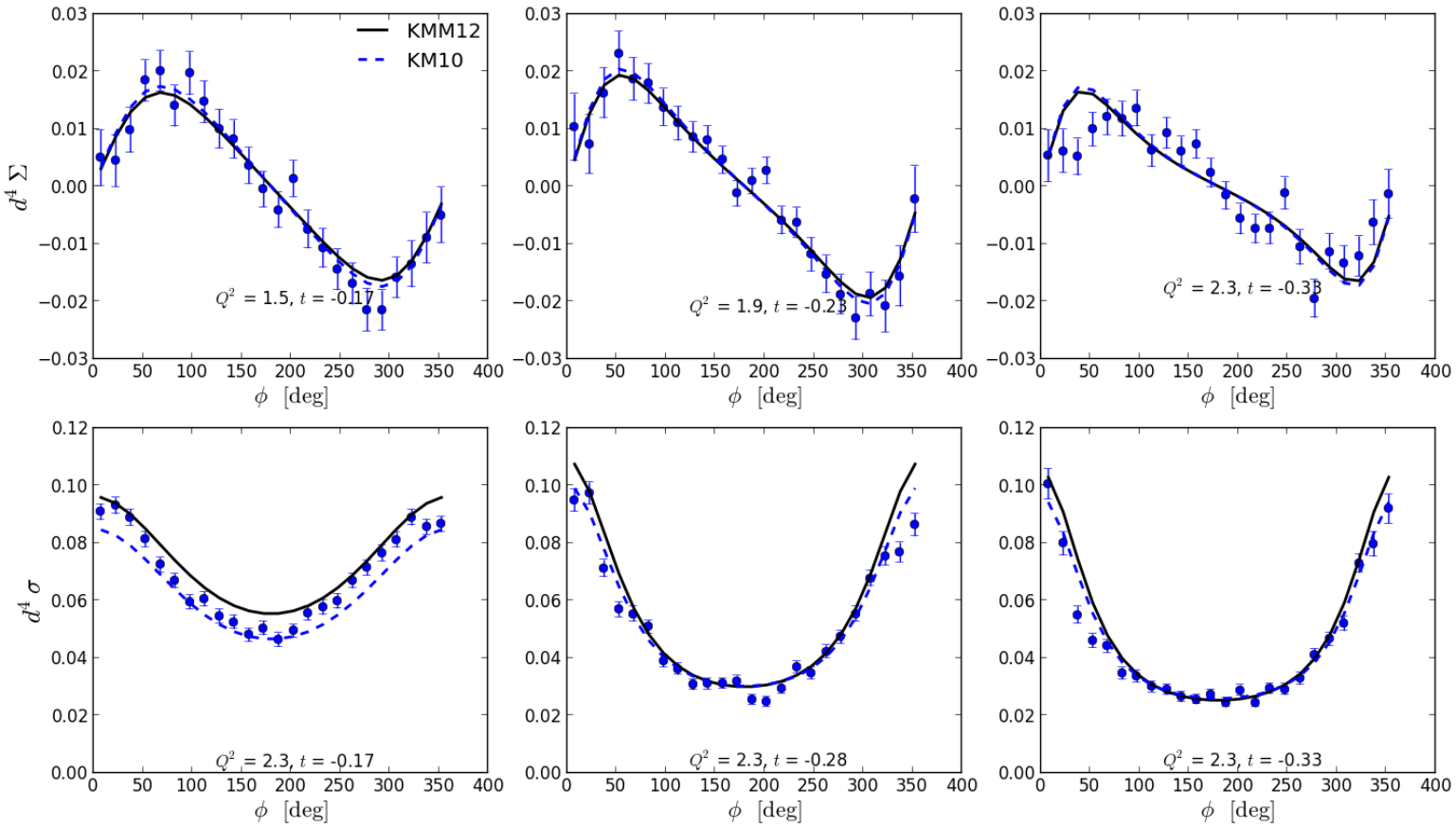
CLAS (2007)

- BSA. (Only data with $|t| \leq 0.3\,\mathrm{GeV}^2$ used for fits.)



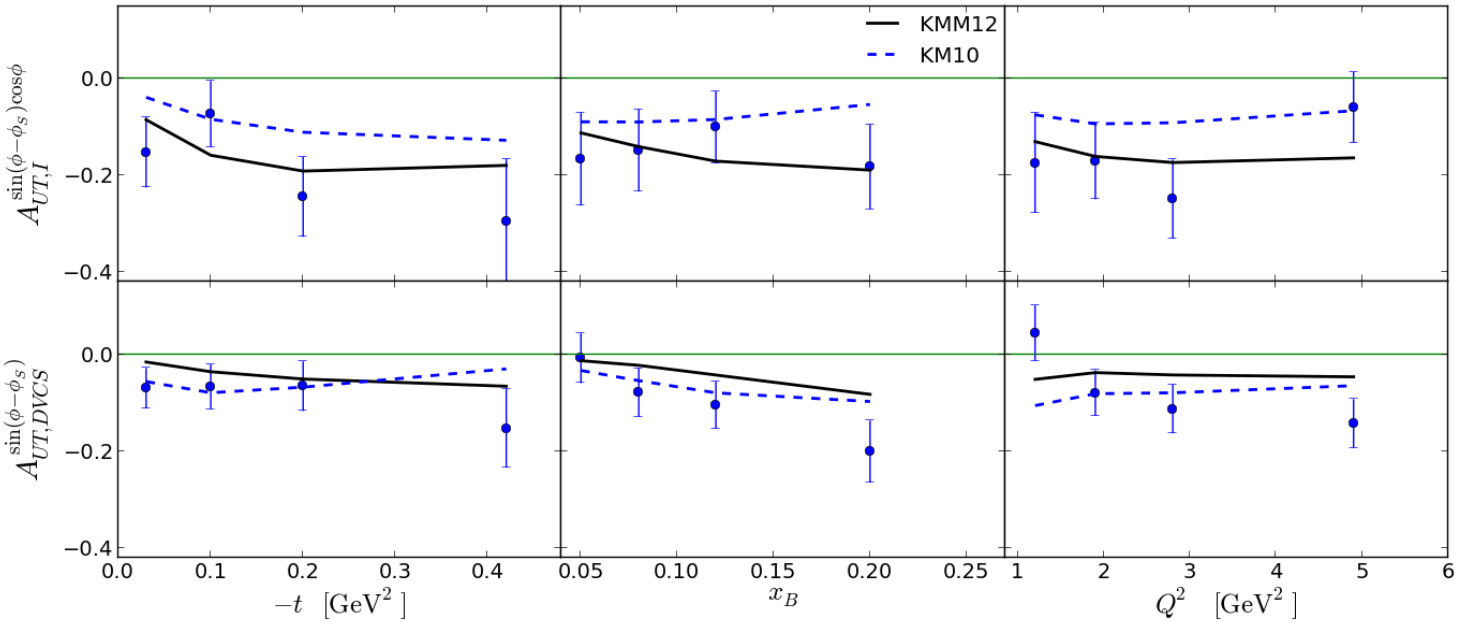
Hall A (2006)

- Fit to unpolarized cross section $d\sigma/(dx_B dQ^2 dt d\phi)$ $\sim \mathfrak{Re}\,\mathcal{H}$
- KM10 fit needs unusually large $\mathfrak{Re}\,\tilde{\mathcal{H}}$.



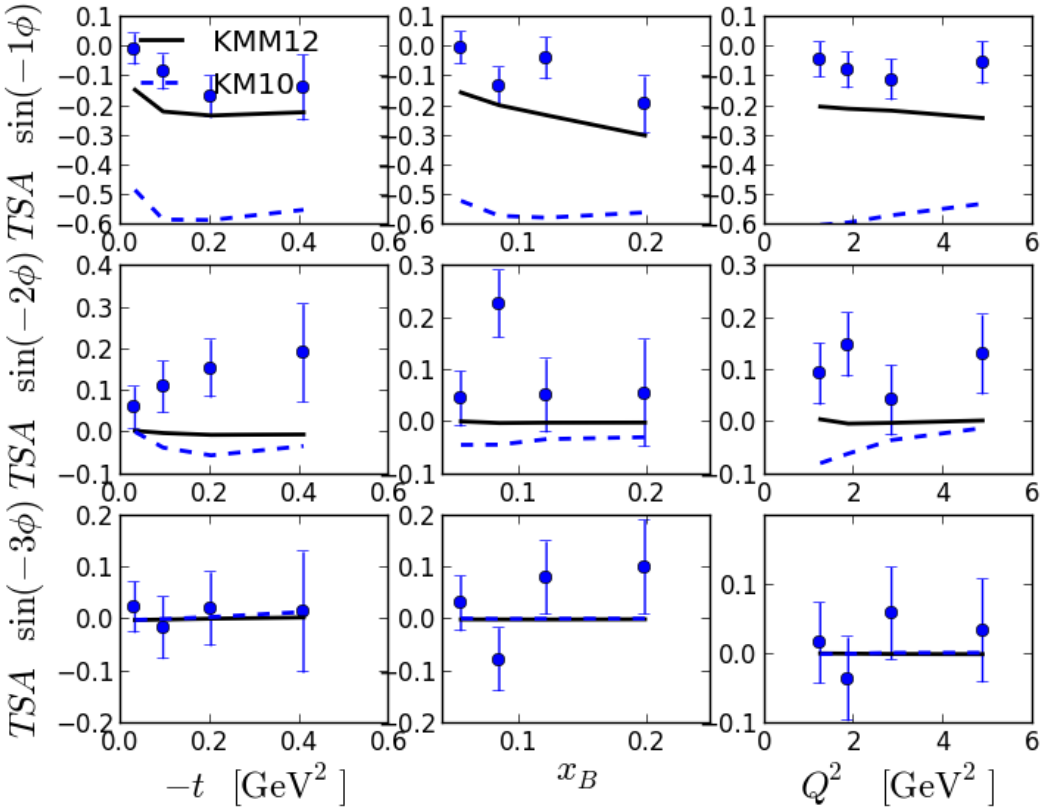
Including data with polarized target

- KMM12: $\chi^2/n_{\mathrm{d.o.f.}} = 124.1/80$, strictly speaking not a good fit, but best what we have at the moment

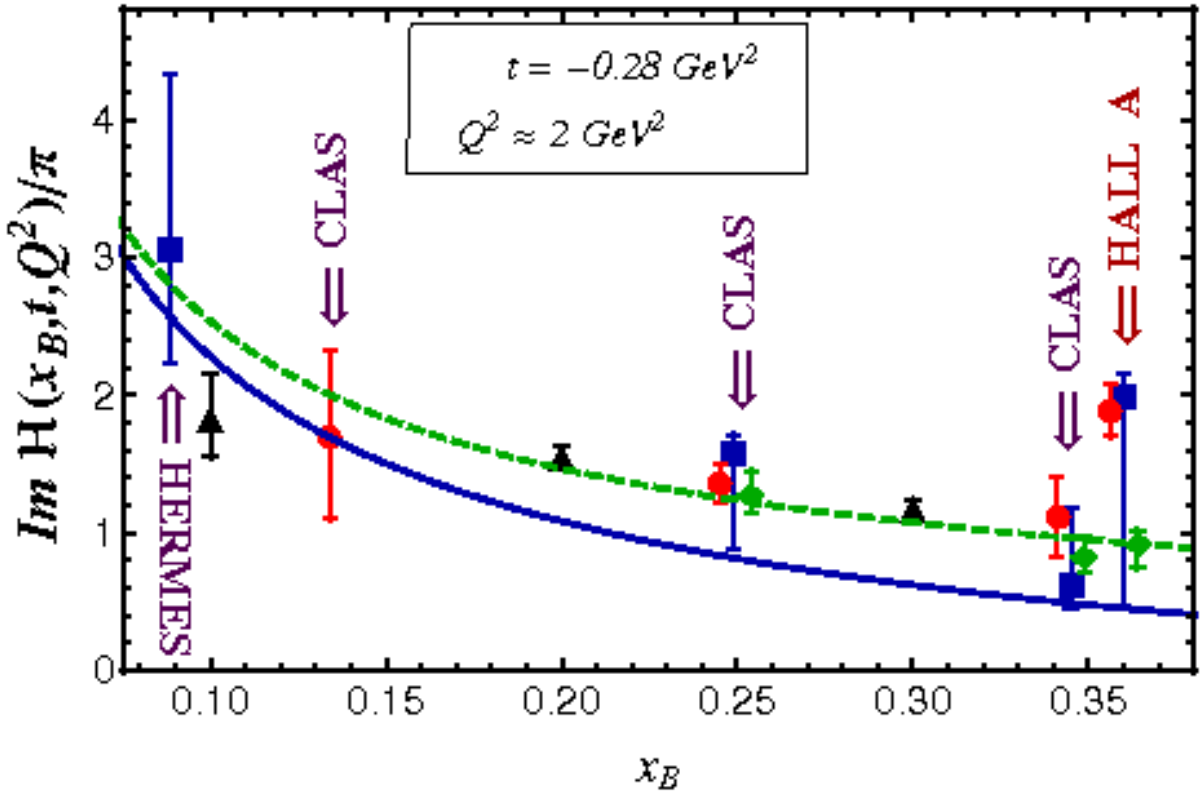


Polarized target (II)

- Surprisingly large $\sin(2\phi)$ harmonic of A_{UL} cannot be described within this leading twist framework

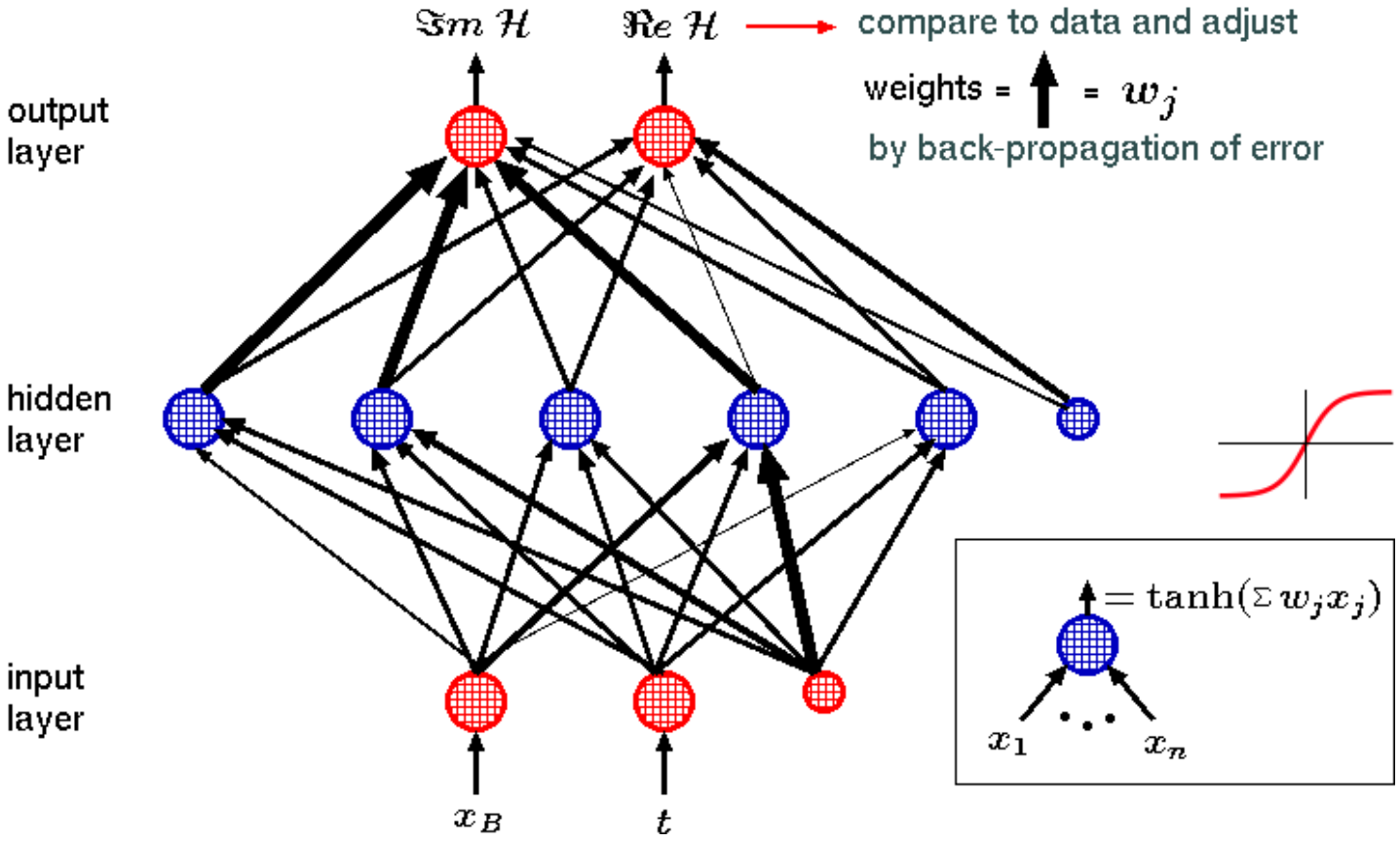


Comparison to others



[Guidal '08, Guidal and Moutarde '09], seven CFF fit (blue squares), [Guidal '10] $\mathcal{H}$, $\tilde{\mathcal{H}}$ CFF fit (green diamonds), [Moutarde '09] H GPD fit (red circles)

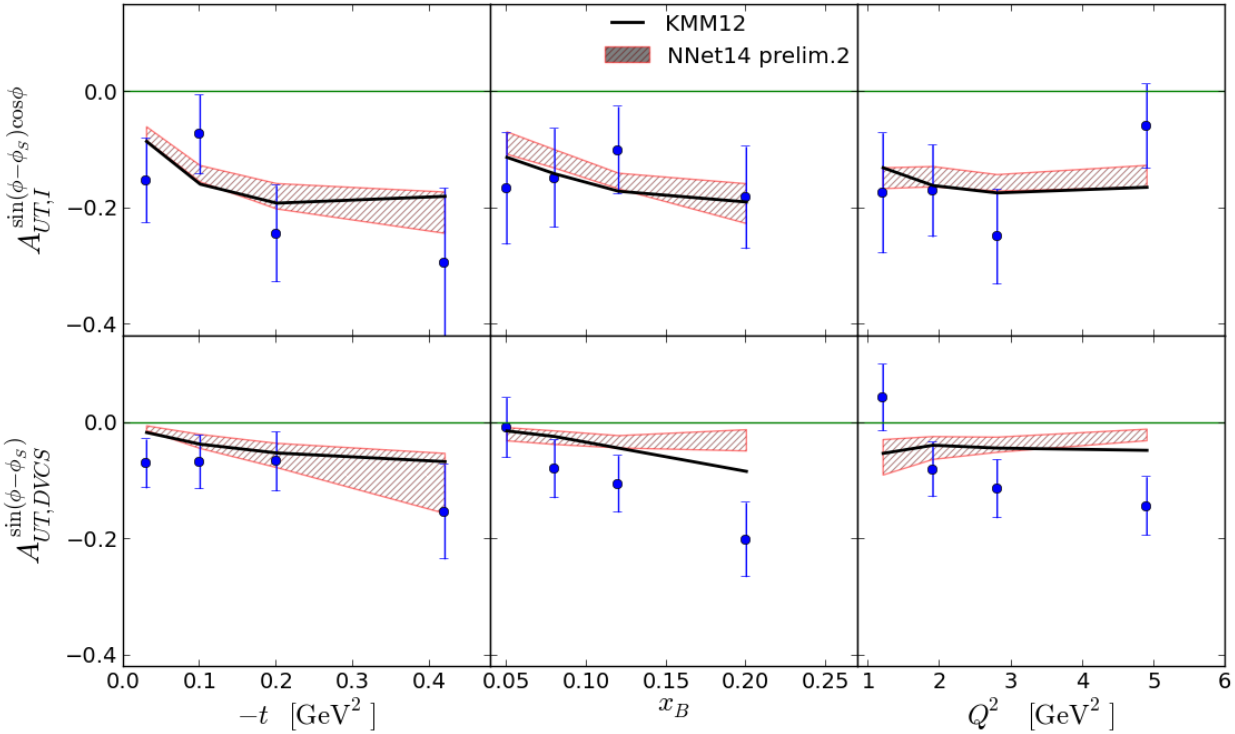
Neural networks



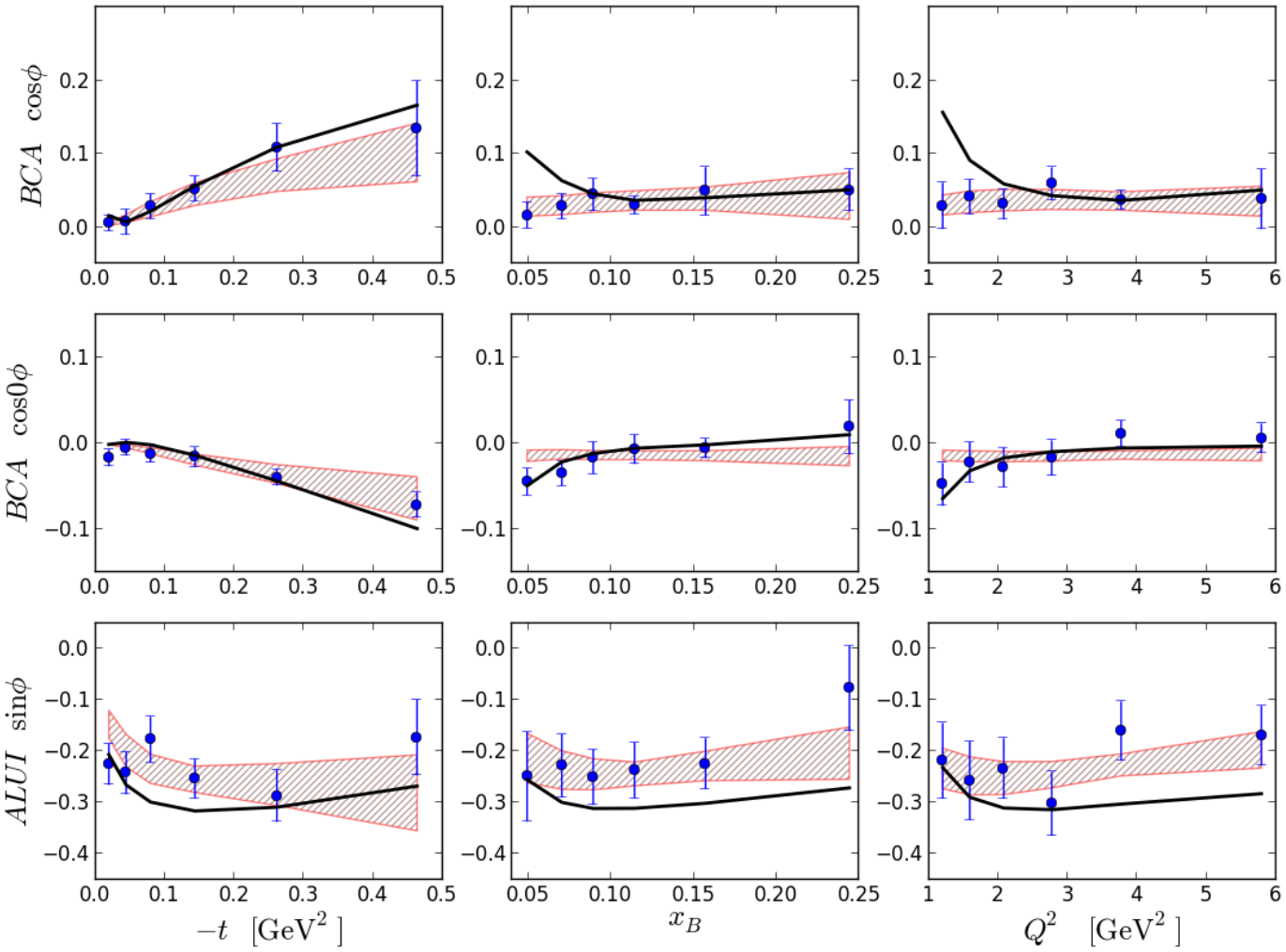
- Essentially a least-squares fit of a complicated many-parameter function. $f(x) = \tanh(\sum w_i \tanh(\sum w_j \cdots))$
  $\Rightarrow$ no theory bias

Preliminary neural Net HERMES fit

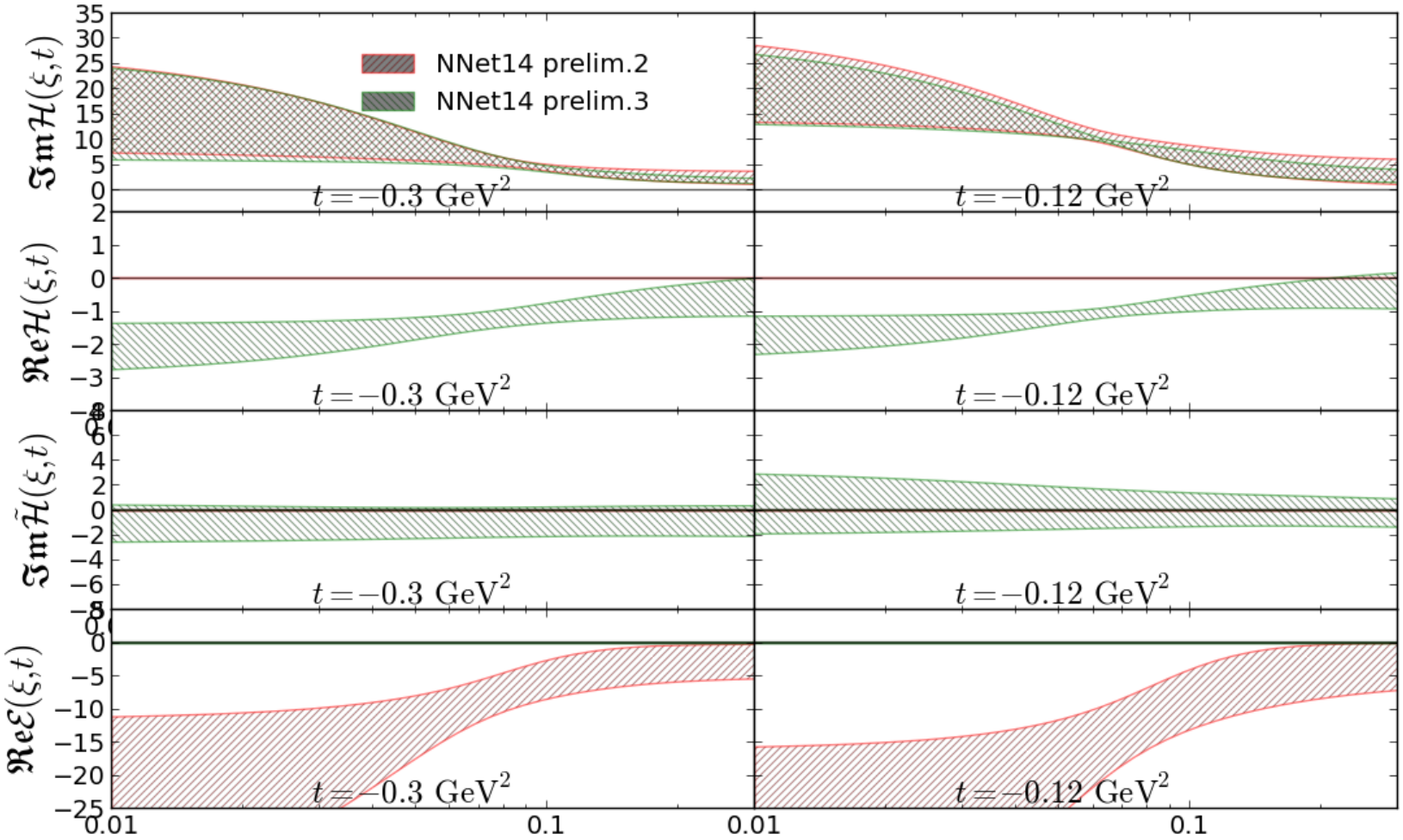
- Fit to all HERMES DVCS data with two types of neural nets
  - (x_B, t) – (7 neurons) – $(\mathfrak{Im}\,\mathcal{H}, \mathfrak{Re}\,\mathcal{H}, \mathfrak{Im}\,\tilde{\mathcal{H}})$: $\chi^2/n_{\rm pts} = 135.4/144$
  - (x_B, t) – (7 neurons) – $(\mathfrak{Im}\,\mathcal{H}, \mathfrak{Re}\,\mathcal{E})$: $\chi^2/n_{\rm pts} = 120.2/144$



Neural Net HERMES fit - BSA/BCA

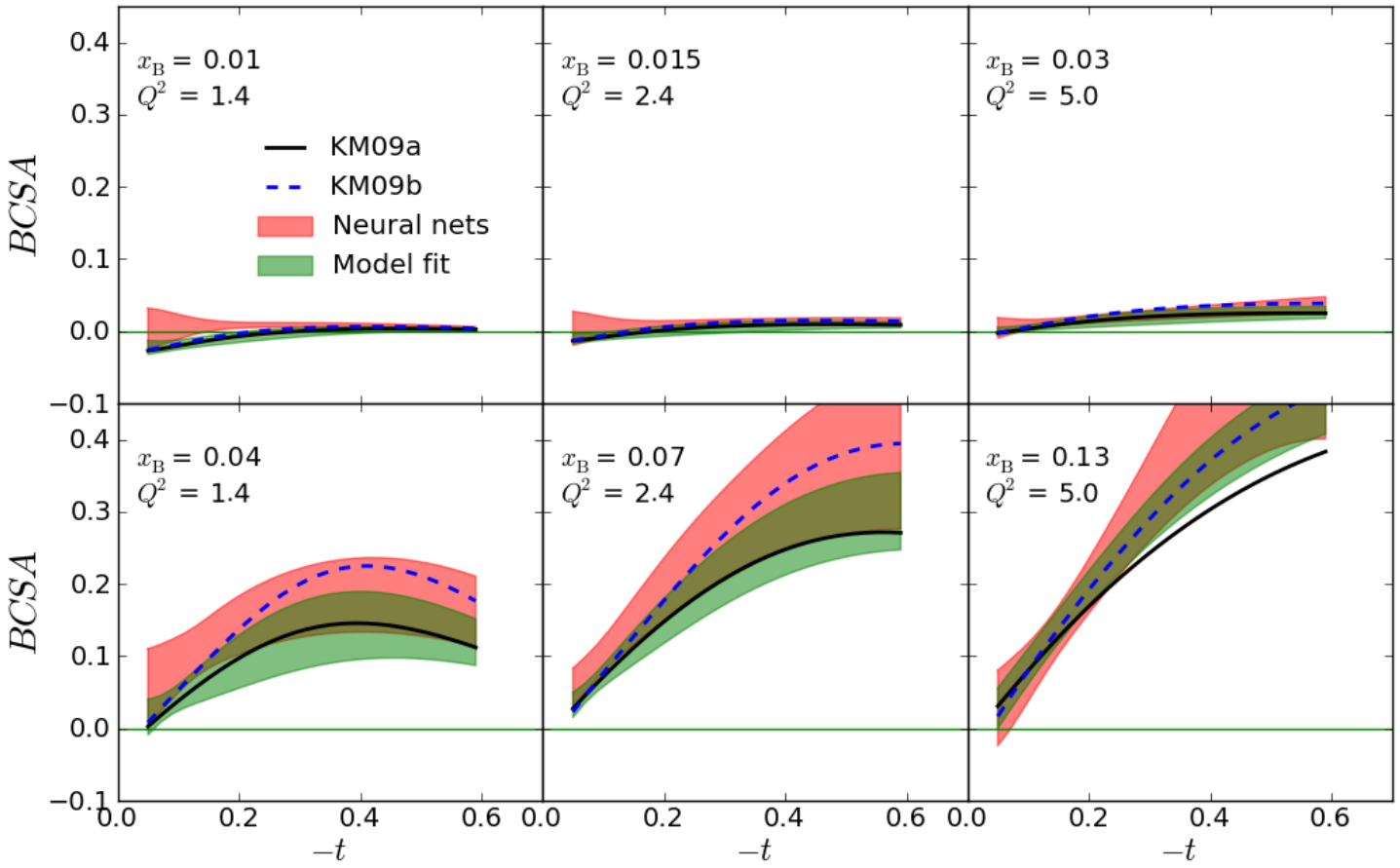


Neural Net HERMES fit - CFFs



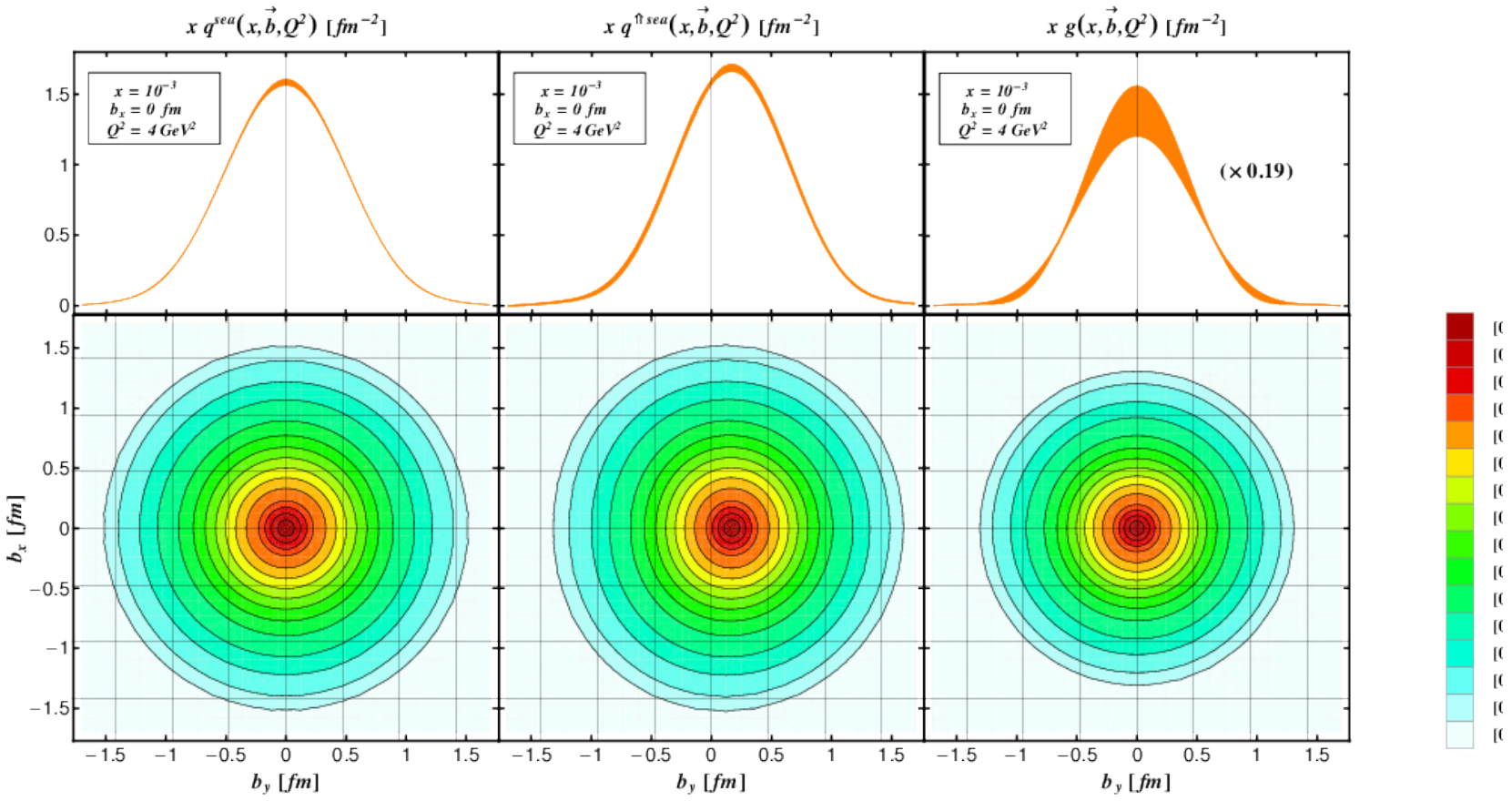
Prediction for COMPASS II BCSA

$$BCSA = \frac{d\sigma_{\mu\downarrow+} - d\sigma_{\mu\uparrow-}}{d\sigma_{\mu\downarrow+} + d\sigma_{\mu\uparrow-}} \qquad (E_\mu = 160\,\mathrm{GeV})$$



Simulation of EIC capabilities

- Parton densities from combined fit to HERA collider and EIC pseudo-data [Aschenauer, Fazio, K.K and Müller '13]



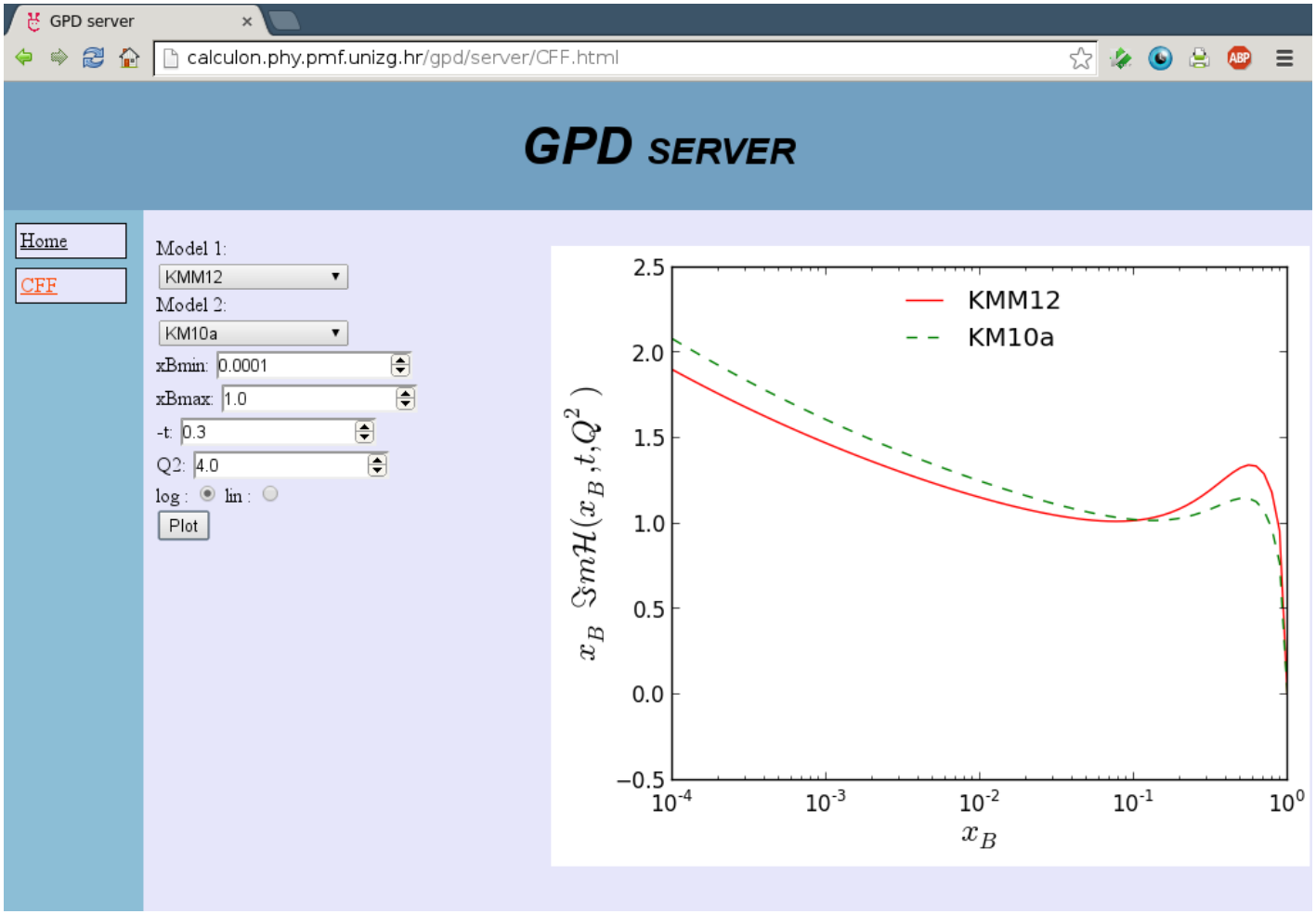
KM models are available at WWW

- Google for "gpd page" — get binary code for cross sections

```
% xs.exe

  xs.exe  ModelID  Charge  Polarization  Ee  Ep  xB  Q2  t  phi

returns cross section (in nb) for scattering of lepton of energy Ee
on unpolarized proton of energy Ep. Charge=-1 is for electron.

ModelID is one of
   0 debug, always returns 42,
   1 KM09a - arXiv:0904.0458 fit without Hall A,
   2 KM09b - arXiv:0904.0458 fit with Hall A,
   3 KM10  - preliminary hybrid fit with LO sea evolution, from Trento presentation,
   4 KM10a - preliminary hybrid fit with LO sea evolution, without Hall A data
   5 KM10b - preliminary hybrid fit with LO sea evolution, with Hall A data

xB Q2 t phi  -- usual kinematics (phi is in Trento convention)

% xs.exe 1 -1 1 27.6 0.938  0.111 3. -0.3 0

  0.18584386497251
```

GPD page and server

- Durham-like CFF/GPD server page



New directions (instead of Summary)

- Improving GPD models
- Adding deeply virtual meson production data and going to NLO [Müller, Lautenschlager, Schäfer '13]
- Including higher twists [Braun, Manashov et al.]
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The end.

Mapping asymmetries to CFFs

- Inverting these relations gives mapping from the set of observables ...

$$A_{\mathrm{LU,I}}^{\sin(1\phi)} \quad A_{\mathrm{C}}^{\cos(1\phi)} \quad A_{\mathrm{C}}^{\cos(0\phi)} \quad A_{\mathrm{UL,+}}^{\sin(1\phi)}$$
$$A_{\mathrm{LL,+}}^{\cos(1\phi)} \quad A_{\mathrm{LL,+}}^{\cos(0\phi)} \quad A_{\mathrm{UT,I}}^{\sin(\varphi)\cos(1\phi)} \quad A_{\mathrm{UT,I}}^{\cos(\varphi)\sin(1\phi)}$$
$$A_{\mathrm{UT,DVCS}}^{\sin(\varphi)\cos(0\phi)} \quad A_{\mathrm{UT,I}}^{\sin(\varphi)\cos(0\phi)} \quad A_{\mathrm{LT,I}}^{\sin(\varphi)\sin(1\phi)} \quad A_{\mathrm{LT,I}}^{\cos(\varphi)\cos(1\phi)}$$
$$A_{\mathrm{LT,BH+DVCS}}^{\cos(\varphi)\cos(0\phi)} \quad A_{\mathrm{LT,I}}^{\cos(\varphi)\cos(0\phi)}$$

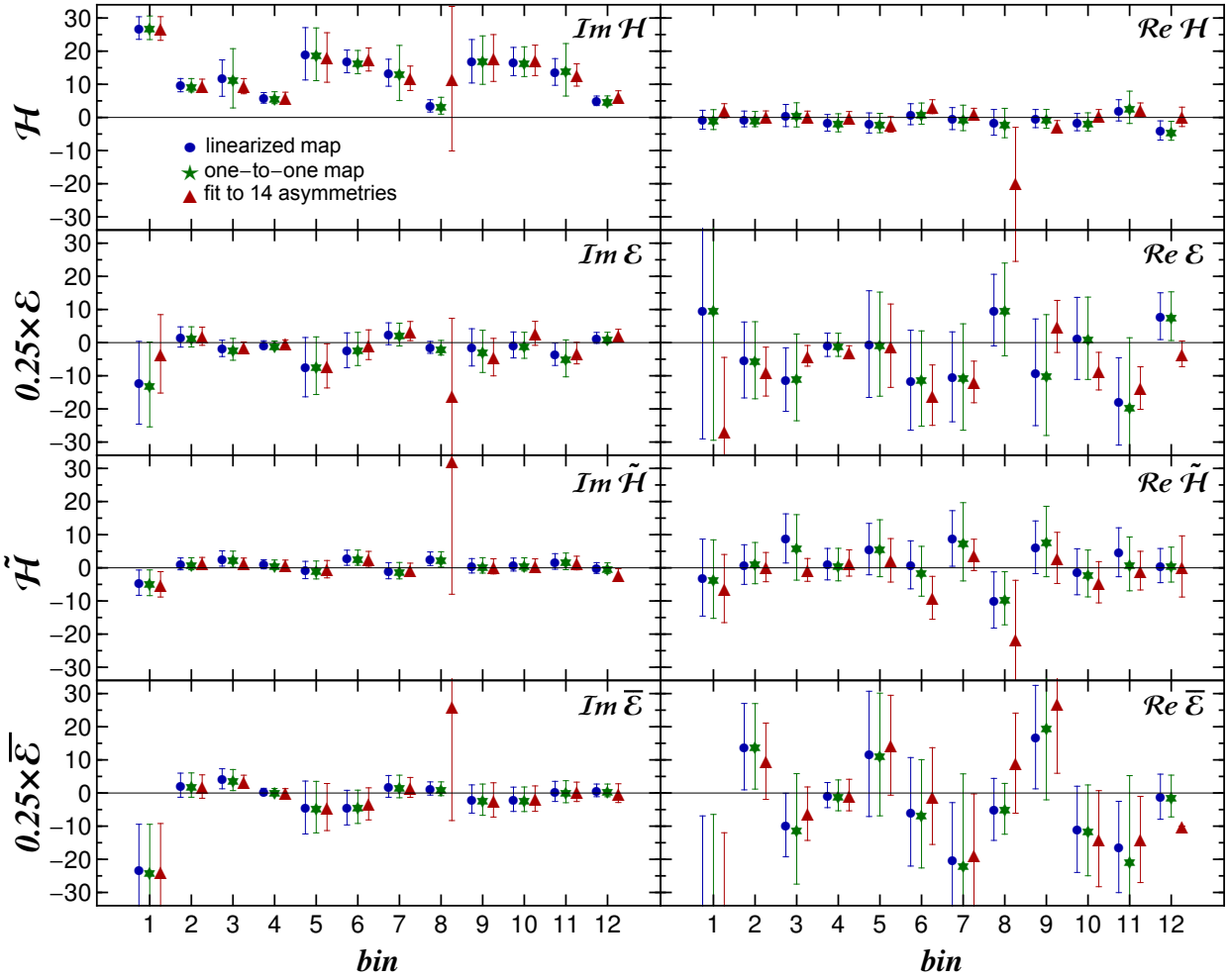
- ... to the set of eight real and imaginary parts of CFFs (sometimes called sub-CFFs or just CFFs) ...

$$\boldsymbol{\mathcal{F}} = \left(\mathfrak{Im}\,\mathcal{H}, \mathfrak{Re}\,\mathcal{H}, \mathfrak{Im}\,\mathcal{E}, \mathfrak{Re}\,\mathcal{E}, \mathfrak{Im}\,\widetilde{\mathcal{H}}, \mathfrak{Re}\,\widetilde{\mathcal{H}}, \mathfrak{Im}\,\widetilde{\mathcal{E}}, \mathfrak{Re}\,\widetilde{\mathcal{E}}\right)$$

- ... where error propagation is straightforward.

Mapping — results

- (Here compared also to standard local least-squares fit)



- Only $\mathfrak{Im}\,\mathcal{H}$, $\mathfrak{Re}\,\mathcal{H}$ and $\mathfrak{Im}\,\widetilde{\mathcal{H}}$ are reliably constrained.

Function fitting by a neural net

- Theorem: Given enough neurons, any smooth function $f(x_1, x_2, \cdots)$ can be approximated to any desired accuracy. Single hidden layer is sufficient (but not always most efficient).

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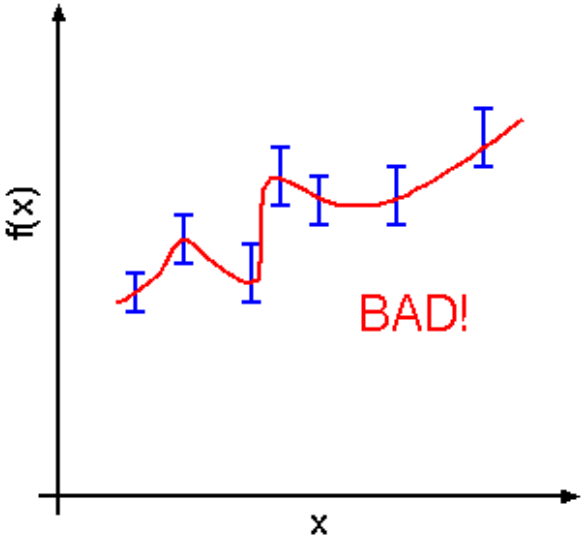

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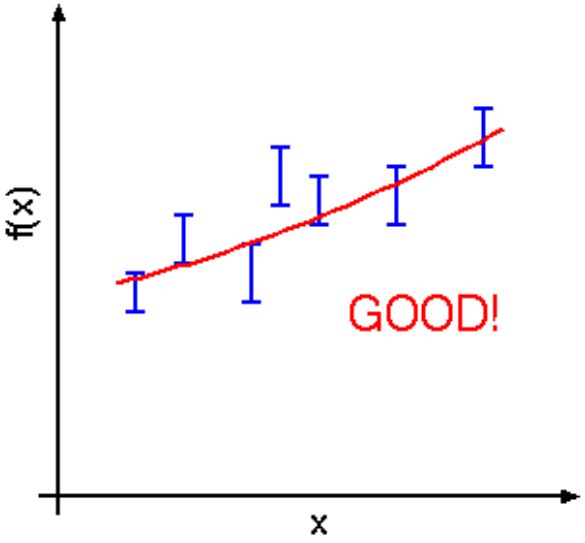

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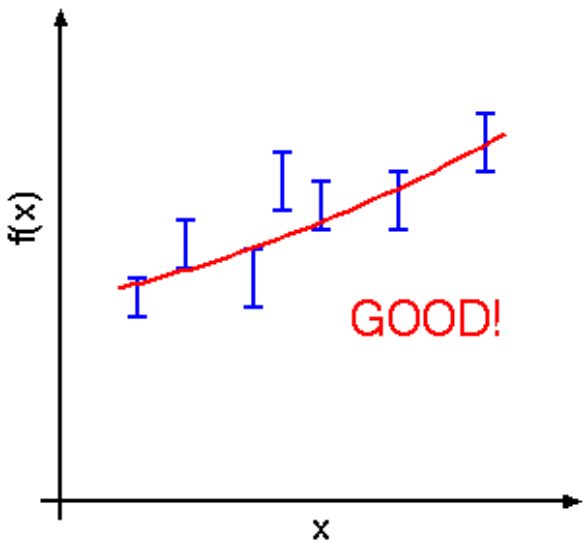
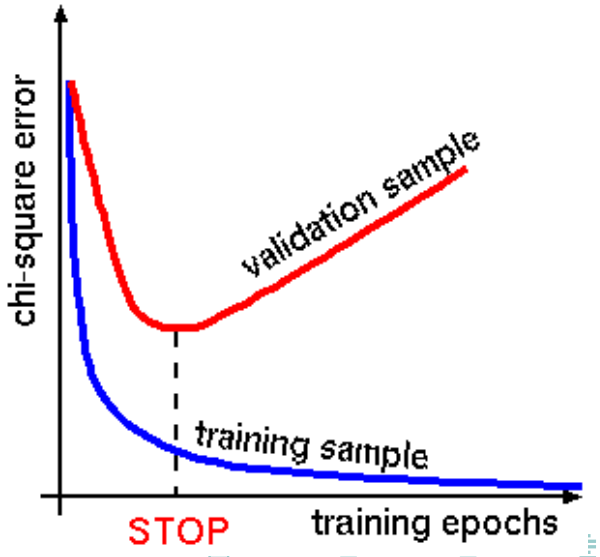
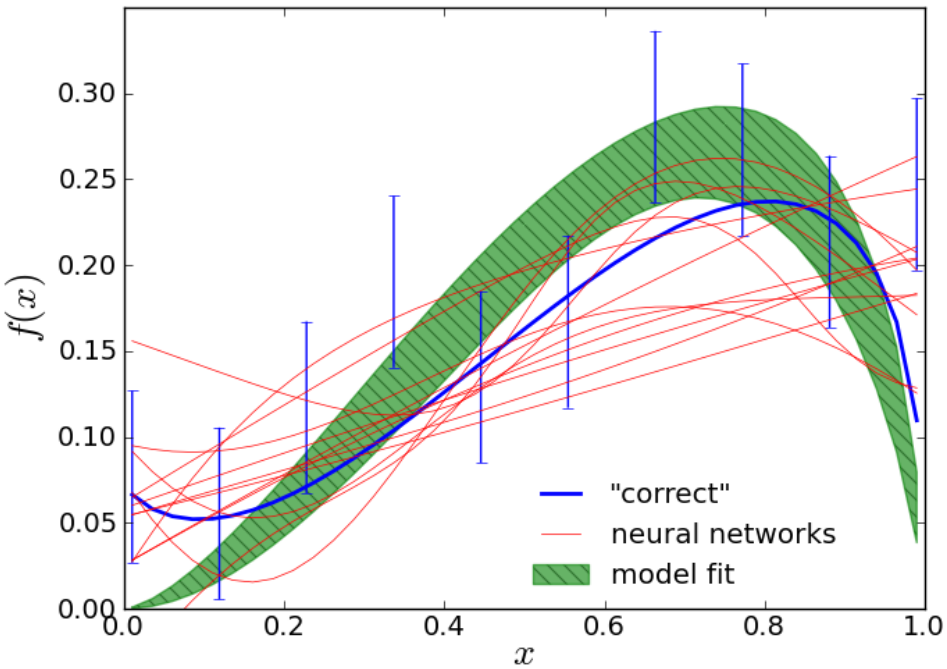
- With simple training of neural nets to data there is a danger of overfitting (a.k.a. overtraining)
- Solution: Divide data (randomly) into two sets: *training* sample and *validation sample*. Stop training when error of validation sample starts increasing.





Toy fitting example

- Fit to data generated according to function (which we pretend not to know).



- Fit with
  1. Standard Minuit fit with ansatz $f(x) = x^a(1-x)^b$
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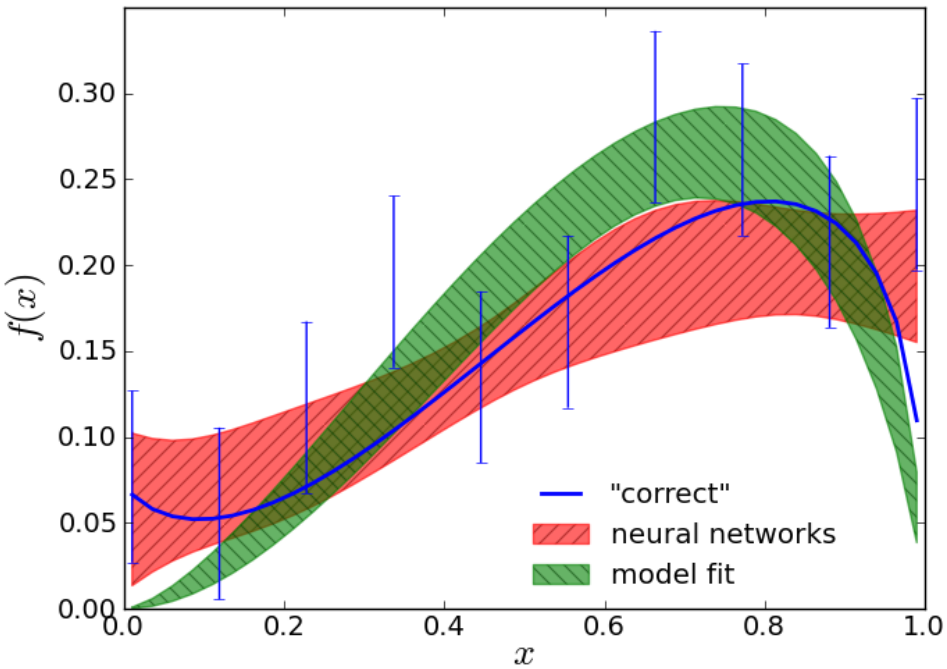

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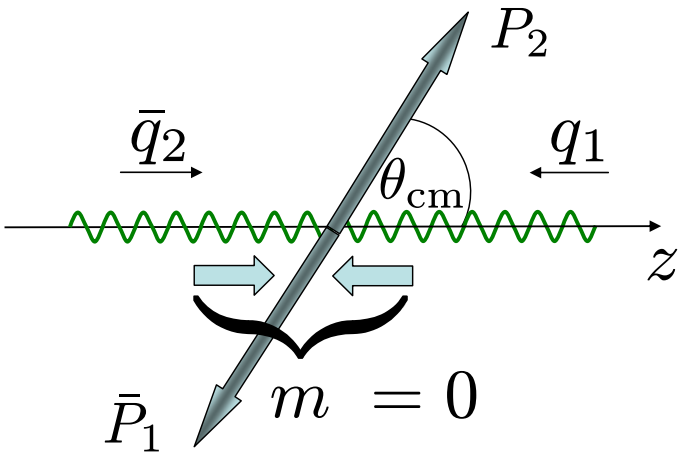
Modelling conformal moments of GPDs (I)

- How to model η-dependence of GPD's $H_j(\eta, t)$?
- Idea: consider crossed t-channel process $\gamma^*\gamma \to p\bar{p}$



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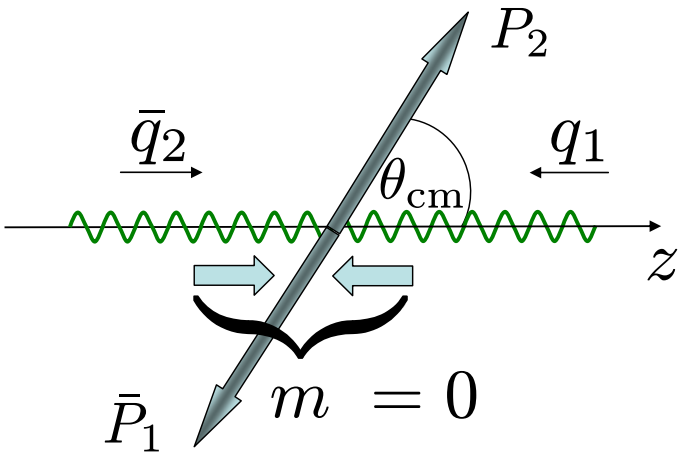


When crossing back to DVCS channel we have:

$$\cos\theta_{\rm cm} \to -\frac{1}{\eta}$$

- ...and dependence on $\theta_{\rm cm}$ in t-channel is given by SO(3) partial wave decomposition of $\gamma^*\gamma$ scattering

$$\mathcal{H}(\eta, \ldots) = \mathcal{H}^{(t)}(\cos\theta_{\rm cm} = -\frac{1}{\eta}, \ldots) = \sum_J (2J+1) f_J(\ldots) d^J_{0,\nu}(\cos\theta)$$

- $d^J_{0,\nu}$ — Wigner SO(3) functions (Legendre, Gegenbauer,...)

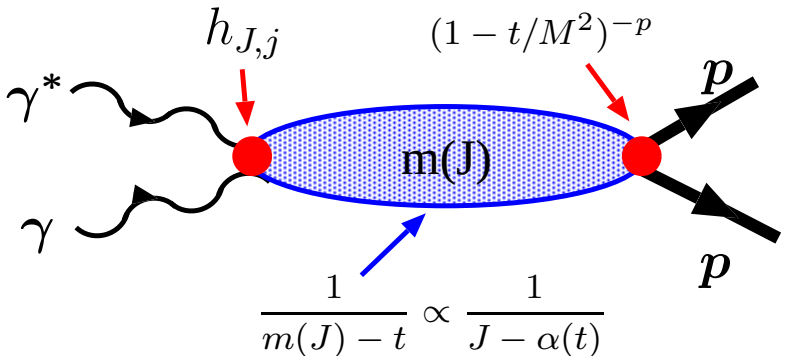
  $\nu = 0, \pm 1$ — depending on hadron helicities

Modelling conformal moments of GPDs (II)

- OPE expansion of both $\mathcal{H}$ and $\mathcal{H}^{(t)}$ leads to

$$H_j(\eta, t) = \eta^{j+1}\, H_j^{(t)}(\cos\theta = -\frac{1}{\eta}, s^{(t)} = t)$$

- and t-channel partial waves are modelled as:



$$H_j(\eta, t) = \sum_J^{j+1} h_{J,j} \frac{1}{J-\alpha(t)} \frac{1}{\left(1-\frac{t}{M^2(J)}\right)^p}\, \eta^{j+1-J} d^J_{0,\nu}$$

- Similar to "dual" parametrization [Polyakov, Shuvaev '02]

Fit results - LO

- For consistency, we don't take standard PDFs, but fit GPDs to DIS data. This determines $N_{\rm sea}$, N_G, $\alpha_{\rm sea}(0)$ and $\alpha_G(0)$, leaving only $M_0^{\rm sea}$, $s_{\rm sea}$ and s_G for DVCS data

- χ^2 values:

model	α_s	χ^2/d.o.f. DIS	χ^2/d.o.f. DVCS	χ^2_t/n.o.p.	χ^2_W/n.o.p.	$\chi^2_{\mathcal{Q}^2}$/n.o.p.
l, dipole	LO	49.7/82	**280./100**	**181./56**	**63.6/29**	**36.2/16**
l, exp.	LO	49.7/82	**316./100**	**192./56**	**79./29**	**44.9/16**
nl, dipole	LO	49.7/82	95.9/98	53.2/56	27./29	15.8/16
nl, exp.	LO	49.7/82	97.9/98	49.1/56	31.2/29	17.7/16
Σ, dipole	LO	49.7/82	101./98	57.7/56	27.4/29	16./16
Σ, exp.	LO	49.7/82	102./98	51./56	32.3/29	18.6/16
l, dipole	LO	**321./182**		**189./56**	**51.1/29**	**27.9/16**

- Parameter values:

model	α_s	$N^{\rm sea}$	$\alpha^{\rm sea}(0)$	$(M^{\rm sea})^2$ [GeV^2]	$s^{\rm sea}$	$\alpha^{\rm G}(0)$	$s^{\rm G}$	$B^{\rm sea}$ [GeV^{-2}]	$b^{\rm eff}$ [GeV^{-2}]	BCA
l, dipole	LO	**0.152**	**1.158**	**0.062**		**1.247**		**33.**	**5.7**	**0.19**
l, exp.	LO	**0.152**	**1.158**			**1.247**		**29.**	**5.1**	**0.23**
nl, dipole	LO	0.152	1.158	0.48	-0.15	1.247	-0.81	4.8	5.5	0.13
nl, exp.	LO	0.152	1.158		-0.18	1.247	-0.86	3.1	5.8	0.14
Σ, dipole	LO	0.152	1.158	0.42	-11.	1.247	-32.	5.4	5.5	0.14
Σ, exp.	LO	0.152	1.158		-13.	1.247	-34.	3.1	5.8	0.15

(boldface numbers = bad fits)

Fit results - NLO

- χ^2 values:

model	α_s	χ^2/d.o.f DIS	χ^2/d.o.f DVCS	χ^2_t/n.o.p	χ^2_W/n.o.p	$\chi^2_{Q^2}$/n.o.p
l	NLO($\overline{\mathrm{MS}}$)	71.6/82	**148./100**	**77.6/56**	**36.8/29**	**33.9/16**
l	NLO($\overline{\mathrm{CS}}$)	71.6/82	105./100	62.9/56	25.1/29	17./16
nl	NLO($\overline{\mathrm{MS}}$)	71.6/82	102./98	60.2/56	23.9/29	17.5/16
nl	NLO($\overline{\mathrm{CS}}$)	71.6/82	104./98	61.4/56	24.9/29	18.1/16
Σ	NLO($\overline{\mathrm{MS}}$)	71.6/82	101./98	60./56	23.9/29	17.5/16
Σ	NLO($\overline{\mathrm{CS}}$)	71.6/82	104./98	61.5/56	24.9/29	18.1/16

- Parameter values:

model	α_s	N^{sea}	$\alpha^{\mathrm{sea}}(0)$	$(M^{\mathrm{sea}})^2$	s^{sea}	$\alpha^{\mathrm{G}}(0)$	s^{G}	B^{sea}	b^{eff}	BCA
l	NLO($\overline{\mathrm{MS}}$)	**0.168**	**1.128**	**0.71**		**1.099**		**3.5**	**5.0**	**0.10**
l	NLO($\overline{\mathrm{CS}}$)	0.168	1.128	0.57		1.099		4.2	5.7	0.09
nl	NLO($\overline{\mathrm{MS}}$)	0.168	1.128	0.59	0.04	1.099	0.02	4.0	5.6	0.09
nl	NLO($\overline{\mathrm{CS}}$)	0.168	1.128	0.58	-0.01	1.099	-0.01	4.1	5.6	0.09
Σ	NLO($\overline{\mathrm{MS}}$)	0.168	1.128	0.60	3.10	1.099	1.10	4.0	5.7	0.09
Σ	NLO($\overline{\mathrm{CS}}$)	0.168	1.128	0.58	-0.42	1.099	-0.58	4.1	5.6	0.09

(boldface numbers = bad fits)

- $s^{\mathrm{sea,G}}$ small $\longrightarrow$ skewness ratio $r \sim 1.5$

Parameter values

KMM12		KM10	
Mv =	0.951 +- 0.282	Mv =	4.00 +- 3.33
rv =	1.121 +- 0.099	rv =	0.62 +- 0.06
bv =	0.400 +- 0.000	bv =	0.40 +- 0.67
C =	1.003 +- 0.565	C =	8.78 +- 0.98
MC =	2.080 +- 3.754	MC =	0.97 +- 0.11
tMv =	3.523 +- 13.17	tMv =	0.88 +- 0.24
trv =	1.302 +- 0.206	trv =	7.76 +- 1.39
tbv =	0.400 +- 0.001	tbv =	2.05 +- 0.40
rpi =	3.837 +- 0.141	rpi =	3.54 +- 1.77
Mpi =	4.000 +- 0.036	Mpi =	0.73 +- 0.37
M02S =	0.462 +- 0.032	M02S =	0.51 +- 0.02
SECS =	0.313 +- 0.039	SECS =	0.28 +- 0.02
THIS =	-0.138 +- 0.012	THIS =	-0.13 +- 0.01
SECG =	-2.771 +- 0.228	SECG =	-2.79 +- 0.12
THIG =	0.945 +- 0.107	THIG =	0.90 +- 0.05