Generalized Parton Distributions: Global analysis with μN and eN

Dieter Müller

- *Preliminaries*
- *Status of Theory and Phenomenology*
- *Conclusions*

some recent and upcoming work in collaboration with:

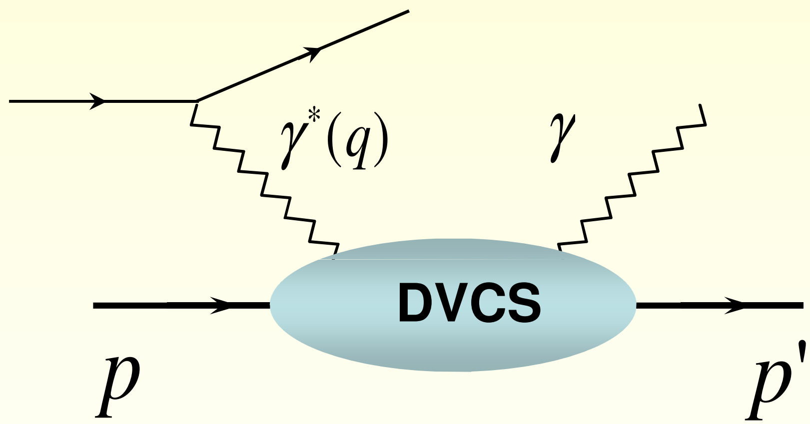
K. Kumerički (KK), E. Aschenauer, S. Firzo, M. Murray
K. Passek-Kumerički (KP-K), T. Lautenschlager, A. Schäfer; M. Meskauskas
A. Belitsky, Y. Ji; V. Braun, A. Manashov, B. Pirnay
D.S. Hwang

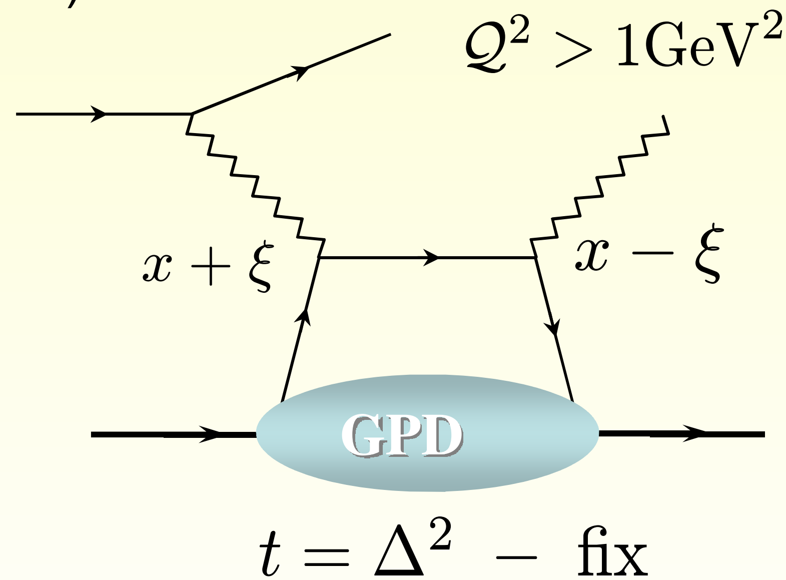
GPDs embed non-perturbative physics

[DM et. al (91/94)
Radyushkin (96)
Ji (96)]

GPDs appear in various hard exclusive processes,

e.g., hard electroproduction of photons (DVCS)





$$\mathcal{F}(\xi, \mathcal{Q}^2, t) = \int_{-1}^{1} dx\, C(x, \xi, \alpha_s(\mu), \mathcal{Q}/\mu) F(x, \xi, t, \mu) + O(\tfrac{1}{\mathcal{Q}^2})$$

CFF
Compton form factor
observable

hard scattering part
perturbation theory
(our conventions/microscope)

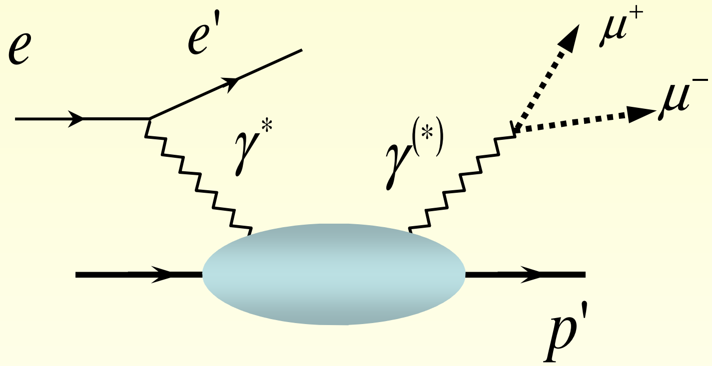
GPD
universal
(conventional)

higher twist
depends on
approximation

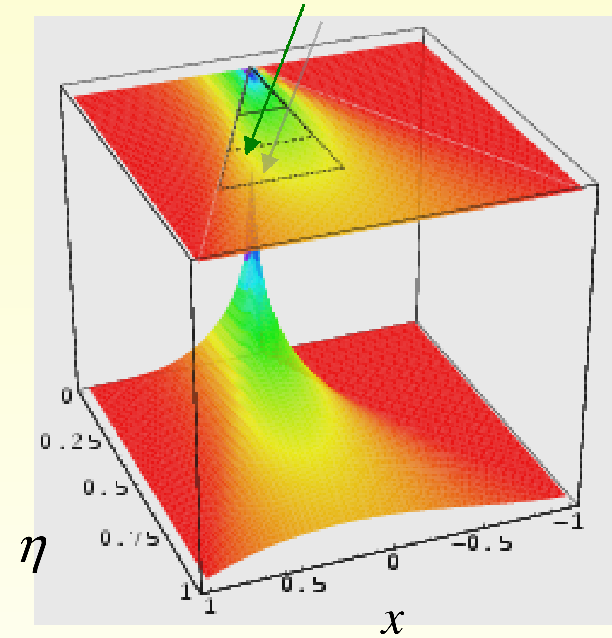
GPD related hard exclusive processes

- Deeply virtual Compton scattering (clean probe)

$$ep \to e'p'\gamma$$
$$ep \to e'p'\mu^+\mu^-$$
$$\gamma p \to p'e^-e^+$$



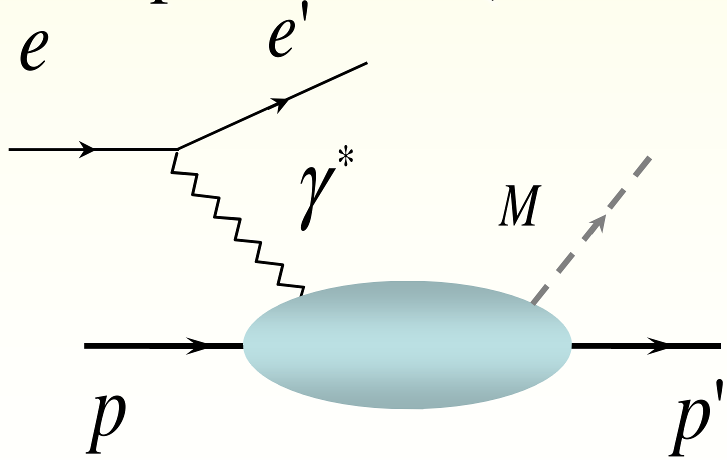
scanned area of the surface as a functions of lepton energy



$ep \to e'p'\mu^+\mu^-$

- Deeply virtual meson production (flavor filter)

$$ep \to e'p'\pi$$
$$ep \to e'p'\rho$$
$$ep \to e'n\pi^+$$
$$ep \to e'n\rho^+$$



<u>twist-two observables:</u>

longitudinal cross sections

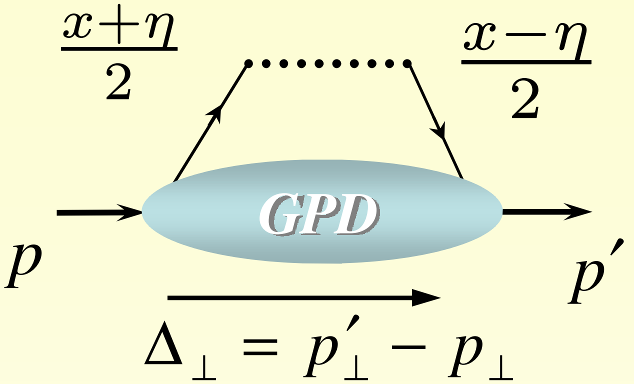
transverse target spin asymmetries

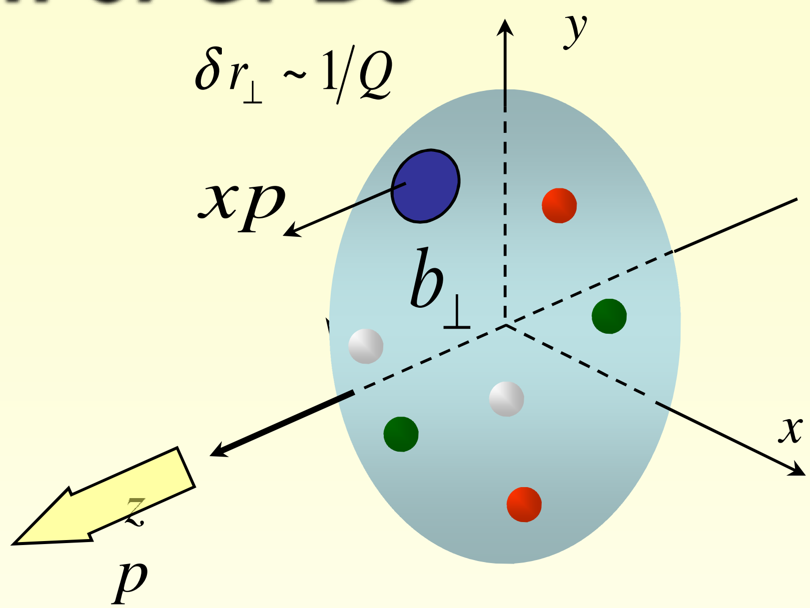
- etc.

factorization proof for longitudinal cross sections

[Collins, Frankfurt, Strikman (96)]

Partonic interpretation of GPDs

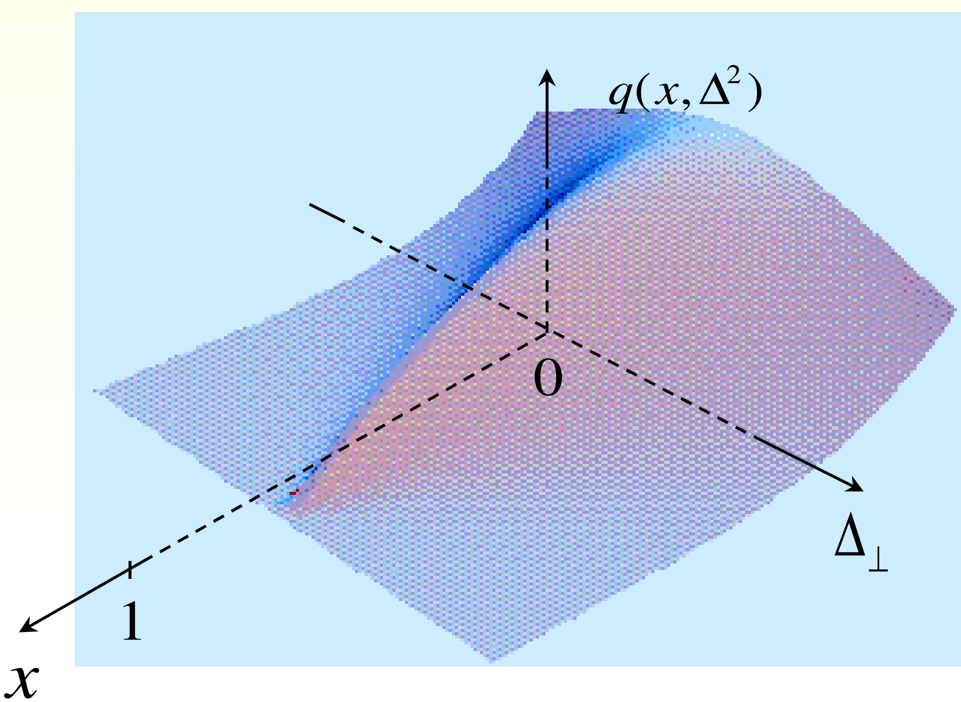




- GPDs simultaneously carry information on *longitudinal* and *transverse* distribution of partons in a proton

for η=0 they have a probabilistic interpretation
(infinite momentum frame) **[Burkhardt (00)]**

$$b_\perp = \sqrt{4\frac{d}{dt}\ln H(x,0,t)}\Big|_{t=0}$$



- GPDs contain also information on partonic angular momentum **[X. Ji (96)]**

$$\frac{1}{2} = \sum_{a=q,G} J_a^z$$

$$J_a^z = \lim_{\Delta\to 0}\frac{1}{2}\int_{-1}^{1} dx\, x\,(H_a+E_a)(x,\eta,\Delta^2)$$

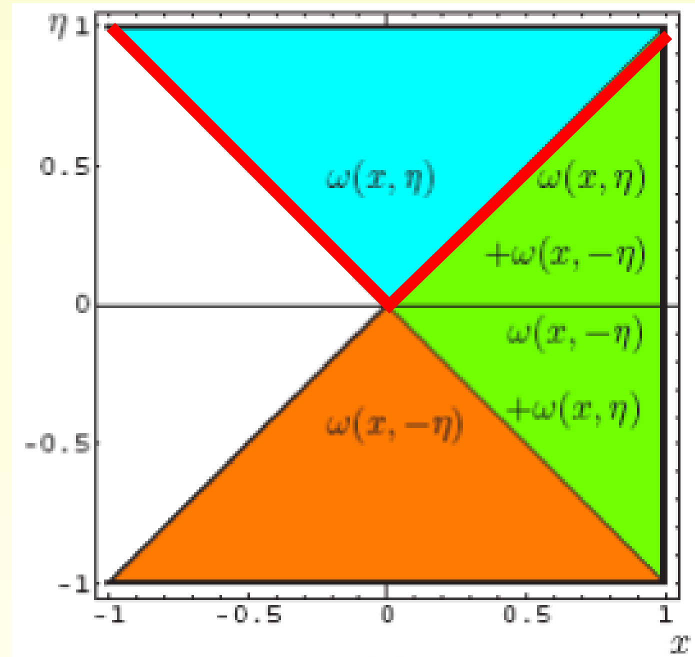
A partonic duality interpretation

quark GPD (anti-quark $x \to -x$):

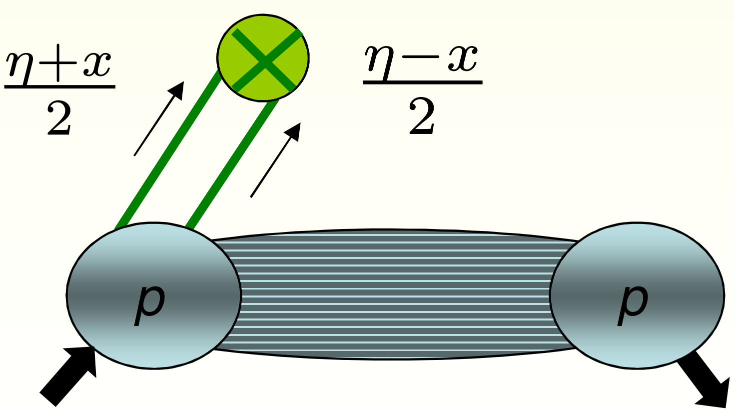
$$F(x,\eta,t) \quad =$$

$$\theta(-\eta \le x \le 1)\,\omega(x,\eta,t) + \theta(\eta \le x \le 1)\,\omega(x,-\eta,t)$$

$$\omega\,(x,\eta,t) = \frac{1}{\eta}\int_0^{\frac{x+\eta}{1+\eta}} dy\,(a-bx)^p f(y,(x-y)/\eta,t)$$

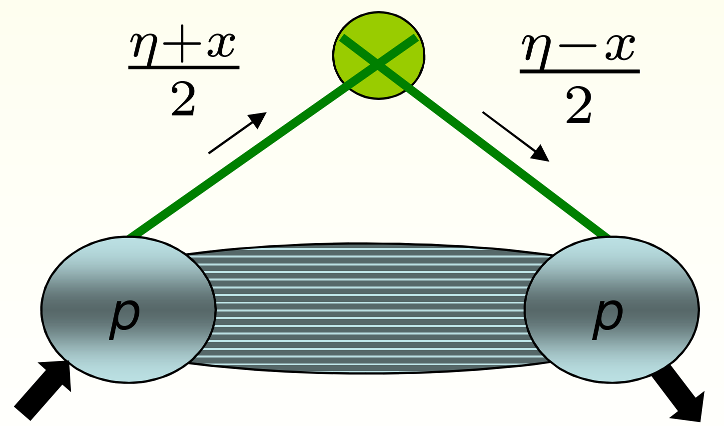


dual interpretation on partonic level:



support extension is unique [DM et al. 92]

ambiguous (*D*-term)
[DM, A. Schäfer (05)
KMP-K (07)]



central region $-\eta < x < \eta$

mesonic exchange in *t*-channel

outer region $\eta < x$

partonic exchange in *s*-channel

Can one `measure' GPDs?

- **CFF** given as **GPD** convolution:

$$\mathcal{F}(\xi, t, \mathcal{Q}^2) \stackrel{\mathrm{LO}}{=} \int_{-1}^{1} dx \left(\frac{1}{\xi - x - i\epsilon} \mp \frac{1}{\xi + x - i\epsilon} \right) F(x, \eta = \xi, t, \mathcal{Q}^2)$$

$$\stackrel{\mathrm{LO}}{=} i\pi F^{\pm}(x = \xi, \eta = \xi, t, \mathcal{Q}^2) + \mathrm{PV} \int_{0}^{1} dx \frac{2x}{\xi^2 - x^2} F^{\pm}(x, \eta = \xi, t, \mathcal{Q}^2)$$

- *F(x,x,t,$\mathcal{Q}^2$)* viewed as **"spectral function"** (*s*-channel cut):

$$F^{\pm}(x, x, t, Q^2) \equiv F(x, x, t, Q^2) \mp F(-x, x, t, Q^2) \stackrel{\mathrm{LO}}{=} \frac{1}{\pi} \Im\mathrm{m} \mathcal{F}(\xi = x, t, Q^2)$$

[Frankfurt et al (97)
Chen (97)
Terayev (05)
KMP-K (07)
Diehl, Ivanov (07)]

- **CFFs** satisfy `**dispersion relations**'
(not the physical ones, threshold ξ_0 set to 1)

$$\Re\mathrm{e} \mathcal{F}(\xi, t, Q^2) = \frac{1}{\pi} \mathrm{PV} \int_{0}^{1} d\xi' \left(\frac{1}{\xi - \xi'} \mp \frac{1}{\xi + \xi'} \right) \Im\mathrm{m} \mathcal{F}(\xi', t, Q^2) + \mathcal{C}(t, Q^2)$$

[Terayev (05)]

access to the **GPD** on the **cross-over line** $\eta = x$ (at LO)

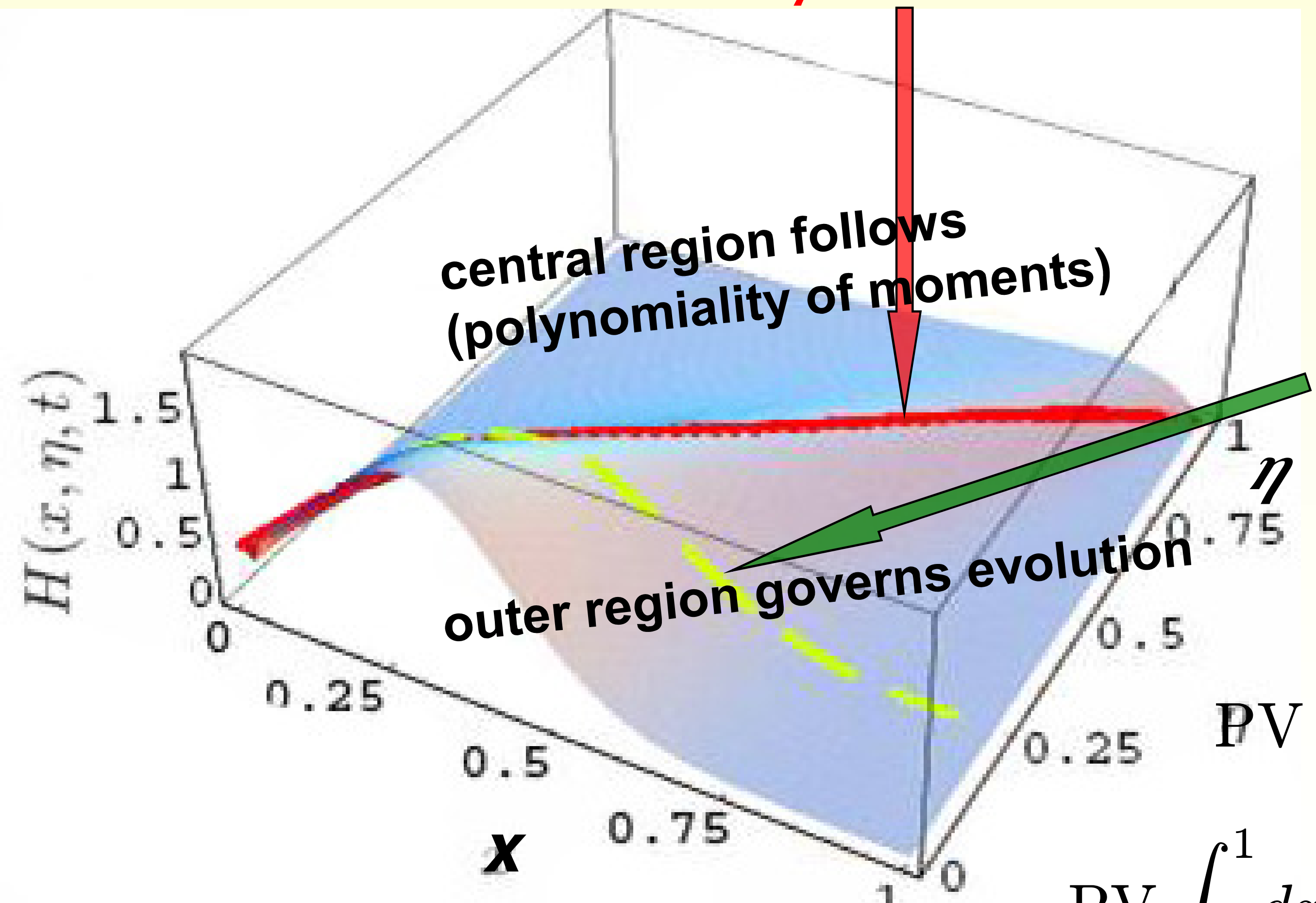
access to the subtraction constant (for *H*,*E* related to `*D*-term')

Modeling & Evolution

outer region governs the evolution at the cross-over trajectory

$$\mu^2 \frac{d}{d\mu^2} F(x,x,t,\mu^2) = \int_x^1 \frac{dy}{x} V(1, x/y, \alpha_s(\mu)) F(y,x,\mu^2)$$

GPD at $\eta = x$ is `measurable' (LO)



net contribution of outer + central region is governed by a sum rule:

$$\mathrm{PV}\int_0^1 dx\, \frac{2x}{\eta^2 - x^2} F^+(x,\eta,t)$$

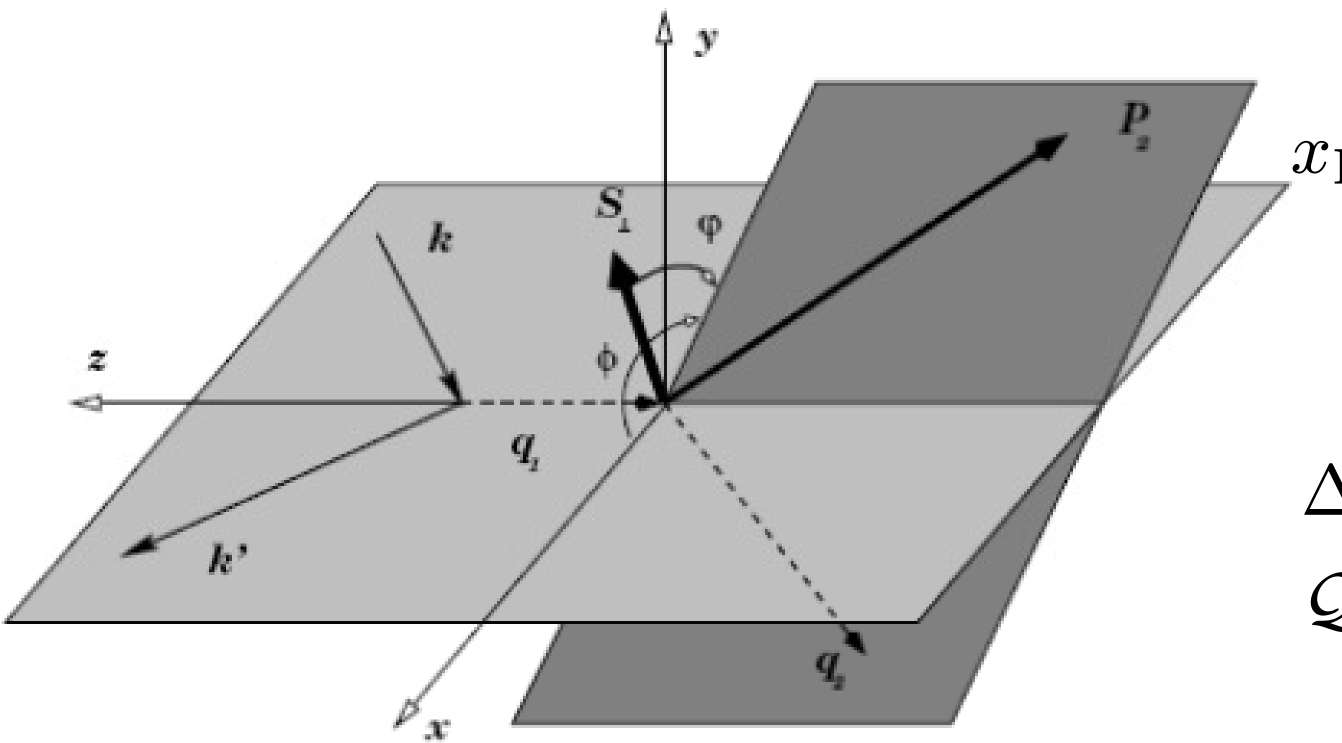
$$= \mathrm{PV}\int_0^1 dx\, \frac{2x}{\eta^2 - x^2} F^+(x,x,t) +{}_7 C(t)$$

Photon leptoproduction $e^{\pm}N \to e^{\pm}N\gamma$

measured by ***H1, ZEUS, HERMES, CLAS, HALL A*** collaborations

planed at ***COMPASS, JLAB@12GeV***, perhaps at ? EIC, ?? LHeC

$$\frac{d\sigma}{dx_{\rm Bj}dyd|\Delta^2|d\phi d\varphi} = \frac{\alpha^3 x_{\rm Bj} y}{16\,\pi^2\,\mathcal{Q}^2}\left(1+\frac{4M^2x_{Bj}^2}{\mathcal{Q}^2}\right)^{-1/2}\left|\frac{\mathcal{T}}{e^3}\right|^2,$$



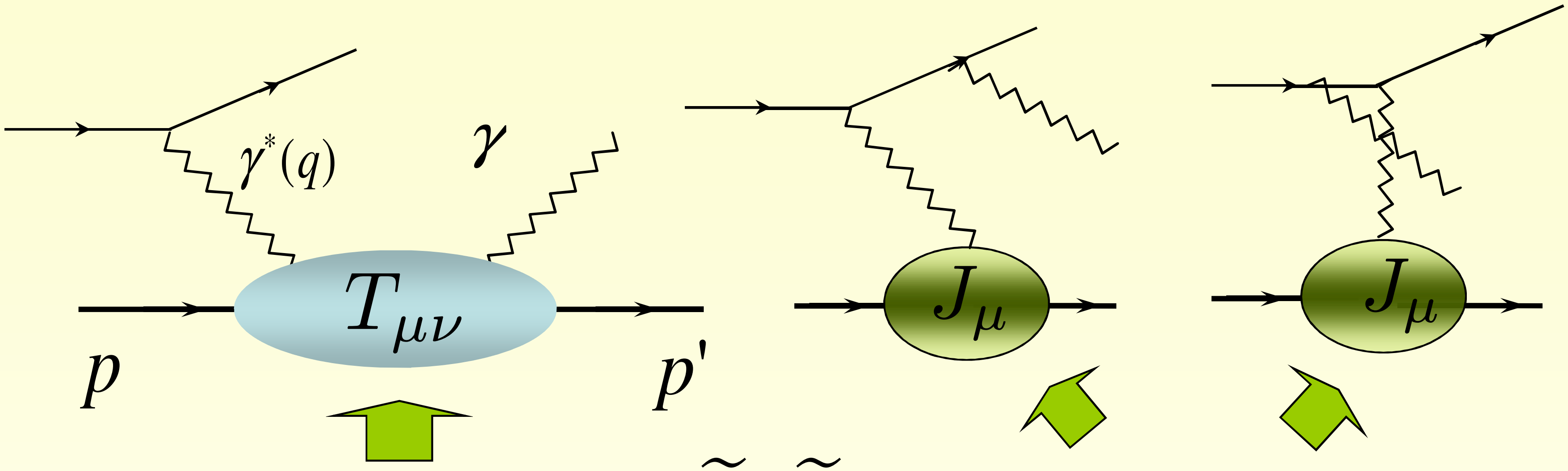
$$x_{\rm Bj} = \frac{\mathcal{Q}^2}{2P_1\cdot q_1} \approx \frac{2\xi}{1+\xi},$$

$$y = \frac{P_1\cdot q_1}{P_1\cdot k},$$

$$\Delta^2 = t \quad (\text{fixed, small}),$$

$$\mathcal{Q}^2 = -q_1^2 \quad (> 1\text{GeV}^2),$$

interference of *DVCS* and *Bethe-Heitler* processes



12 Compton form factors $\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}, \cdots$ elastic form factors F_1, F_2

(helicity amplitudes)

$$|\mathcal{T}_{\mathrm{BH}}|^2 = \frac{e^6(1+\epsilon^2)^{-2}}{x_{\mathrm{Bj}}^2 y^2 \Delta^2 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\mathrm{BH}} + \sum_{n=1}^{2} c_n^{\mathrm{BH}} \cos(n\phi) \right\},$$

exactly known
(LO, QED)

$$|\mathcal{T}_{\mathrm{DVCS}}|^2 = \frac{e^6}{y^2 \mathcal{Q}^2} \left\{ c_0^{\mathrm{DVCS}} + \sum_{n=1}^{2} \left[c_n^{\mathrm{DVCS}} \cos(n\phi) + s_n^{\mathrm{DVCS}} \sin(n\phi) \right] \right\},$$

harmonics $\Updownarrow$ 1:1 helicity ampl.

$$\mathcal{I} = \frac{\pm e^6}{x_{\mathrm{Bj}} y^3 \Delta^2 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\mathcal{I}} + \sum_{n=1}^{3} \left[c_n^{\mathcal{I}} \cos(n\phi) + s_n^{\mathcal{I}} \sin(n\phi) \right] \right\}.$$

harmonics $\Updownarrow$ 1:1 helicity ampl.

relations among **harmonics** and **GPDs** are not more based on $1/\mathcal{Q}$ expansion:
(all harmonics are expressed by twist-2 and -3 GPDs) **[Diehl et. al (97)
Belitsky, DM, Kirchner (01)
Belitsky, DM, Ji (12)]**

$$\begin{Bmatrix} c_1 \\ s_1 \end{Bmatrix}^{\mathcal{I}} \propto \frac{\Delta}{\mathcal{Q}}\ \text{tw-2(GPDs)} + O(1/\mathcal{Q}^3), \qquad c_0^{\mathcal{I}} \propto \frac{\Delta^2}{\mathcal{Q}^2}\ \text{tw-2(GPDs)} + O(1/\mathcal{Q}^4),$$

$$\begin{Bmatrix} c_2 \\ s_2 \end{Bmatrix}^{\mathcal{I}} \propto \frac{\Delta^2}{\mathcal{Q}^2}\ \text{tw-3(GPDs)} + O(1/\mathcal{Q}^4), \qquad \begin{Bmatrix} c_3 \\ s_3 \end{Bmatrix}^{\mathcal{I}} \propto \frac{\Delta \alpha_s}{\mathcal{Q}} (\text{tw-2})^{\mathrm{T}} + O(1/\mathcal{Q}^3),$$

$$c_0^{\mathrm{CS}} \propto\ (\text{tw-2})^2, \qquad \begin{Bmatrix} c_1 \\ s_1 \end{Bmatrix}^{\mathrm{CS}} \propto \frac{\Delta}{Q}\ (\text{tw-2})\ (\text{tw-3}), \qquad \begin{Bmatrix} c_2 \\ s_2 \end{Bmatrix}^{\mathrm{CS}} \propto \alpha_s (\text{tw-2})(\text{tw-2})^{\mathrm{GT}}$$

setting up the **perturbative framework:** **[Belitsky, DM (97);
Mankiewicz et. al (97);
Ji,Osborne (97/98);
Pire, Szymanowski, Wagner (11)
DM, Pire, Szymanowski, Wagner]**

✓ ***twist-two*** coefficient functions at ***next-to-leading*** order

✓ anomalous dimensions and evolution kernels at ***next-to-leading*** order **[Belitsky, DM (98) + Freund (01)]**

✓ ***next-to-next-to-leading*** order in a specific conformal subtraction scheme **[KMP-K & Schaefer 06]**

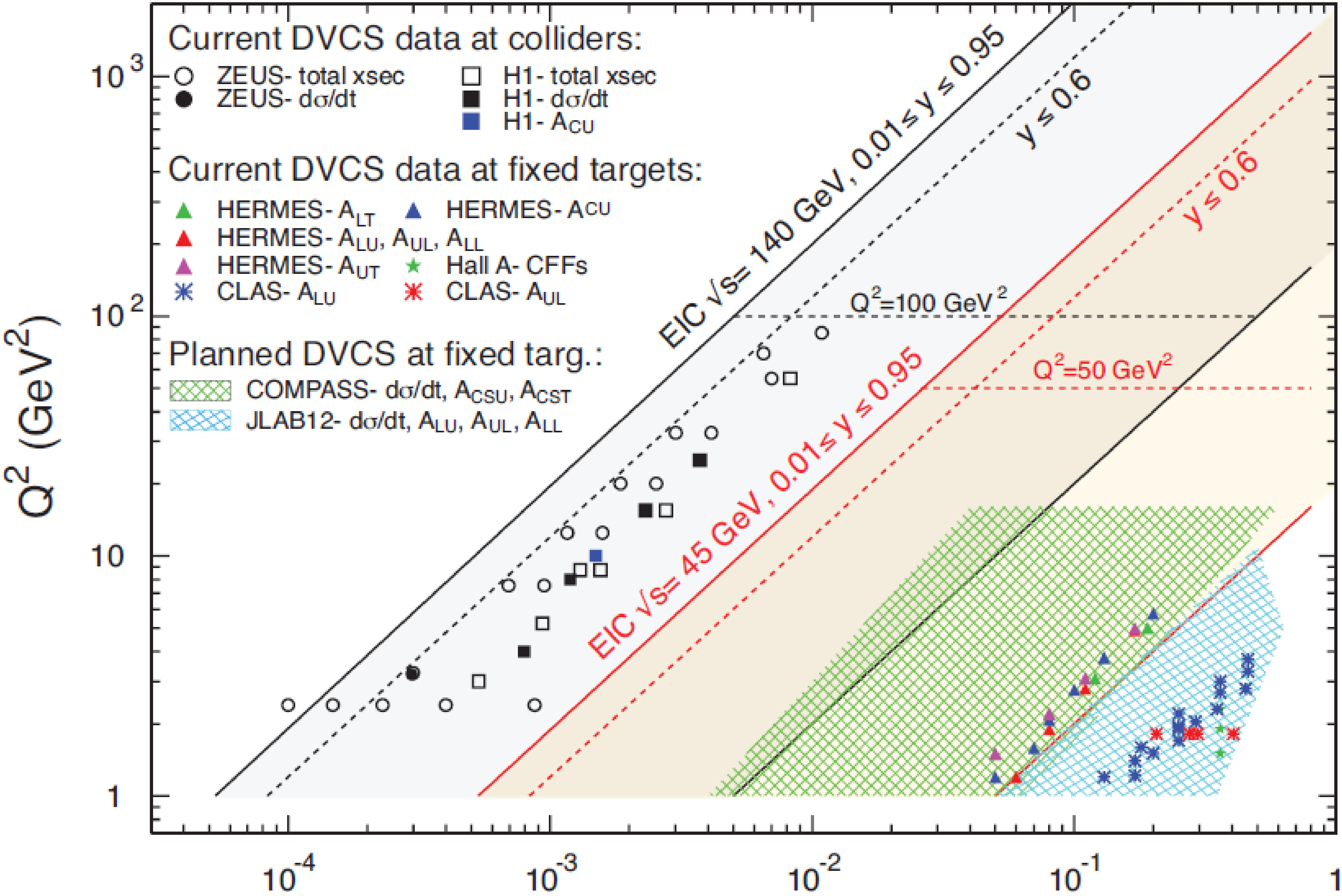
✓ ***twist-three*** including quark-gluon-quark correlation at LO **[Anikin,Teryaev, Pire (00); Belitsky DM (00); Kivel et. al]**

✓ partially, ***twist-three*** sector at ***next-to-leading*** order **[Kivel, Mankiewicz (03)]**

? \`target mass corrections' (not understood) **[Belitsky DM (01)]**

✓ **kinematical twist-four** corrections **[Braun, Manashov, (11)]**

DVCS world data set



Strategies to analyze DVCS data

(ad hoc) modeling: VGG code [Goeke et. al (01) based on Radyushkin's DDA]
BMK model [Belitsky, DM , Kirchner (01) based on RDDA]
`aligned jet' model [Freund, McDermott, Strikman (02)]
Goloskokov/Kroll (05) based on RDDA (pinned down by DVMP)
`dual' model [Polyakov,Shuvaev 02;Guzey,Teckentrup 06;Polyakov 07]
" -- " [KMP-K (07) in MBs-representation]
polynomials [Belitsky et al. (98), Liuti et. al (07), Moutarde (09)]

dynamical models: not applied [Radyushkin et.al (02); Tiburzi et.al (04); Hwang DM (07)]…
(respecting Lorentz symmetry)

flexible models: any representation by including *unconstrained* degrees of freedom
(for fits) KMP-K (07/08) for H1/ZEUS in MBs-integral-representation

CFFs (real and imaginary parts) and GPD fits/predictions

i. CFF extraction with formulae (local) [BMK (01), HALL-A (06)] and [KK,DM, Murray]
least square fits (local) [Guidal, Moutarde (08…)]
neural networks – a start up [KMS (11)]
ii. `dispersion integral' fits [KMP-K (08),KM (08…)]
iii. flexible GPD modeling [KM (08…)]
vi. model comparisons VGG code, however also BMK01 (up to 2005)
& predictions Goloskokov/Kroll (07) model based on RDDA

Asking for CFFs (physics case)

- CFFs are defined in the whole kinematical region [Belitsky, DM, Ji (12)]
- contain (generalized) polarizabilities
- their access requires a complete measurement

toy example DVCS off a scalar target [KK, DM, Murray (13)]

- for the first step we use *s*-channel helicity conservation hypothesis (neglecting twist-three and transversity associated CFFs)

- linearized set of equations (approximately valid)

$$A_{\mathrm{LU,I}}^{\sin(1\phi)} \approx N c_{\Im\mathfrak{m}}^{-1} \mathcal{H}^{\Im\mathfrak{m}} \quad \text{and} \quad A_{\mathrm{C}}^{\cos(1\phi)} \approx N c_{\Re\mathfrak{e}}^{-1} \mathcal{H}^{\Re\mathfrak{e}}$$

- normalization *N* is bilinear in CFFs

$$0 \lesssim N(\boldsymbol{A}) \approx \frac{1}{1+\frac{k}{4}|\mathcal{H}|^2} \approx \frac{\int_{-\pi}^{\pi} d\phi\, \mathcal{P}_1(\phi)\mathcal{P}_2(\phi) d\sigma_{\mathrm{BH}}(\phi)}{\int_{-\pi}^{\pi} d\phi\, \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)\left[d\sigma_{\mathrm{BH}}(\phi) + d\sigma_{\mathrm{DVCS}}(\phi)\right]} \lesssim 1$$

- cubic equation for *N* with two non-trivial solutions

$$N(\boldsymbol{A}) \approx \frac{1}{2}\left(1 \pm \sqrt{1 - k\, c_{\Im\mathfrak{m}}^2 \left(A_{\mathrm{LU,I}}^{\sin(1\phi)}\right)^2 - k\, c_{\Re\mathfrak{e}}^2 \left(A_{\mathrm{C}}^{\cos(1\phi)}\right)^2}\right)$$

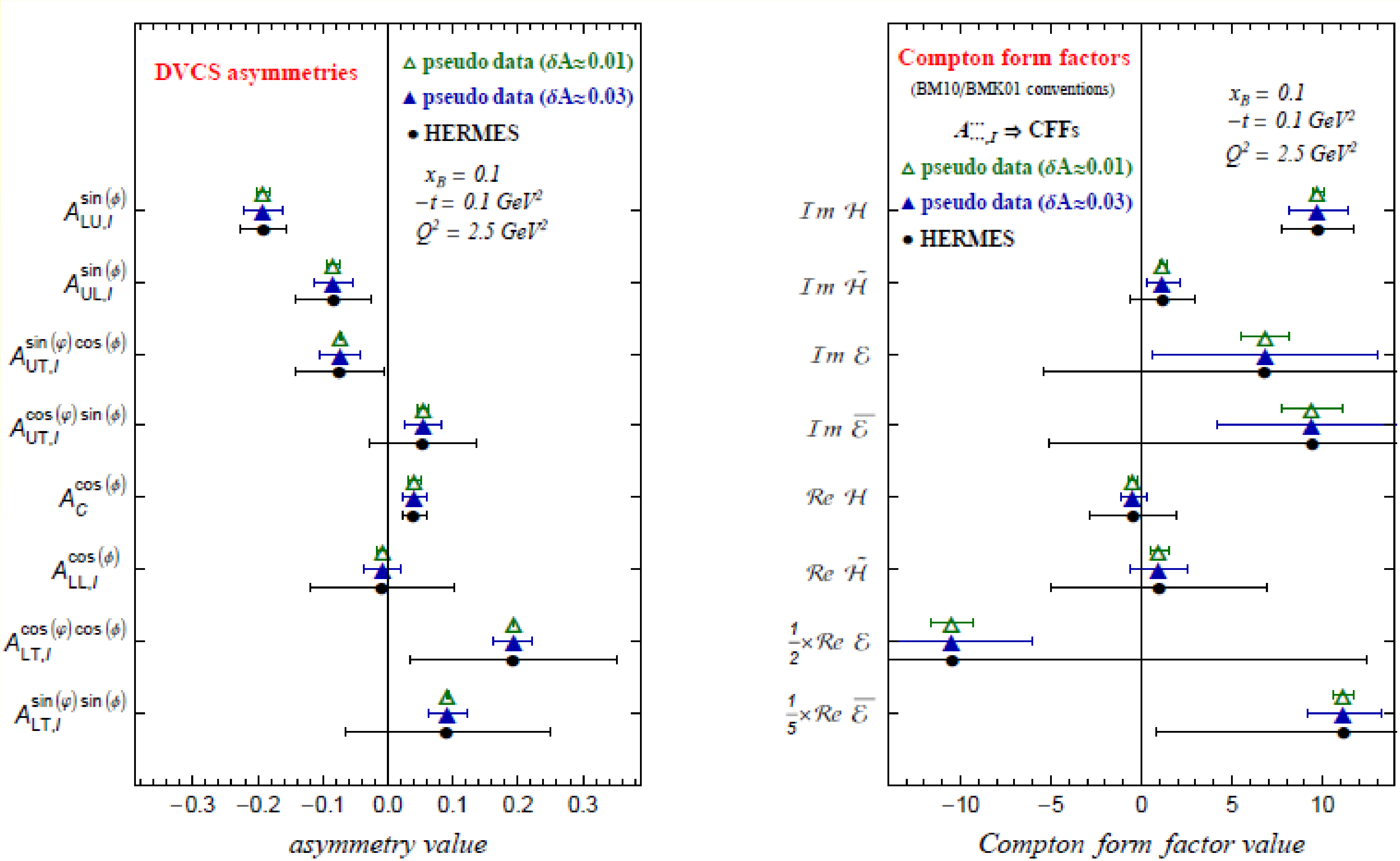
+ BH regime
- DVCS regime

- standard error propagation

NOTE: there is no need to linearize, one can do mapping numerically

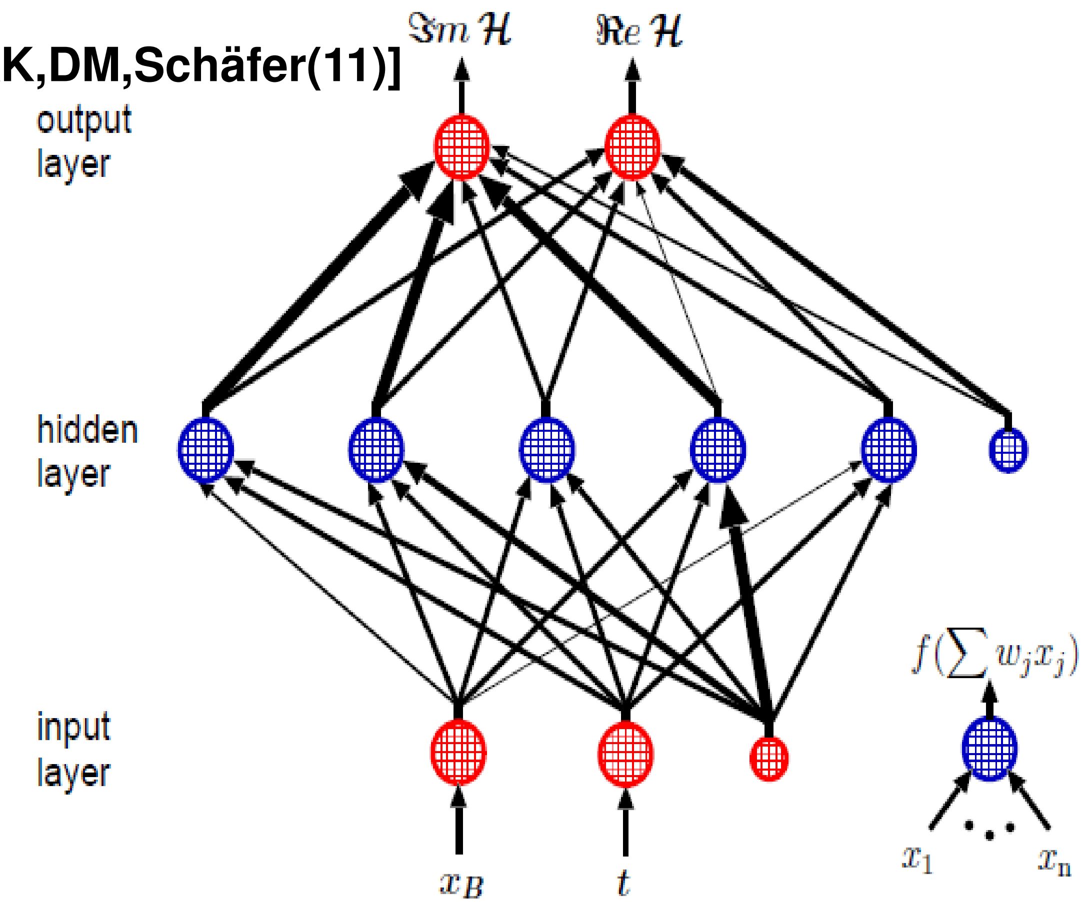
➢ a complete measurement allows in principle to pin down all CFFs **KK, DM, Murray (13)**

➢ missing information in incomplete measurements can be filled with noise



➢ larger statistics:
some *E* CFF constraint might have been obtained by HERMES

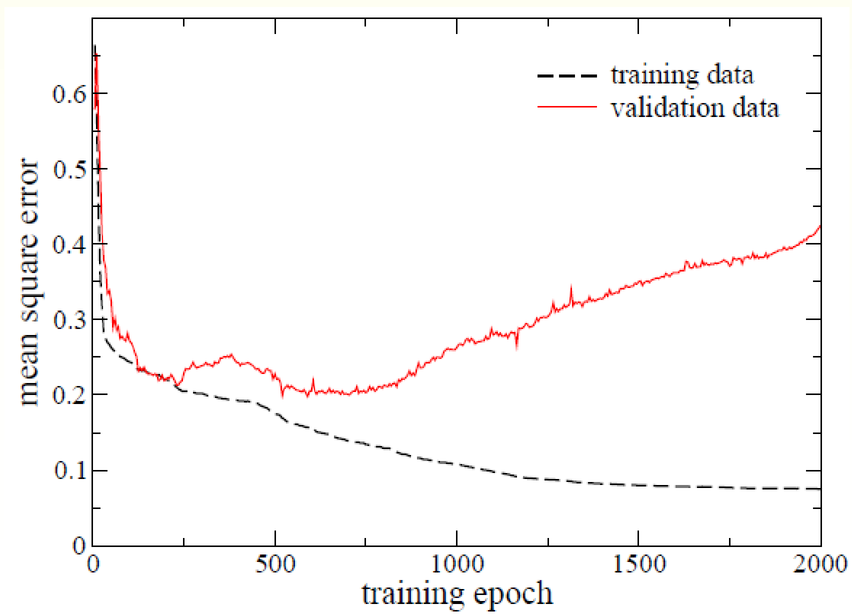
Neural Networks [KK,DM,Schäfer(11)]



- kinematical values are represented by the input layer
- propagated trough the network, where weights are set randomly
- random values for $Im\mathcal{H}$ and $Re\mathcal{H}$
- calculation of χ^2
- backwards propagation (PyBrain)
- adjusting weights so that error decreases
- repeat procedure
- taking next kinematical point

Monte Carlo procedure to propagate errors, i.e., generating a replica data set

avoiding over fitting (fitting to noise), dividing data set, taking a control example if error increases after decreasing – one stops

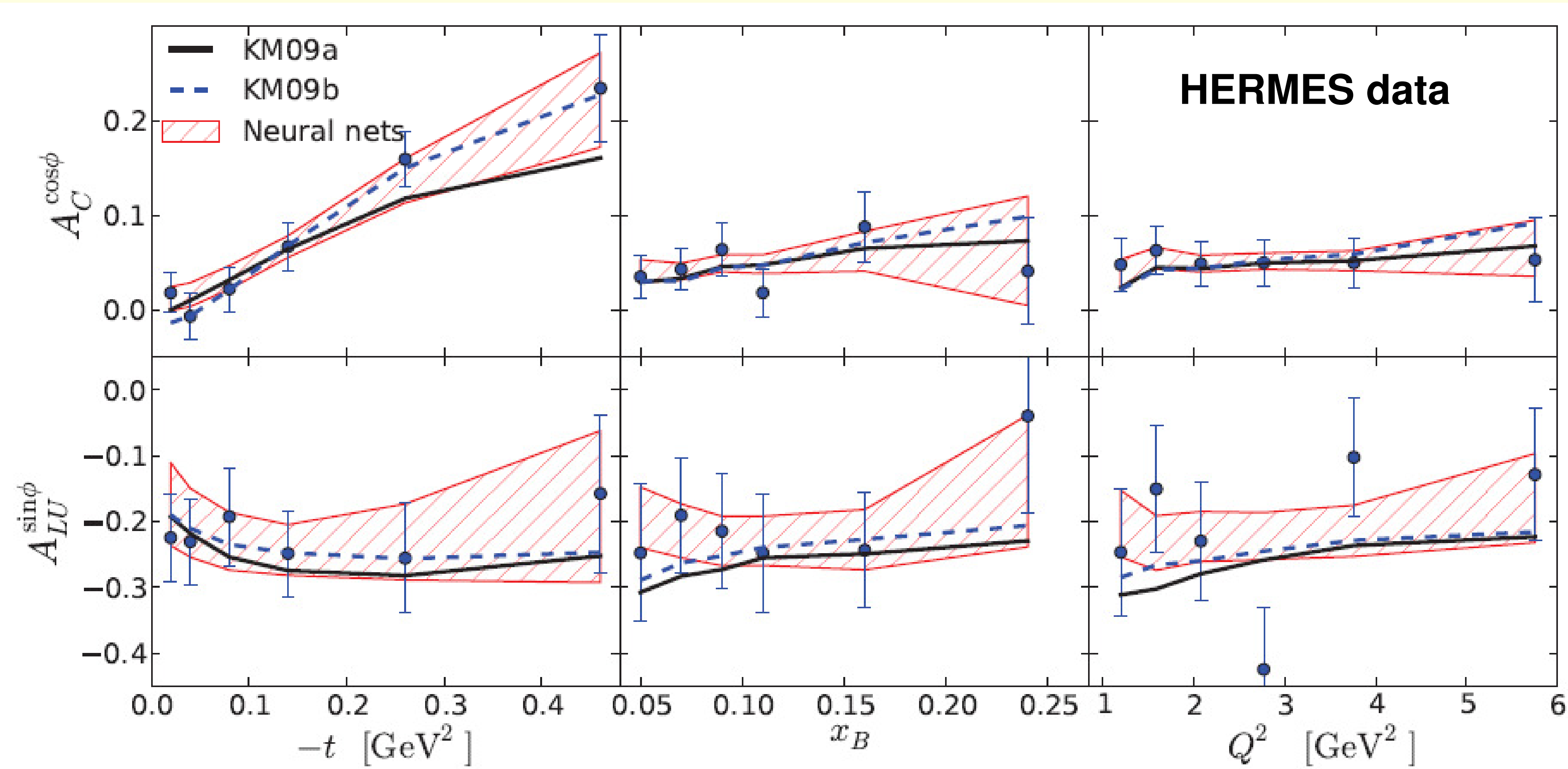


A first use of neural network fits

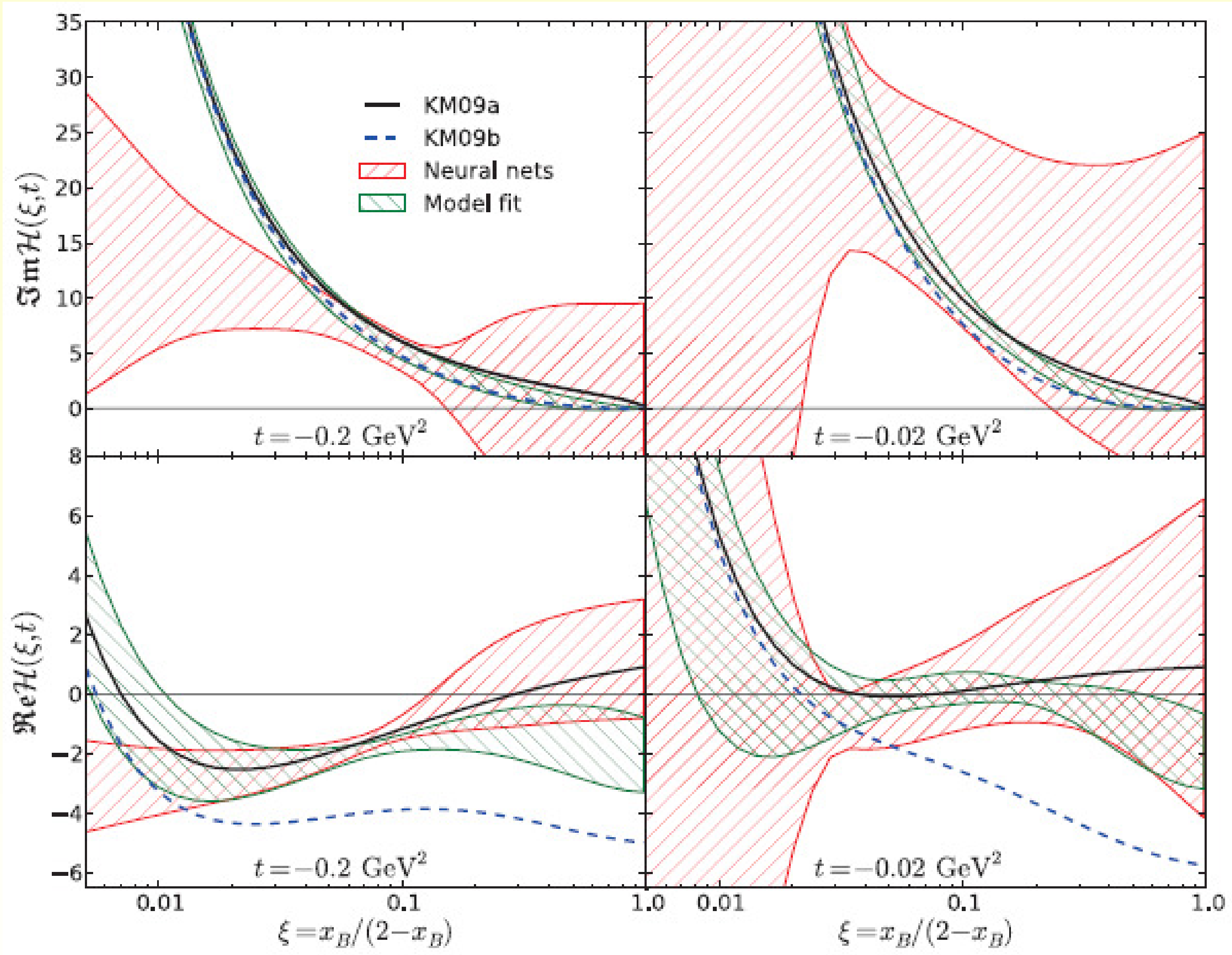
(ideal) tool for error propagation and quantifying model uncertainties

used to access real and imaginary part of $\mathcal{H}$ CFF from HERMES

results are compatible to model, CFF fits, and mapping

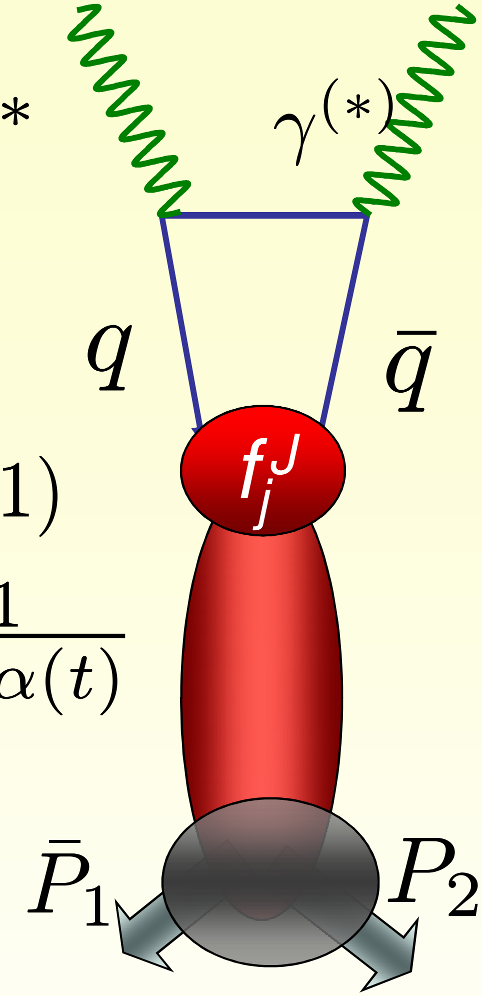


Model prediction versus unbiased error propagation



- model fits and neural networks are complimentary
- meaning of error bands should be properly understood
- error propagation is practically an art (full information is not given)

GPD ansatz from t-channel view



- at short distance a quark/anti-quark state is produced, labeled by ***conformal spin*** *j+2*
- they form an intermediate mesonic state with total angular momentum ***J*** strength of ***coupling*** is

$$f_j^J,\ J \le j\,(+1)$$

- mesons propagate with

$$\frac{1}{m^2(J)-t} \propto \frac{1}{J-\alpha(t)}$$

- decaying into nucleon anti-nucleon pair with given angular momentum *J*, described by an ***impact form factor***

$$\frac{1}{(1-\frac{t}{M^2(J)})^p}$$

➢ (conformal) GPD moments expanded in Wigner`s rotation matrices

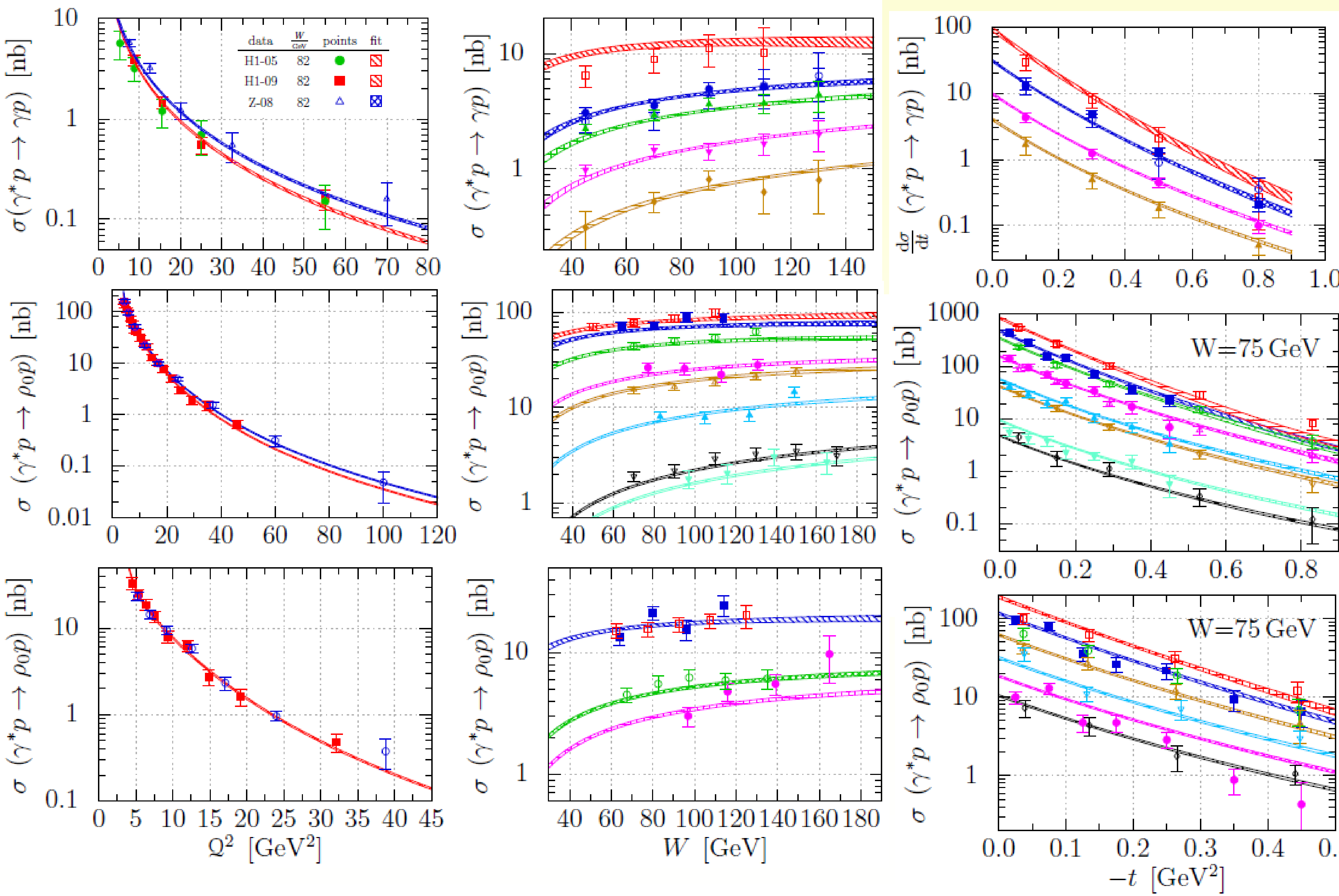
$$F_j(t,\eta) = \sum_J^{j\,(+1)} \frac{f_j^J}{J-\alpha(t)} \frac{1}{(1-\frac{t}{M^2(J)})^p}\, \eta^{j\,(+1)-J}\, \hat{d}_J^F(\eta)\,, \quad \hat{d}_J^F(\eta=0)=1$$

- labeling by *t*-channel quantum numbers J^{PC} **[Lebed, Ji (00),...]**
- so-called D-term arises from *0 ⁺⁺ , (f^0 or σ) 2⁺⁺, 4⁺⁺, ...,* has even *J=j+1 (or j = -1 in DR)* pole (*J (=0)* has multiple meanings **[KMP-K(07&08)]**)
- usable for large *x* (employing effective rotation matrices)

DIS+DVCS+DVMP phenomenology at small-x_B (H1,ZEUS)

works somehow without DIS at LO **[T. Lautenschlager, DM, A. Schäfer (soon)]**

works at NLO ($Q^2 > 4\ GeV^2$), done with Bayes theorem (probability distribution function)

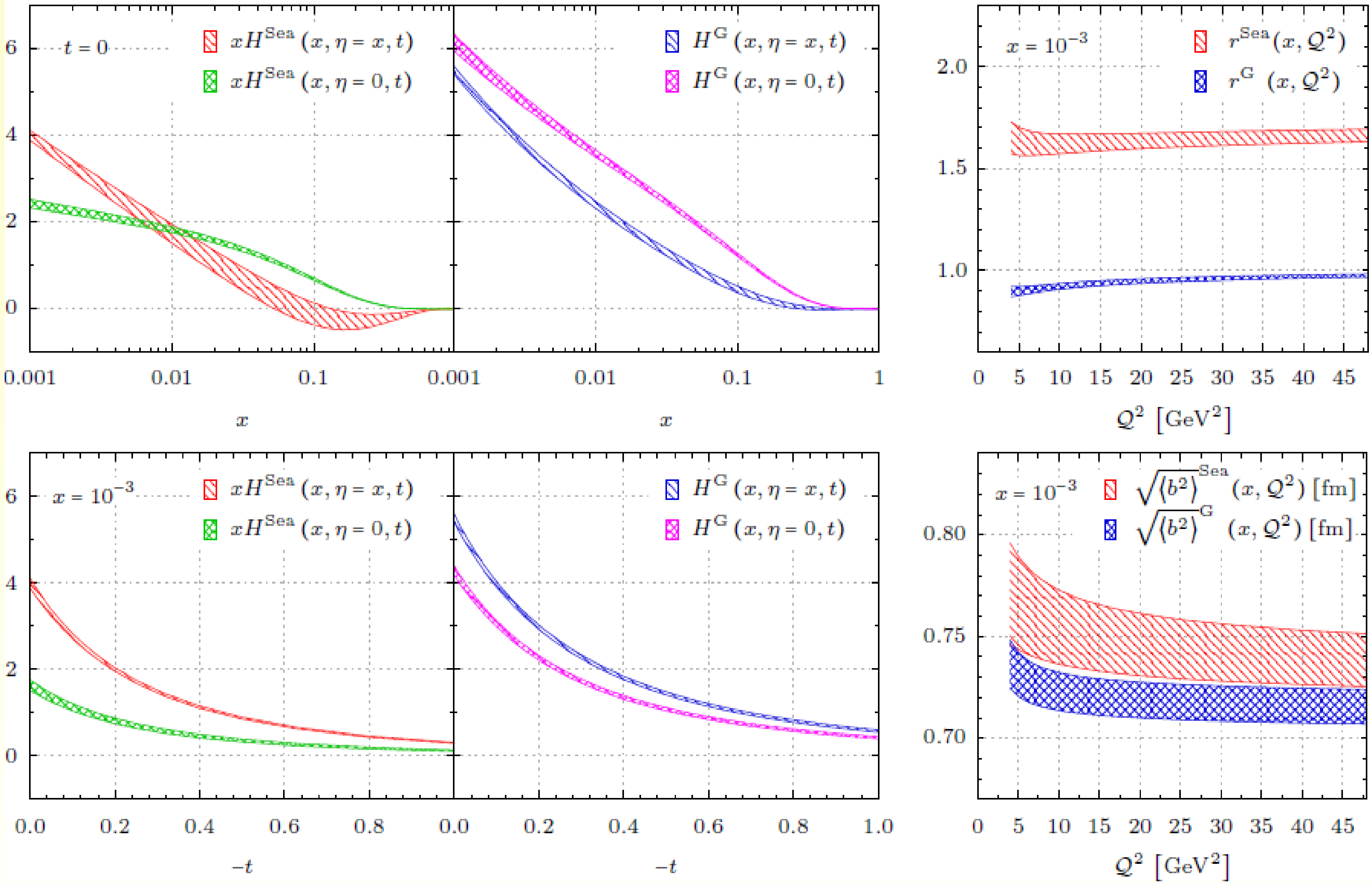


fixed:

meson DA
flavor content

errors might be perhaps larger

entirely model dependency for x> 10^{-2}



- going from LO to NLO increases the skewness ratios (known since `ever', **[KMP-K(07)]**)
- gluons are more centralized as sea quarks (expected from DVCS & *J/ψ* interpretation)
- cross-talk of skewness and *t*-dependency has been addressed by pdf
- NLO GPDs look rather compatible to Goloskokov/Kroll and Martin et. al finding
- there is also DVCS beam charge and perhaps **beam spin** data are coming up

A simple valence quarks GPD model

- model of GPD *H(x,x,t) within DD motivated ansatz at* $Q^2=2$ GeV^2

fixed: *PDF normalization* *eff. Reage pole* *large t-counting rules*

$$H(x,x,t) = \frac{n\, r\, 2^{\alpha}}{1+x}\left(\frac{2x}{1+x}\right)^{-\alpha(t)}\left(\frac{1-x}{1+x}\right)^{b}\frac{1}{\left(1-\frac{1-x}{1+x}\frac{t}{M^2}\right)^{p}}.$$

free parameters: r-ratio at small x *large x-behavior* *p-pole mass*

- unpolarized valence quarks : asking for *r, b, M* parameters

$$n = 1.0, \quad \alpha(t) = 0.43 + 0.85t/\mathrm{GeV}^2, \quad p = 1$$

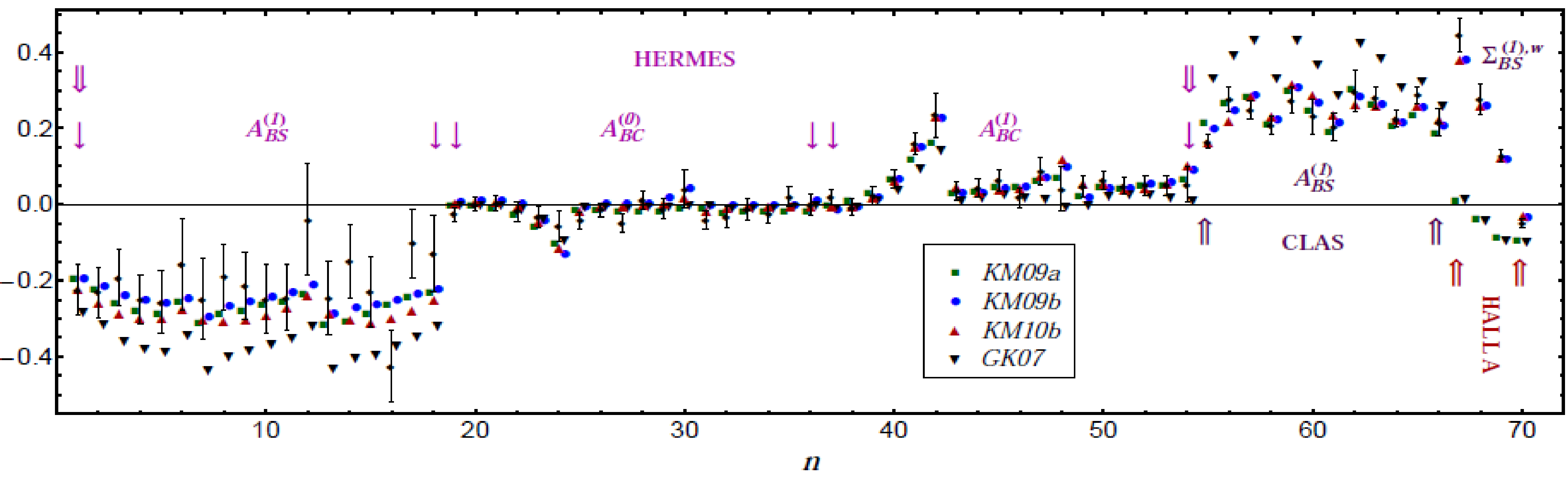
- flexible parameterization of subtraction constant (so-called D-term convoluted with hard amplitude)

$$\mathcal{D}(t) = \frac{-C}{(1-t/M_c^2)^2}$$

- analogous ansatz for porlarized quark GPD + pion-pole contribution
- no E(x,x,t) nor Ê(x,x,t) is set up
- KM...> 2010 hybrid models GPD evolution for sea /gluon + DR for valence

Fixed target DVCS data

- HERMES(02-12) 12x34 asymmetries (+few bins) $0.05 \le \langle x_B \rangle \le 0.2$, $\langle |t| \rangle \le 0.6$ GeV^2
  [$sin(\varphi)$, ..., $cos(3\varphi)$, $\langle Q^2 \rangle \approx 2.5$ GeV^2
  two kinds of electrons, all polarization options]
- HERMES(12) A_{LU} with recoil detector
  (compatible with old data, differences in GPD interpretation)
- CLAS(07) 12x12 [$A_{LU}(\varphi)$] $0.14 \le \langle x_B \rangle \le 0.35$, $\langle |t| \rangle \le 0.3$ GeV^2
  40x12 [$A_{LU}(\varphi)$] (large |t| or bad sta.) $\langle Q^2 \rangle \approx 1.8$ GeV^2
  (06,08) A_{UL} and A_{LU}
- HALL A(06) 12x24 [$\Delta\sigma(\varphi)$] $\langle x_B \rangle = 0.36$, $\langle |t| \rangle \le 0.33$ GeV^2
  3x24 [$\sigma(\varphi)$] $\langle Q^2 \rangle \approx 1.8$ GeV^2



KM10 fits to DVCS off unpolarized proton

- a hybrid model: three effective SO(3) PWs for sea quarks/gluons
  dispersion relations for valence
  still *E* GPD is neglected (only D-term)
  still $\hat{E}$ GPD only flexible pion pole contribution
- asking for GPD *H* and `*D-term*' *($\hat{H}$ is considered as effective d.o.f.)*

leading order, including evolution for sea quarks/ gluons
quark twist-two dominance hypothesis within CFF convention **[BM10]**

- data selection (taking moments of azimuthal angle harmonics)

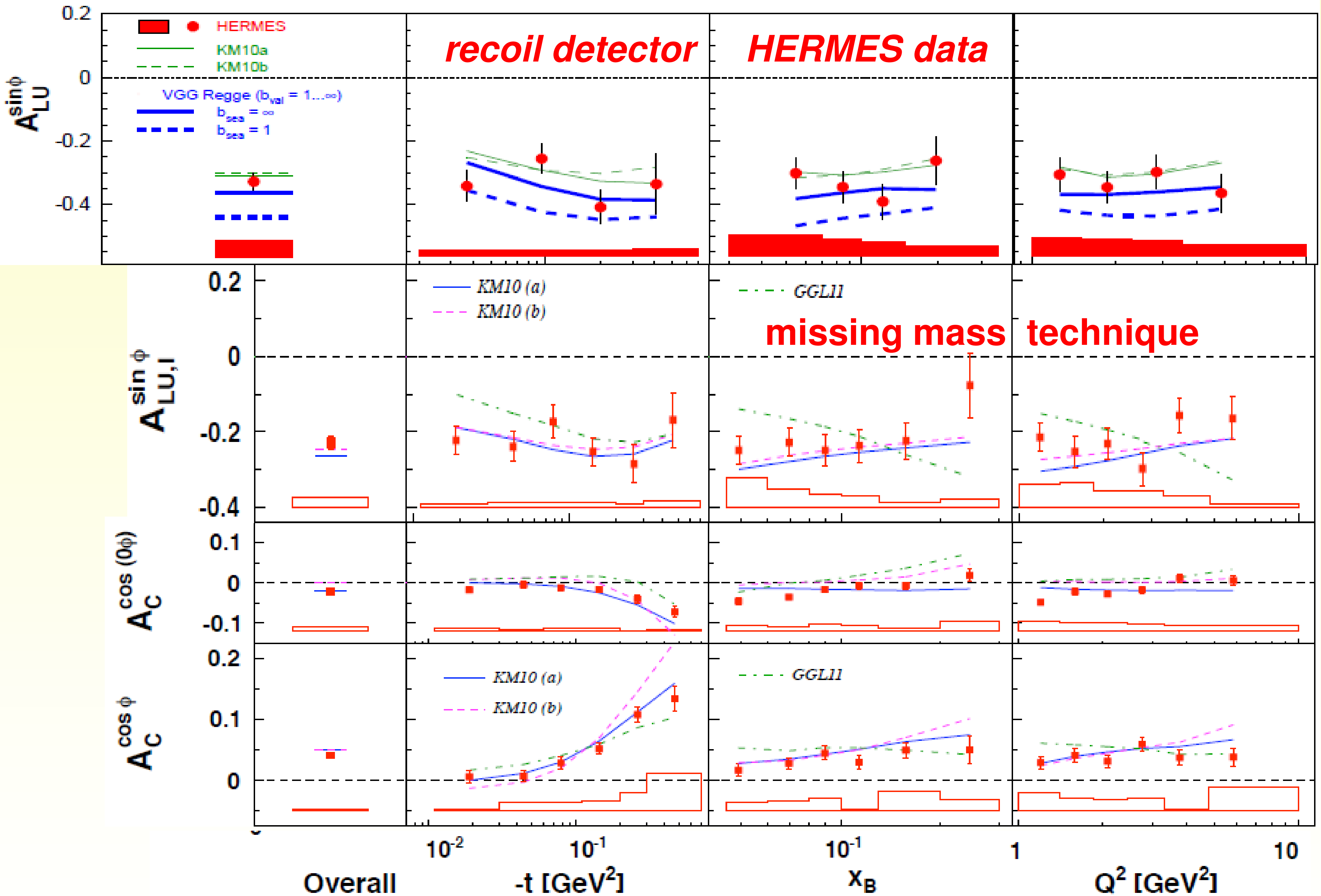
KM10a: neglecting HALL-A data
KM10b: forming ratios of moments
KM10: original HALL-A data
neglecting large $-t$ BSA CLAS data

15 parameter fit, e.g.,
including all HALL-A data

175 data points
$\chi^2/d.o.f. = 132/165$

```
-----------------------------
M02S  =  0.51  +-  0.02
SECS  =  0.28  +-  0.02
SECG  = -2.79  +-  0.12
THIS  = -0.13  +-  0.01
THIG  =  0.90  +-  0.05
  Mv  =  4.00  +-  3.33  (edge)
  rv  =  0.62  +-  0.06
  bv  =  0.40  +-  0.67
   C  =  8.78  +-  0.98
  MC  =  0.97  +-  0.11
 tMv  =  0.88  +-  0.24
 trv  =  7.76  +-  1.39
 tbv  =  2.05  +-  0.40
 rpi  =  3.54  +-  1.77
 Mpi  =  0.73  +-  0.37
-----------------------------
```

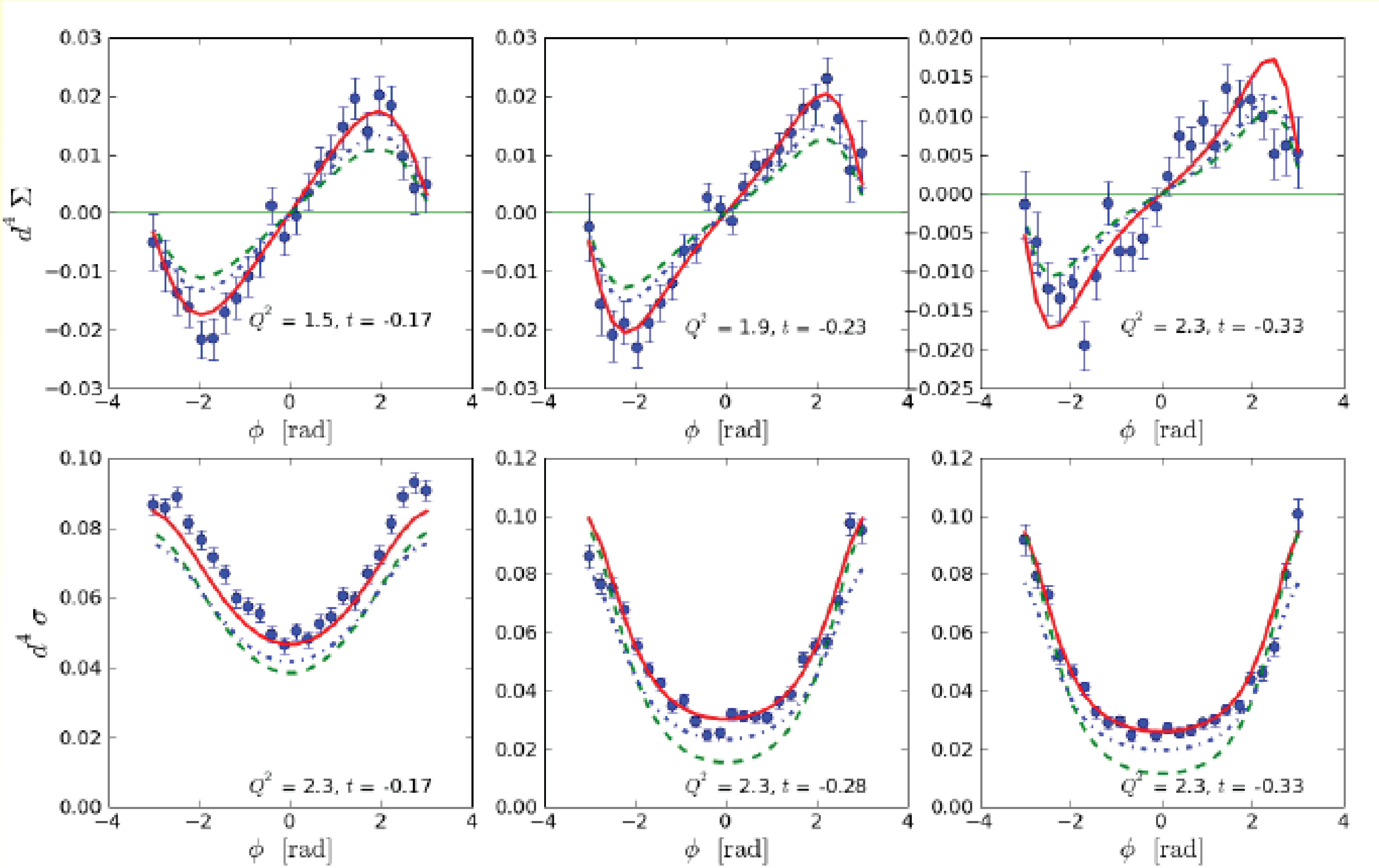

- results are given as **xs.exe** on ***http://calculon.phy.hr/gpd/***



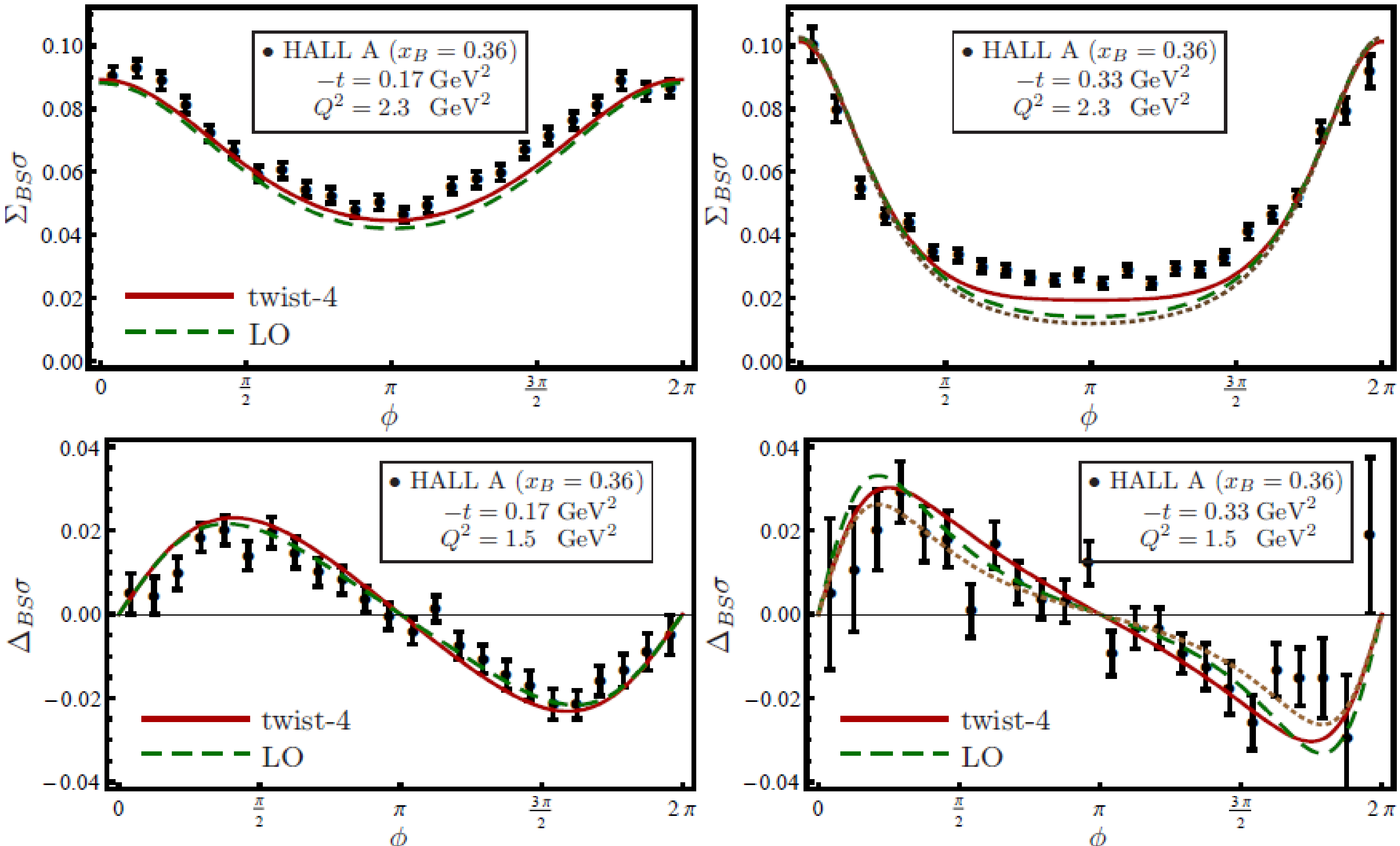
- recoil detector data are compatible with missing mass technique ones
- fits produce curves were data are scattered around
- recoil data: RDDA is not so much disfavored as it was before the case

HALL A φ-dependence

- φ-dependence is described (if we fit to it)



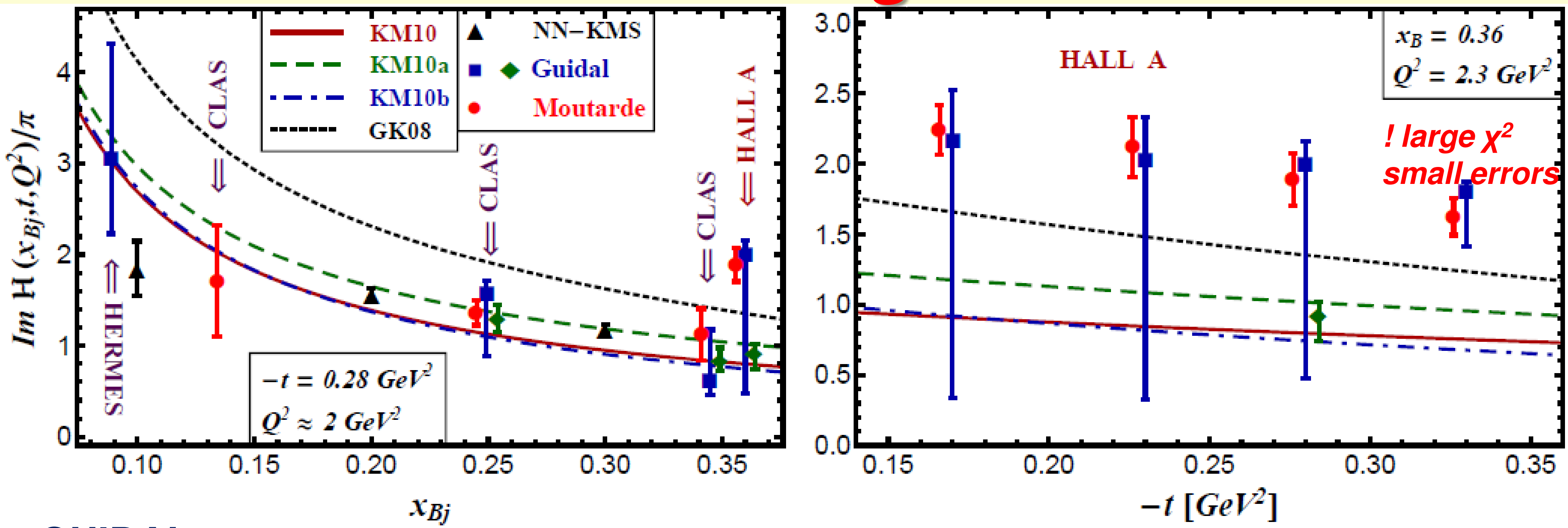
How to understand Hall A data?



[Braun, Manashov, Pirnay, DM]

- do higher twist corrections explain data?
  e.g., with GPD standard models such as Goloskokov-Kroll based on RDDA
- wrong understanding on CFF hierarchy?
- exclusivity issue in all other fixed target data?
- Is (QED) correction procedure understood?
- naive understanding of `power corrections' **[VGG (99)]** is entirely misleading

KM... versus CFF fits & large-x "model" fit



GUIDAL twist-two dominance hypothesis
7 parameter fit to all harmonics of unpolarized cross section
propagated errors + "theoretical" error estimate

GUIDAL same + longitudinal TSA

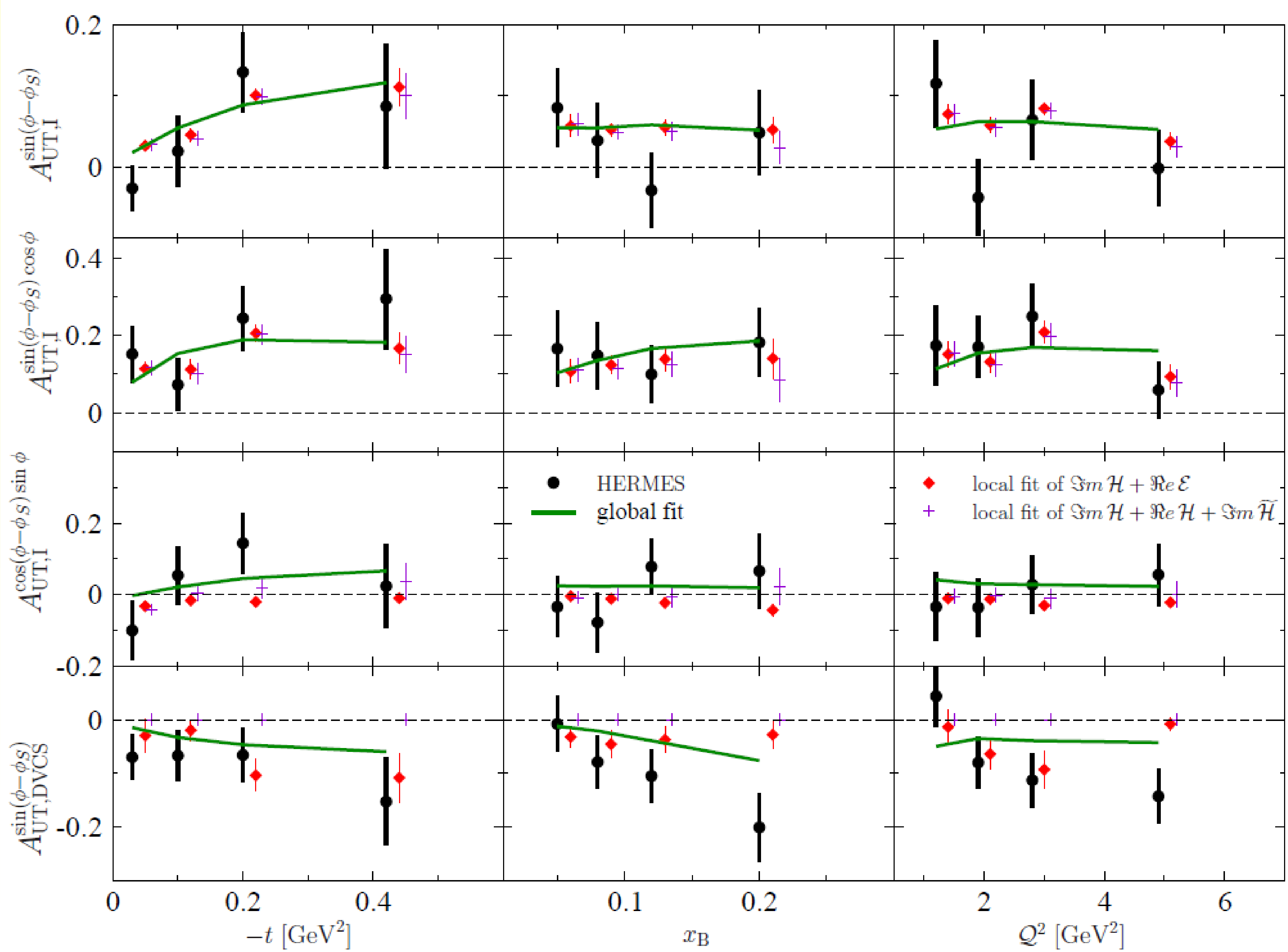
Moutarde H dominance hypothesis within a smeared polynomial expansion
propagated errors + "theoretical" error estimate

NN neural network within H dominance hypothesis
green (blue) [red] curves (KM10...) without (with) HALL A data (ratios)

GK08 black curve GPDs (based on RDDA) obtained from handbag approach to DVMP

- reasonable agreement for HERMES and CLAS kinematics
- large *x*-region and real part remains unsettled

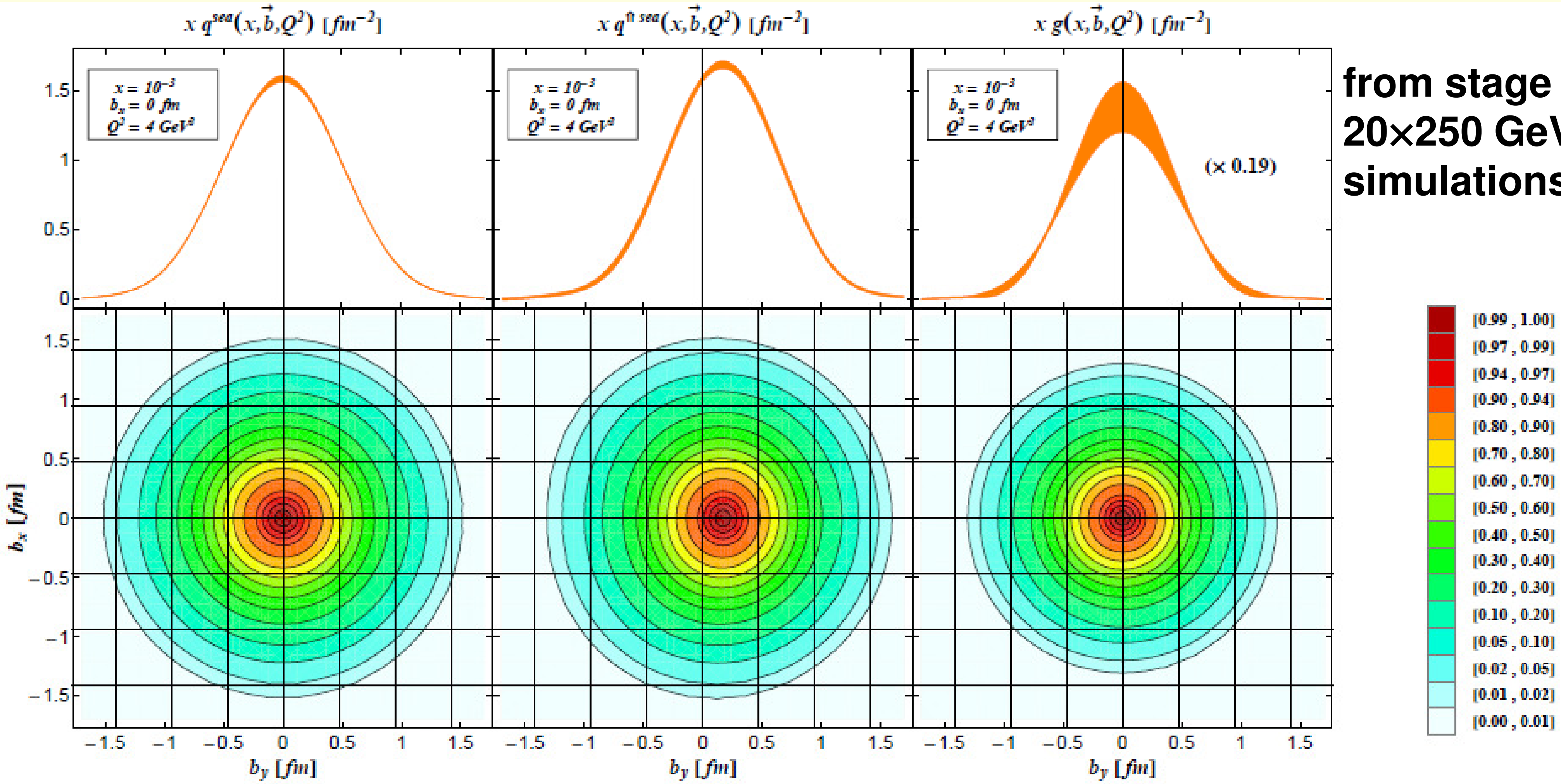
- KMM12 (KM10 type model) includes polarized target DVCS data (global fit to most of data , $\chi^2/d.o.f \approx 1.6$ - best what is there at present e.g., transverse polarized HERMES asymmetries looks as)



The Future

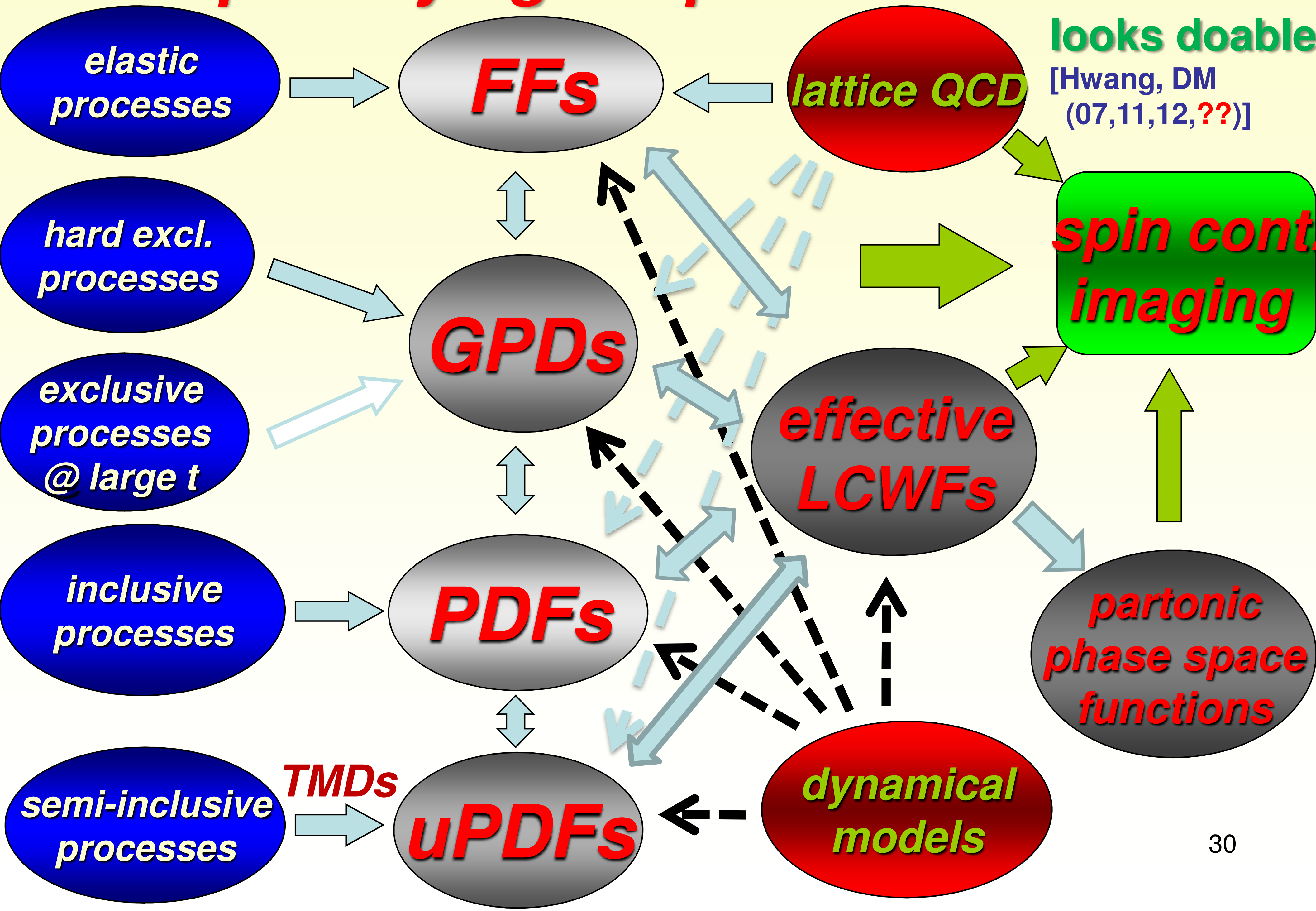
- ✓ Compass
- ✓ JLAB@12 GeV
- ? ENC@GSI
- ? LHeC@CERN
- ? EIC@BNL or EIC@JLAB

Aschenauer, Firzo KK, DM (13)



from stage II 20×250 GeV^2 simulations

quantifying the partonic content



Summary

GPDs are intricate and (thus) a promising tool

- to reveal the transverse distribution of partons (to some extend done at small x_B)
- to address the spin content of the nucleon (not possible at present)
- providing a bridge to LCWFs & non-perturbative methods (e.g., lattice)
- modeling in terms of effective LCWFs is doable (require efforts)

first decade of hard exclusive leptoproduction measurements

- CFFs have their own interest, bridging low and high virtuality regimes
- should be straightforward to improve global (flexible) model fits to DVCS
- DVCS and DVMP data are describable in global fits at small x
- moving on: to NLO, kinematical twist, full GPD models, DVCS+DVMP+...
- covering the kinematical region between HERA (COMPASS) experiments within a high luminosity machine and dedicated detectors is needed to quantify exclusive and inclusive QCD phenomena: handle on GPD E & 3D

need :

tools/technology for global NLO QCD fits (inclusive + exclusive)

theory development ***(desired but not urgent needed for phenomenology)***

back ups

Field theoretical GPD definition

GPDs are defined as matrix elements of **renormalized** ***light-ray*** operators:

DM, Robaschik, Geyer, Dittes, Hořejši (94)

$$\boxed{F(x,\eta,\Delta^2,\mu^2) = \int_{-\infty}^{\infty} d\kappa\, e^{i\kappa\, x\, n\cdot P} \langle P_2 | \mathcal{R}T :\phi(-\kappa n)[(-\kappa n),(\kappa n)]\phi(\kappa n): | P_1\rangle, \; n^2 = 0}$$

momentum fraction x , skewness $\eta = \frac{n\cdot\Delta}{n\cdot P}$ $\Delta = P_2 - P_1$ $P = P_1 + P_2$ $\Delta^2 \equiv t$

For a nucleon target we have four chiral even twist-two GPDs:

$$\bar\psi_i \gamma_+ \psi_i \quad \Rightarrow \quad {}^i q^V = \bar U(P_2,S_2)\gamma_+ U(P_1,S_1) H_i + \bar U(P_2,S_2)\frac{i\sigma_{+\nu}\Delta^\nu}{2M} U(P_1,S_1) E_i$$

$$\bar\psi_i \gamma_+ \gamma_5 \psi_i \quad \Rightarrow \quad {}^i q^A = \bar U(P_2,S_2)\gamma_+\gamma_5 U(P_1,S_1) \widetilde H_i + \bar U(P_2,S_2)\frac{\gamma_5 \Delta_+}{2M} U(P_1,S_1) \widetilde E_i$$

shorthands:

chiral even GPDs: $F = \{H, E, \widetilde H, \widetilde E\}$ *& CFFs:* $\mathcal{F} = \{\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}\}$

chiral odd GPDs: $F_T = \{H_T, E_T, \widetilde H_T, \widetilde E_T\}$ $\mathcal{F}_T = \{\mathcal{H}_T, \mathcal{E}_T, \widetilde{\mathcal{H}}_T, \widetilde{\mathcal{E}}_T\}$

[Belitsky, DM, Kirchner (01) -- BM Ji (12)]

$$\mathcal{T}^{\rm VCS}_{ab}(\phi) = (-1)^{a-1}\varepsilon_2^{\mu *}(b) T_{\mu\nu}\varepsilon_1^{\nu}(a)$$

$$\mathcal{T}^{\rm VCS}_{ab} = \mathcal{V}(\mathcal{F}_{ab}) - b\,\mathcal{A}(\mathcal{F}_{ab}) \quad \text{for} \quad a \in \{0,+,-\},\ b \in \{+,-\}$$

$$\mathcal{V}(\mathcal{F}_{ab}) = \bar{u}_2\left(\not{m}\mathcal{H}_{ab} + i\sigma_{\alpha\beta}\frac{m^\alpha\Delta^\beta}{2M}\,\mathcal{E}_{ab}\right)u_1$$

$$\mathcal{A}(\mathcal{F}_{ab}) = \bar{u}_2\left(\not{m}\gamma_5\,\widetilde{\mathcal{H}}_{ab} + \gamma_5\frac{m\cdot\Delta}{2M}\,\widetilde{\mathcal{E}}_{ab}\right)u_1\,, \qquad m^\mu = \frac{q_1^\mu + q_2^\mu}{(p_1+p_2).(q_1+q_2)}$$

parameterization of (DV)CS helicity amplitudes

$$\begin{aligned}
T_{\mu\nu} = & -\widetilde{g}_{\mu\nu}\frac{q\cdot V_{\rm T}}{p\cdot q} + i\widetilde{\varepsilon}_{\mu\nu}\frac{q\cdot A_{\rm T}}{p\cdot q} + \left(q_{2\,\mu} - \frac{q_2^2}{p\cdot q}p_\mu\right)\left(q_{1\,\nu} - \frac{q_1^2}{p\cdot q}p_\nu\right)\frac{q\cdot V_{\rm L}}{p\cdot q} \\
& + \left(q_{1\,\nu} - \frac{q_1^2}{p\cdot q}p_\nu\right)\left(g_{\mu\rho} - \frac{p_\mu\, q_{2\,\rho}}{p\cdot q}\right)\left[\frac{V^\rho_{\rm LT}}{p\cdot q} + \frac{i\epsilon^\rho{}_{qp\sigma}}{p\cdot q}\frac{A^\sigma_{\rm LT}}{p\cdot q}\right] \\
& + \left(q_{2\,\mu} - \frac{q_2^2}{p\cdot q}p_\mu\right)\left(g_{\nu\rho} - \frac{p_\nu\, q_{1\,\rho}}{p\cdot q}\right)\left[\frac{V^\rho_{\rm TL}}{p\cdot q} + \frac{i\epsilon^\rho{}_{qp\sigma}}{p\cdot q}\frac{A^\sigma_{\rm TL}}{p\cdot q}\right] \\
& + \left(g_\mu{}^\rho - \frac{p_\mu\, q_2^\rho}{p\cdot q}\right)\left(g_\nu{}^\sigma - \frac{p_\nu\, q_1^\sigma}{p\cdot q}\right)\left[\frac{\Delta_\rho\Delta_\sigma + \widetilde{\Delta}^\perp_\rho\widetilde{\Delta}^\perp_\sigma}{2M^2}\frac{q\cdot V_{\rm TT}}{p\cdot q} + \frac{\Delta_\rho\widetilde{\Delta}^\perp_\sigma + \widetilde{\Delta}^\perp_\rho\Delta_\sigma}{2M^2}\frac{q\cdot A_{\rm TT}}{p\cdot q}\right]
\end{aligned}$$

(one) parameterization of (DV)CS tensor

equivalent to *Tarrach*'s one

relations of CFFs to helicity dependent CFFs are easily calculated:

$$\begin{aligned}
\mathcal{F}_{+b} = & \left[\frac{1+b\sqrt{1+\epsilon^2}}{2\sqrt{1+\epsilon^2}} + \frac{(1-x_{\rm B})x_{\rm B}^2(4M^2-t)\left(1+\frac{t}{\mathcal{Q}^2}\right)}{\mathcal{Q}^2\sqrt{1+\epsilon^2}\left(2-x_{\rm B}+\frac{x_{\rm B}t}{\mathcal{Q}^2}\right)^2}\right]\mathcal{F}_{\rm T} \\
& + \frac{1-b\sqrt{1+\epsilon^2}}{2\sqrt{1+\epsilon^2}}\frac{\widetilde{K}^2}{M^2\left(2-x_{\rm B}+\frac{x_{\rm B}t}{\mathcal{Q}^2}\right)^2}\mathcal{F}_{\rm TT} + \frac{2x_{\rm B}\widetilde{K}^2}{\mathcal{Q}^2\sqrt{1+\epsilon^2}\left(2-x_{\rm B}+\frac{x_{\rm B}t}{\mathcal{Q}^2}\right)^2}\mathcal{F}_{\rm LT} \\
\mathcal{F}_{0+} = & \frac{\sqrt{2}\,\widetilde{K}}{\sqrt{1+\epsilon^2}\mathcal{Q}\left(2-x_{\rm B}+\frac{x_{\rm B}t}{\mathcal{Q}^2}\right)}\left\{\left[1+\frac{2x_{\rm B}^2\left(4M^2-t\right)}{\mathcal{Q}^2\left(2-x_{\rm B}+\frac{x_{\rm B}t}{\mathcal{Q}^2}\right)}\right]\mathcal{F}_{\rm LT}\right. \\
& \left. + x_{\rm B}\left[1+\frac{2x_{\rm B}(4M^2-t)}{\mathcal{Q}^2\left(2-x_{\rm B}+\frac{x_{\rm B}t}{\mathcal{Q}^2}\right)}\right]\mathcal{F}_{\rm T} + x_{\rm B}\left[2-\frac{4M^2-t}{M^2\left(2-x_{\rm B}+\frac{x_{\rm B}t}{\mathcal{Q}^2}\right)}\right]\mathcal{F}_{\rm TT}\right\}
\end{aligned}$$

usable for DVCS - RCS, extendable to timelike (D)VCS, double(D)VCS or DIS

GPD phenomenology lessons: first decade

- qualitatively GPD formalism works in DVCS (from the start up)
- first look: no serious problems in DVMP (apart from ? about very large x_B data)
  also supported by hand-bag model description of Goloskokov/Kroll
- description of present DVCS data is reached/feasible with flexible models
  for unpolarized target– but GPD understanding induces tension among data
  large unidentified contribution called Ĥ is disfavored by polarized target data
- many uncertainties: exclusivity, correction procedure, assumptions
- HERMES gave proof of principle that on can go for a complete measurement

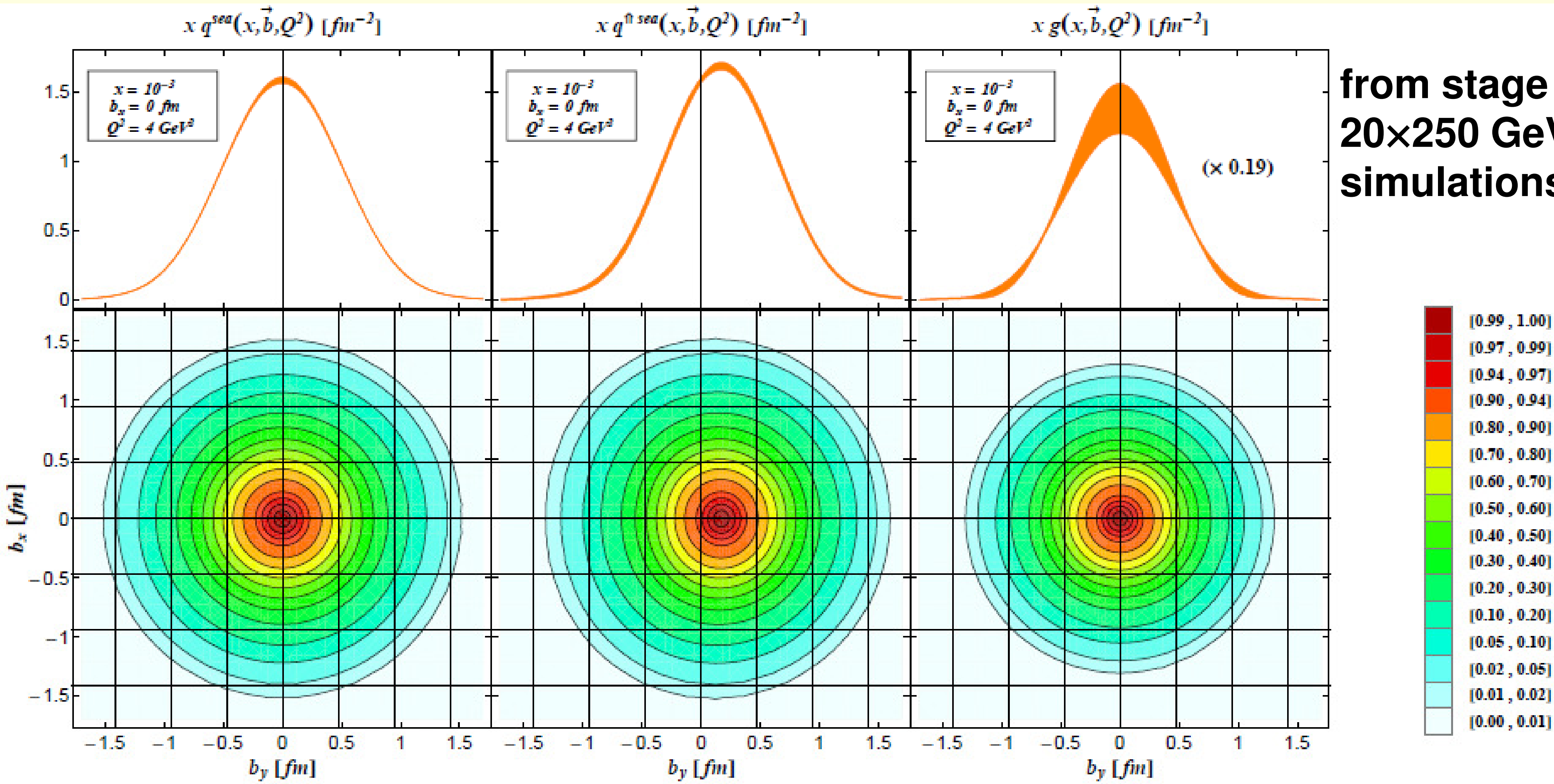
partonic interpretation:

- RDDA (GVP01,BMK01, VGG code in its many versions, GK07, ...)
  a bit disfavored at LO can not reach a χ^2/dof ~ *1...1.6* (its like χ^2/nop ~5...10)
  should work at NLO **[Freund, McDermott (02)]**
- GPD H is dominant (? 15% accuracy), tomography at small-x_B
- GPD $\hat{H}$ is constrained
- no access to GPD E from present data, pion pole model for $\hat{E}$ is disfavored
- D-term related subtraction constant comes out negative (& sizable)
  Goke et. al model prediction (perhaps fit result might be not stable)

The Future

- ✓ Compass
- ✓ JLAB@12 GeV
- ? ENC@GSI
- ? LHeC@CERN
- ? EIC@BNL or EIC@JLAB

Aschenauer, Firzo KK, DM (13)

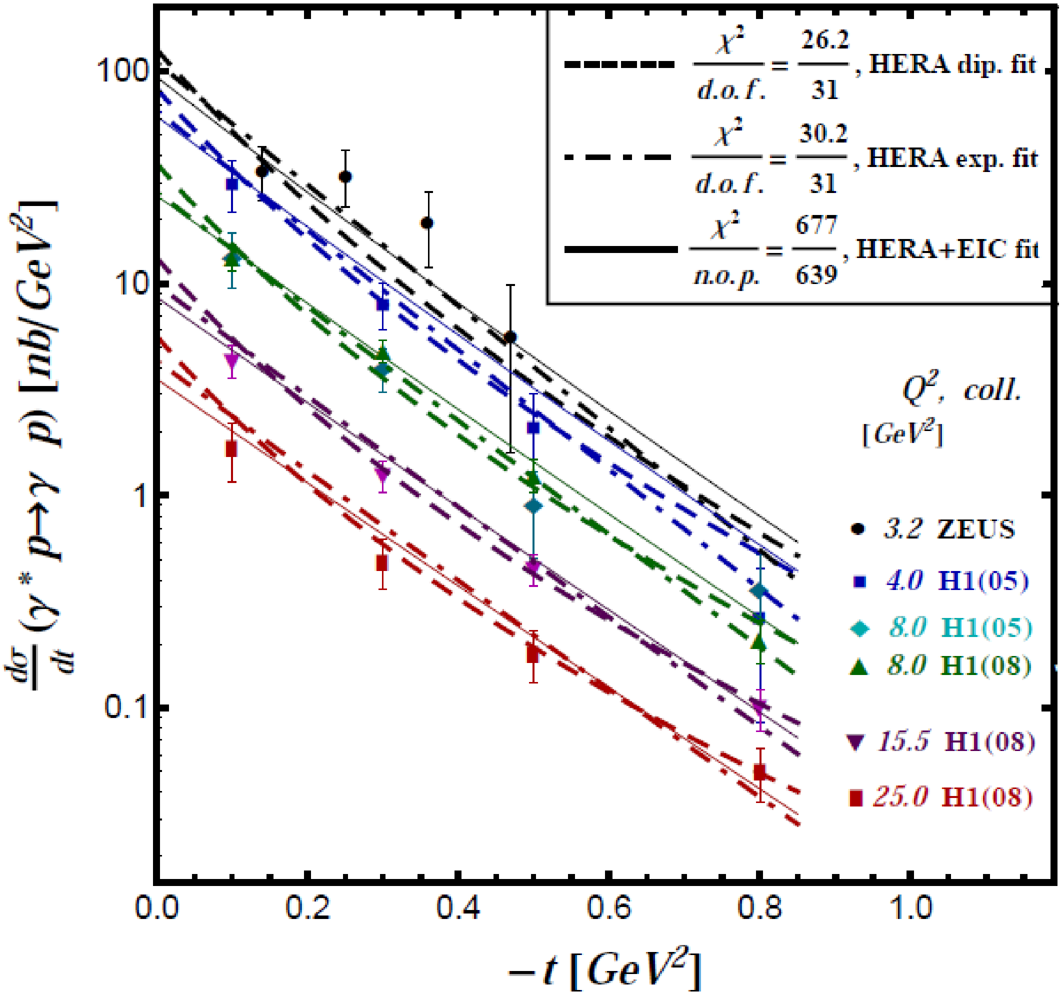


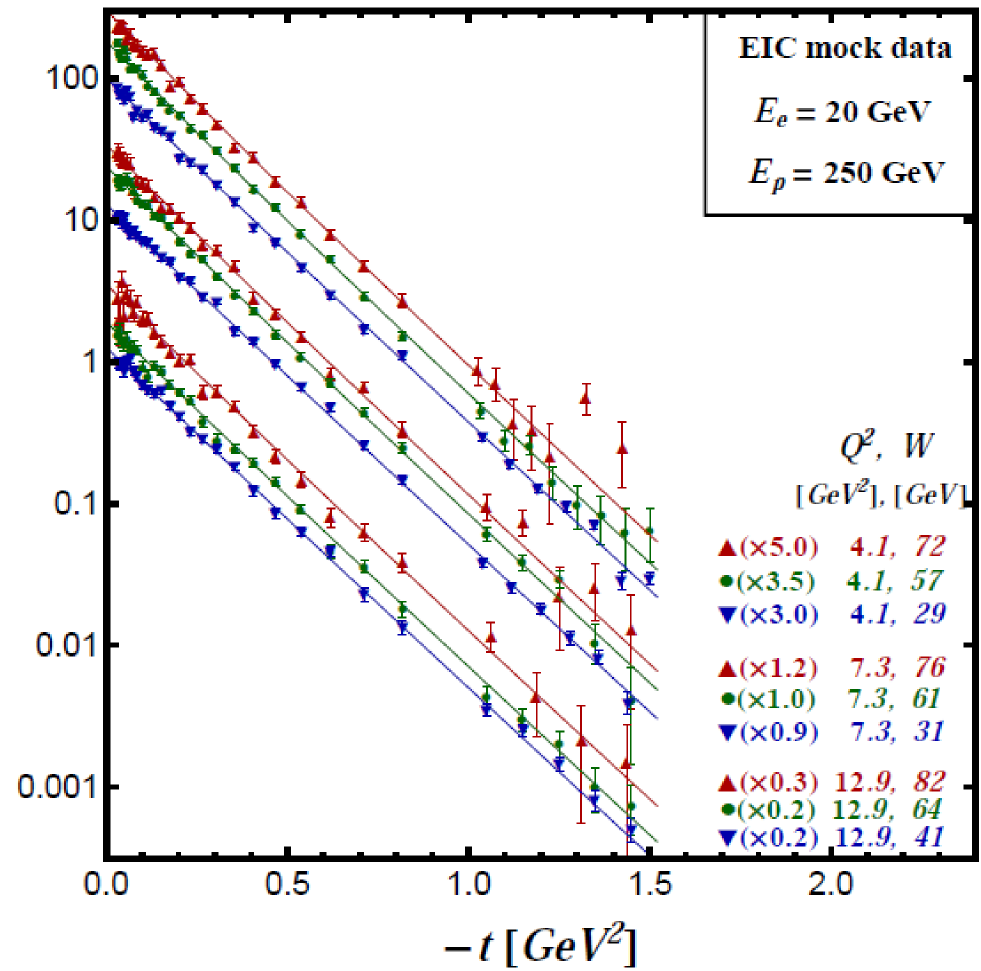
from stage II 20×250 GeV^2 simulations

Impact of EIC data to extract GPD H

two simulations from S. Fazio for DVCS cross section ~ 650 data points
-t < ~0.8 GeV^2 for ~ 10/fb
1 GeV^2 < *–t < 2* GeV^2 for ~ 100/fb (cut: *–t < 1.5* GeV^2 , *4* GeV^2 < *Q2* to ensure *–t < Q^2*)

pseudo data are re-generated with GeParD
statistical errors rescaled
5% systematical errors added in quadrature, 3% Bethe-Heitler uncertainty





Imaging (probabilistic interpretation)

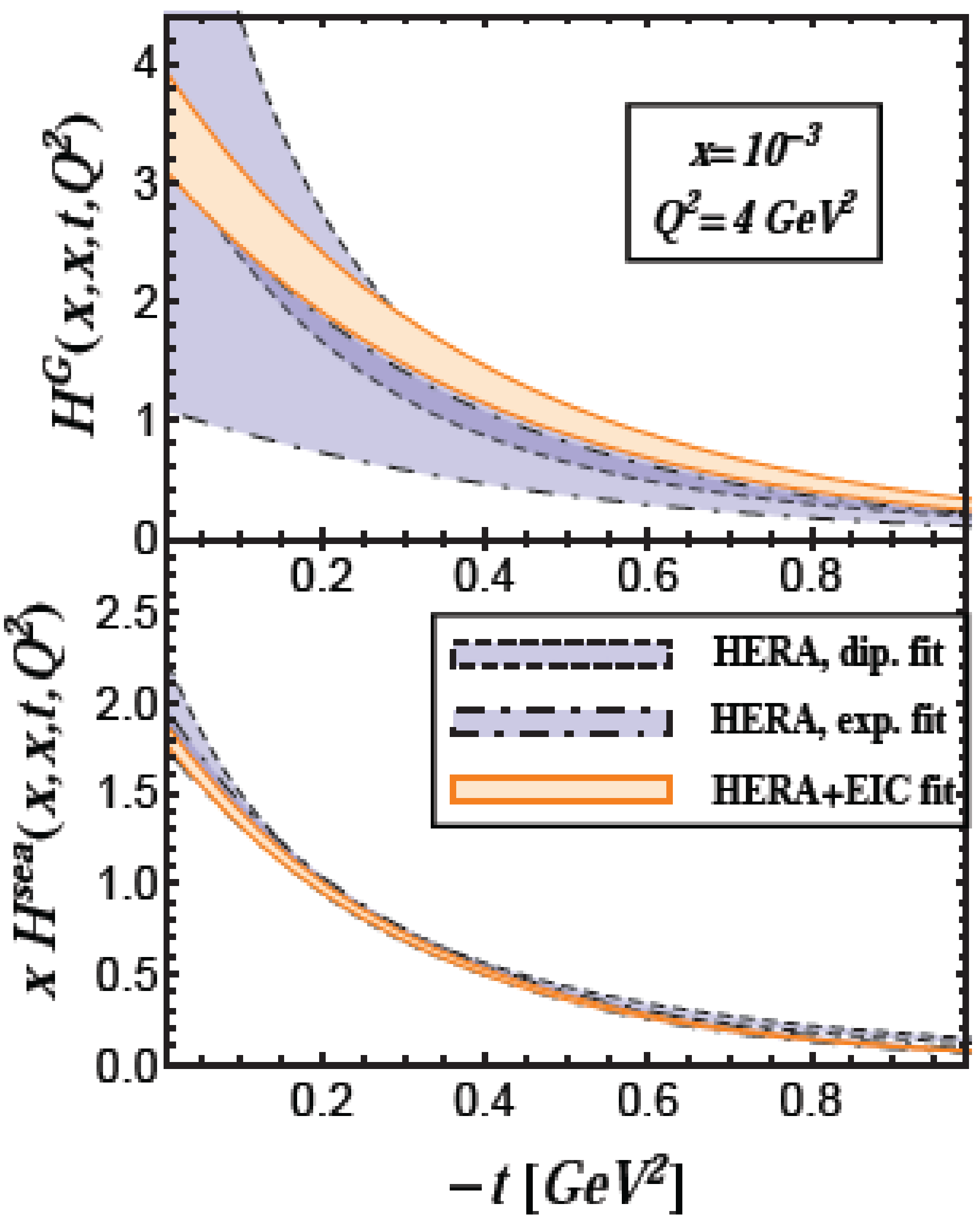
$$q(x,\vec{b},\mu^2) = \frac{1}{4\pi}\int_0^\infty d|t|\, J_0(|\vec{b}|\sqrt{|t|}) H(x,\eta=0,t,\mu^2)$$

skewness effect vanishes (s_2 , $s_4 \to 0$)

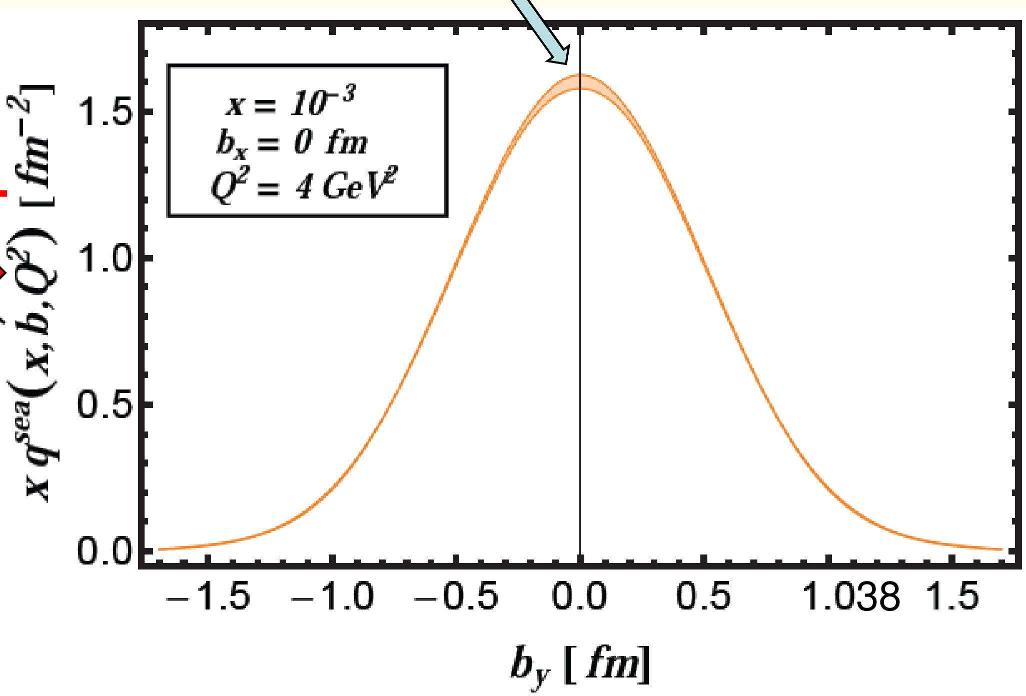
- reduce fit uncertainties
- increase model uncertainties

extrapolation errors for $-t \to 0$
(large b uncertainties – small effect)

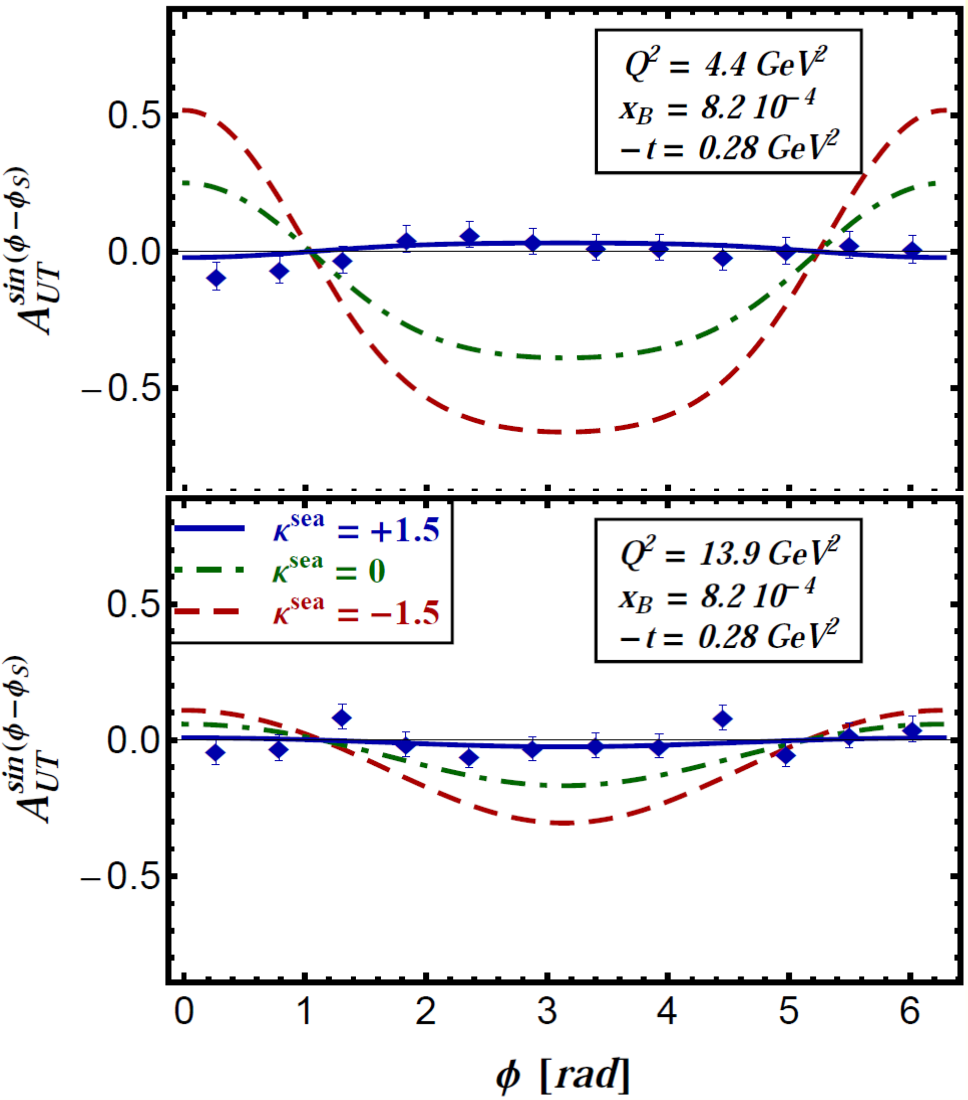
extrapolation errors into $-t > 1.5$ GeV^2
(small b uncertainties)



FT



Single transverse target spin asymmetry



20x250 2x5/fb mock data
(~1200 data points with statistical errors
+ 5% systematics at cross section level)

flexible GPD model for E^{sea} and E^G

normalization (and t-dependency) of E^{sea} is reasonable constraint

E^G is essentially unconstraint