Color Propagation in Nuclei JLAB12, EIC, RHIC, LHC

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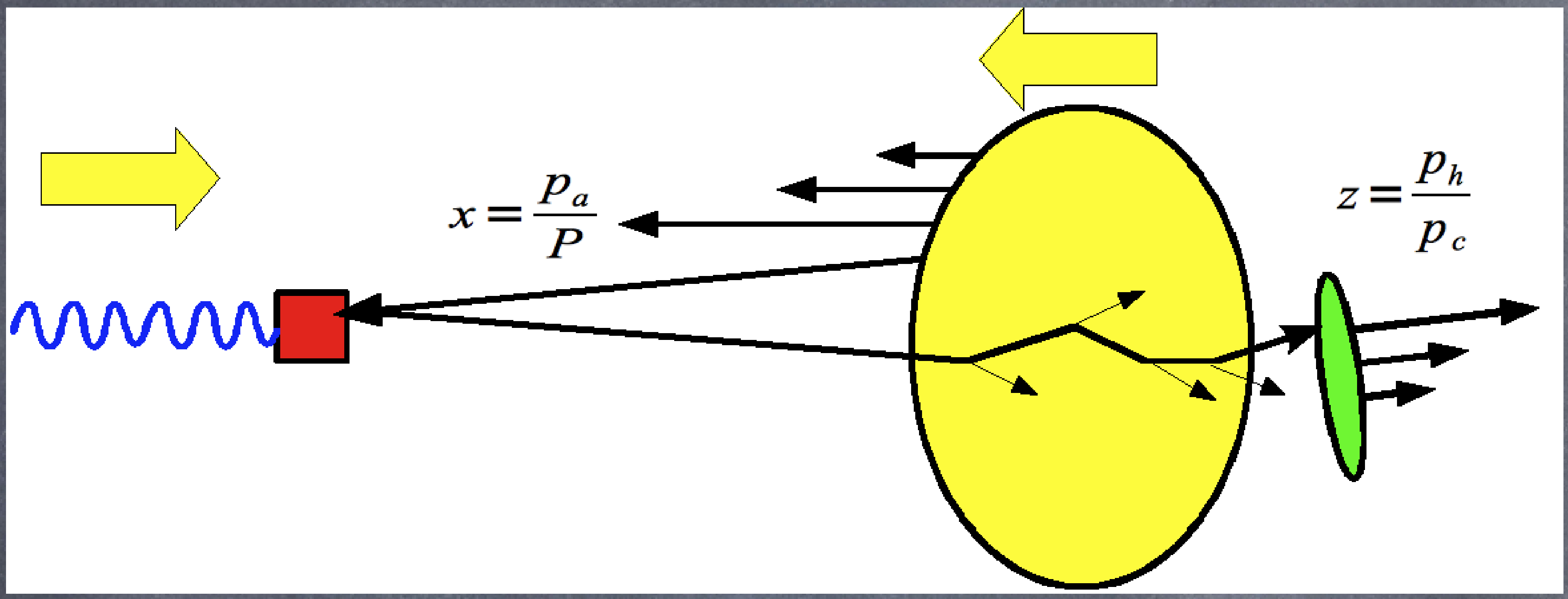
QCD Frontier workshop, JLAB Oct. 21-22, 2013

outline

- A universal formalism for Jets in DIS and Heavy-Ion collisions
- Jets as a window on high Q^2 structure of nucleon and QGP
- Change in sub-structure with T and μ
- Monte-Carlo ``improvements"
- Connections with Lattice.

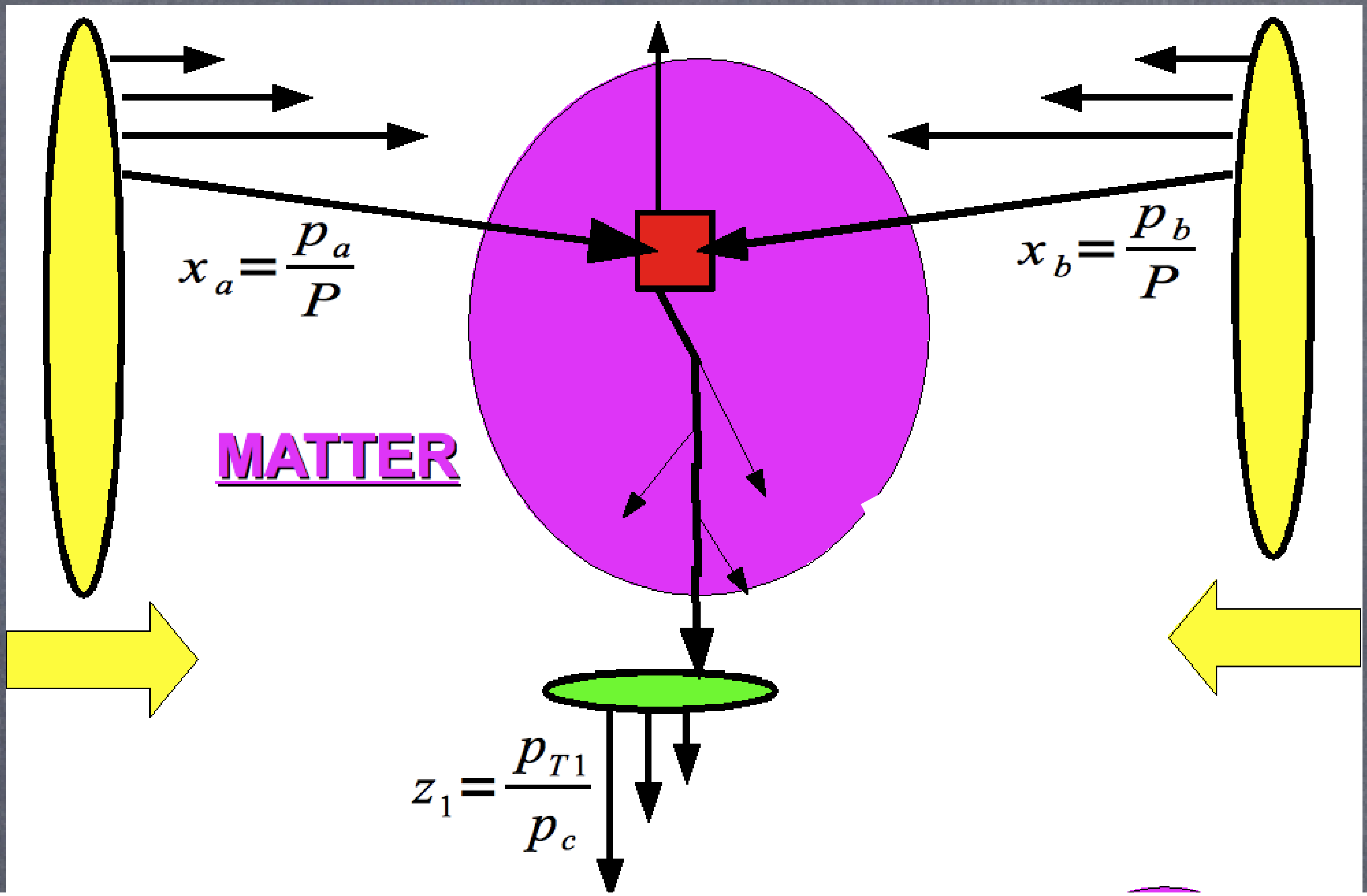
The base set-up



$$d\sigma = \int dx \; G(x) \; d\hat{\sigma} \; \tilde{J}$$

$$\tilde{J} = J_{vac} + \int dL f(\hat{q} \ldots, L, Q^2) \times J_{vac}$$

Same factorized set up in heavy-ion collisions



$$d\sigma = \int dx_a dx_b \; G(x_a) \; G(x_b) \; d\hat{\sigma} \; \tilde{J}$$

$$\tilde{J} = J_{vac} + \int dL f(\hat{q} \ldots, L, Q^2) \times J_{vac}$$

We will set up the formalism in A-DIS and then extend it to HIC

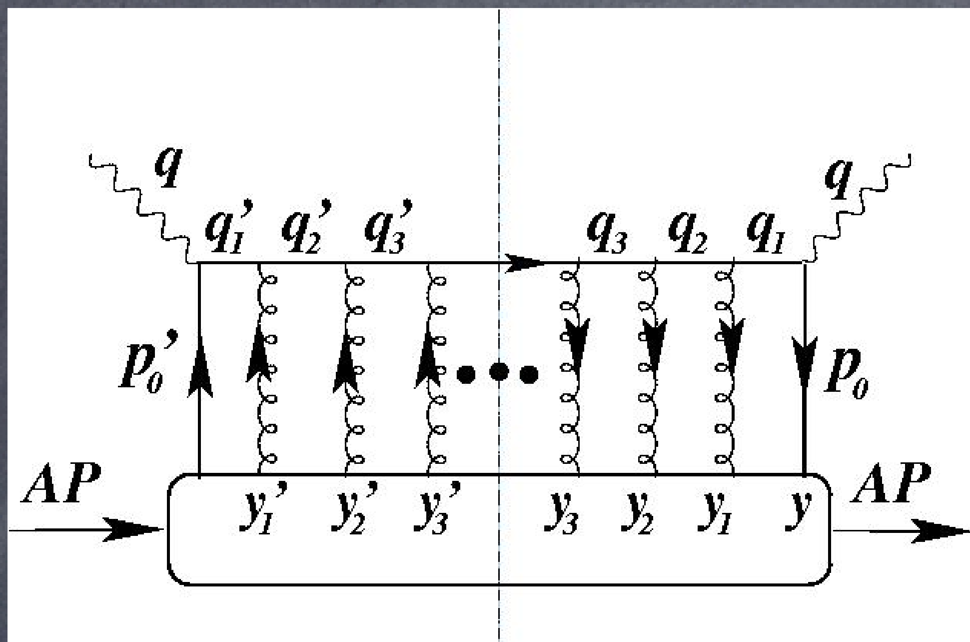
Proton $$P = (P^+, P^-, \mathbf{P}_\perp) = (\frac{Q}{x_B\sqrt{2}}, 0, \mathbf{0})$$

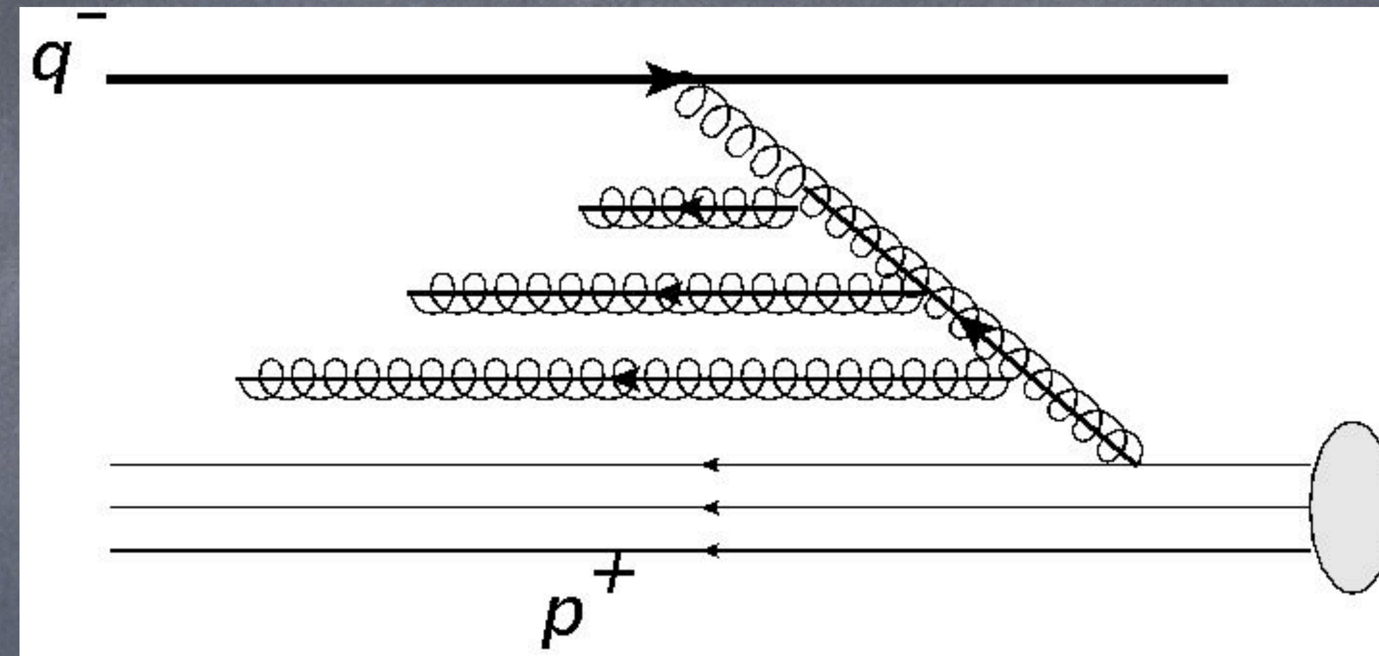
in coming quark $$p \equiv (p^+, p^-, p_\perp) = (\frac{Q}{\sqrt{2}}, \frac{k_\perp^2}{2p^+}, \mathbf{k}_\perp)$$

photon $$q \equiv (q^+, q^-, \mathbf{0}) = (\frac{Q}{\sqrt{2}}, -\frac{Q}{\sqrt{2}}, \mathbf{0})$$

outgoing quark $$p_f = (0, q^-, \mathbf{k}_\perp)$$

How is a single hard parton modified





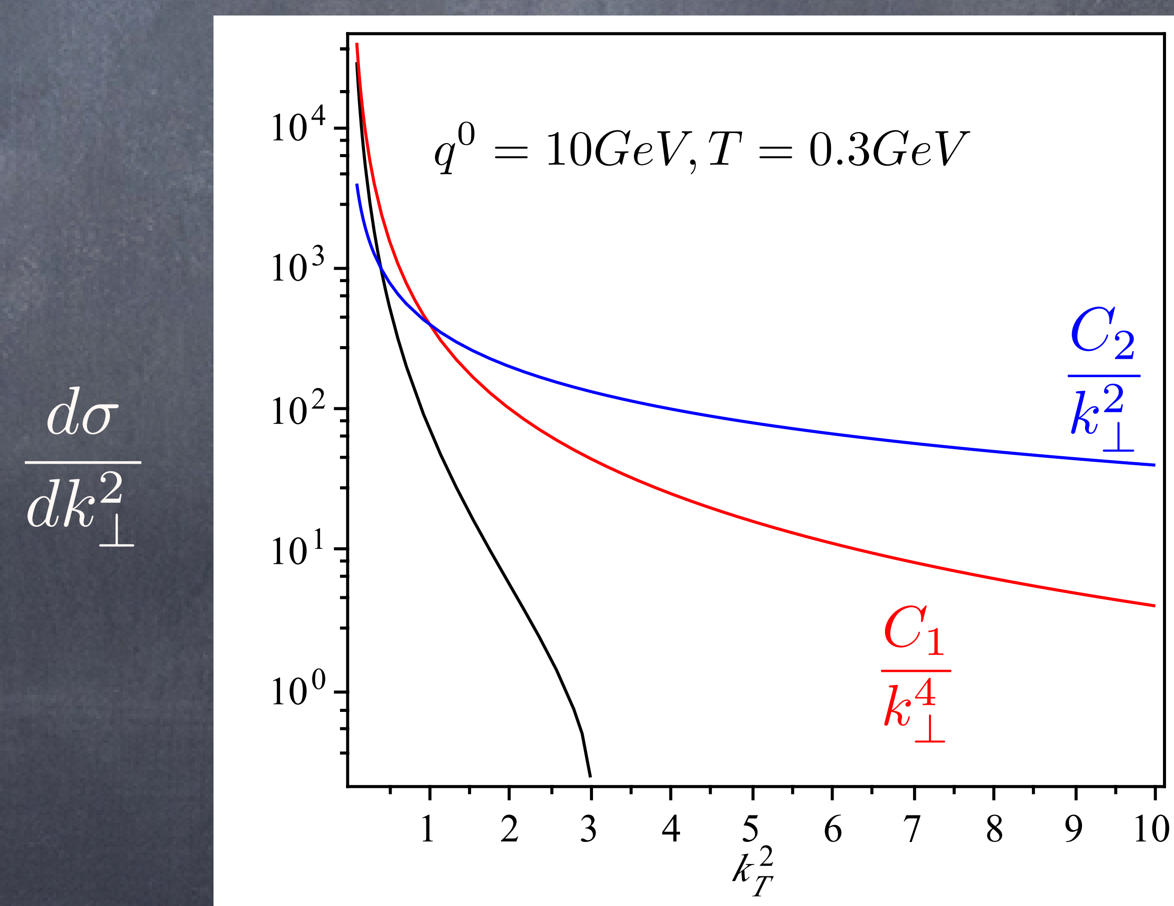
Struck quark has,

energy ~ Q and
virtuality ~ λQ

hence, gluons have

$$k_\perp \sim \lambda Q, \qquad k^+ \sim \lambda^2 Q$$

could also have $k^- \sim \lambda Q$

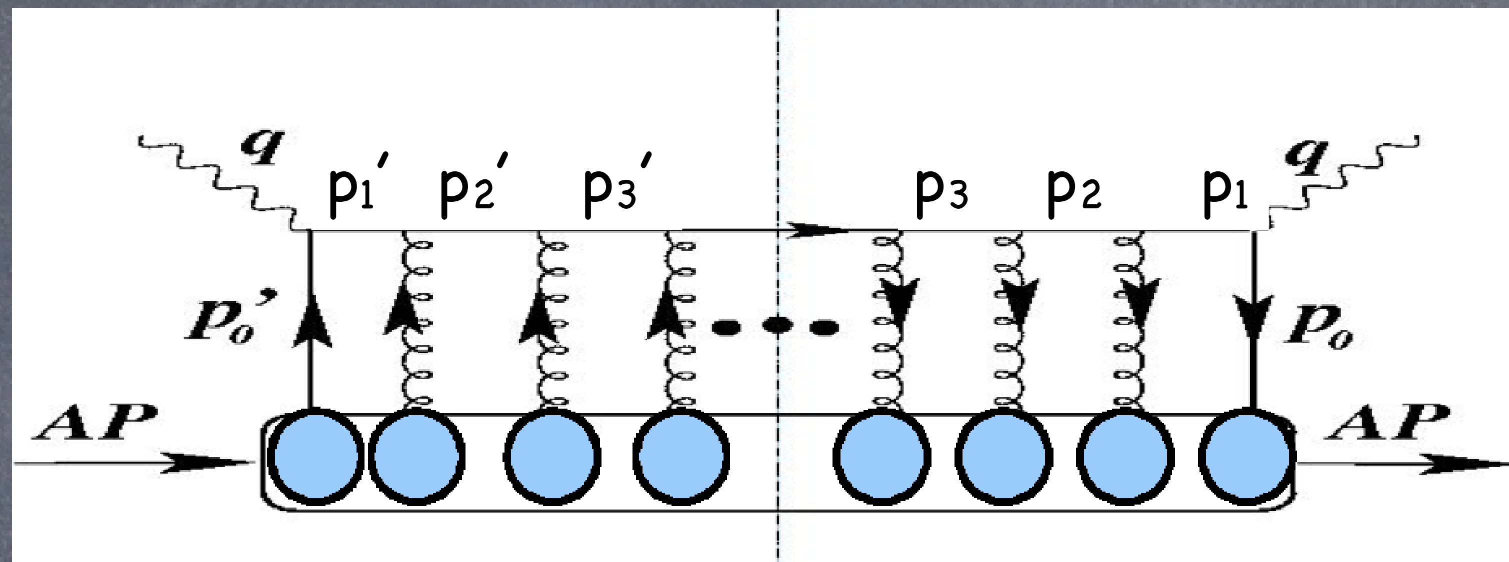


Calculate in negative light-cone gauge $A^- = 0$

So what do we get from resumming ?

a) transverse broadening

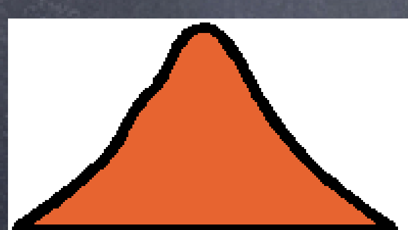
$$p^+ = \frac{p^0 + p_z}{\sqrt{2}}$$



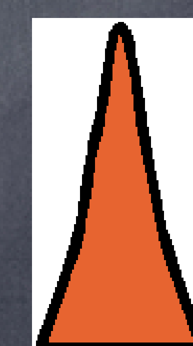
$$p^- = \frac{p^0 - p_z}{\sqrt{2}}$$

Assuming independent scattering of nucleons gives a diff. equation

These cannot be soft, they must have transverse momentum, Glauber gluons.



$$\frac{\partial f(p_\perp, t)}{\partial t} = \nabla_{p_\perp} \cdot D \cdot \nabla_{p_\perp} f(p_\perp, t)$$



$$\langle p_\perp^2 \rangle = 4Dt$$

$$\hat{q} = \frac{p_\perp^2}{t} = \frac{2\pi^2 \alpha_s C_R}{N_c^2 - 1} \int d\tilde{t} \left\langle F^{\mu\alpha}(\tilde{t}) v_\alpha F_\mu^{\beta}(0) v_\beta \right\rangle$$

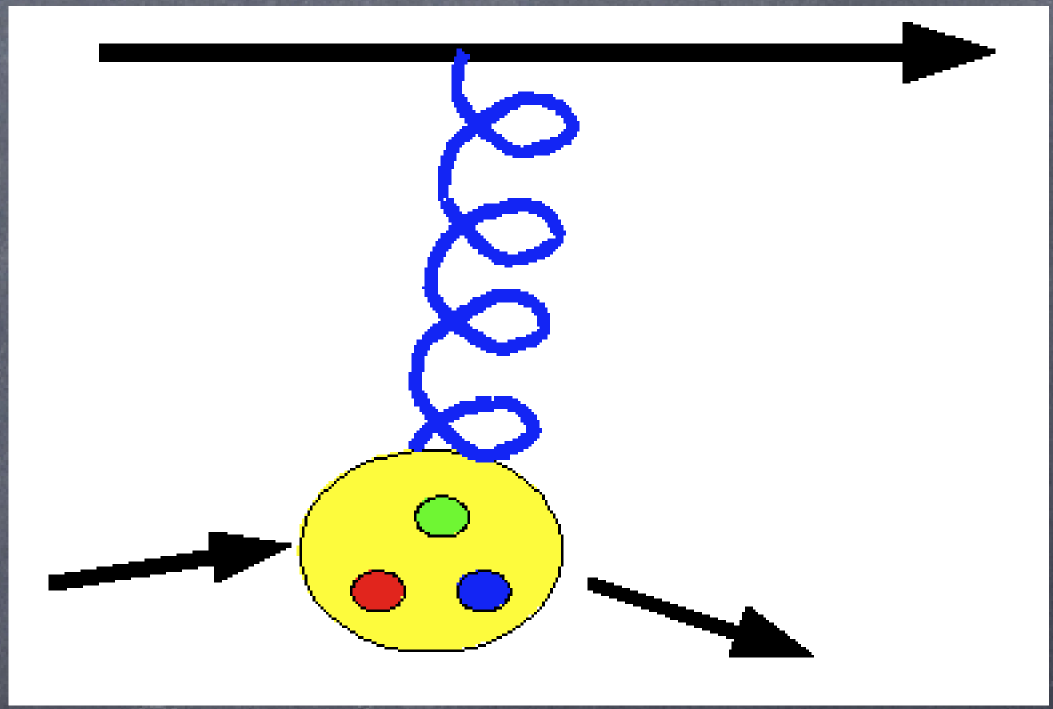
A. Majumder and B. Muller, Phys. Rev. C 77, 054903 (2008)

b) Longitudinal drag and diffusion

A close to on shell parton has a 3-D distribution

$$p^{+} = \frac{p_{\perp}^{2}}{2p^{-}}$$

$$f(\vec{p}) \equiv \delta^{2}(p_{\perp}^{2})\delta(p^{-} - q^{-} + k^{-})$$



Using the same analysis, we get a drag. and diff. term

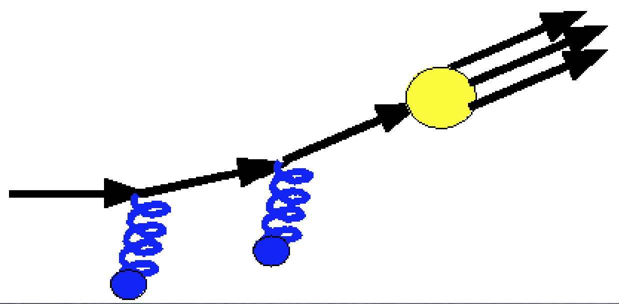
$$\frac{\partial f(p^{-}, L^{-})}{\partial L^{-}} = c_{1}\frac{\partial f}{\partial p^{-}} + c_{2}\frac{\partial^{2} f}{\partial^{2} l^{-}}$$

c_1 is dE/dL,

A. Majumder, Phys. Rev. C 80, 031902 (2009)

There are a bunch of medium properties which modify the parton and frag. func.

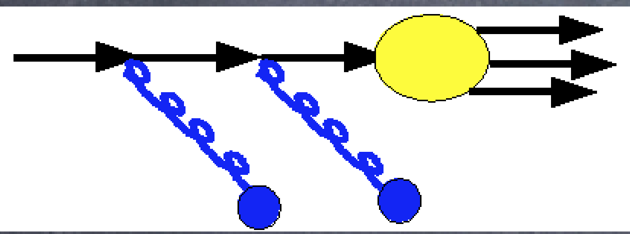
$\hat{q}$, $\hat{e}$ = dE/dL and $\hat{f}$ = dN/dL



$$D\left(\frac{\vec{p}_h}{|\vec{p}+\vec{k}_\perp|}, m_J^2\right)$$

$$\hat{q} = \frac{\langle p_T^2 \rangle_L}{L}$$

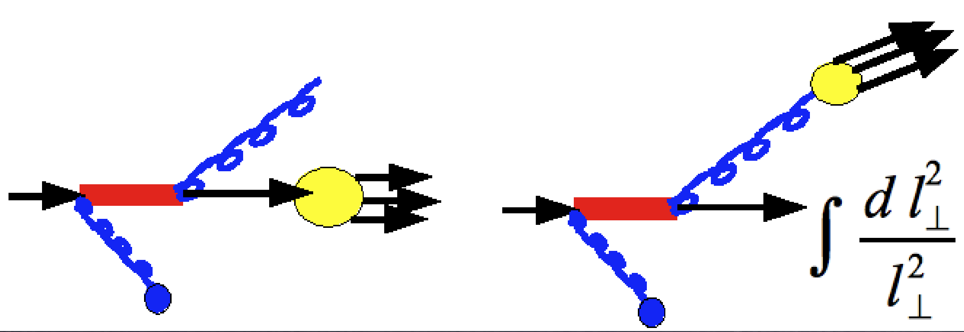
Transverse momemtum diffusion rate



$$D\left(\frac{p_h}{p-k}, m_J^2\right)$$

$$\hat{e} = \frac{\langle \Delta E \rangle_L}{L}$$

Elastic energy loss rate also diffusion rate e_2

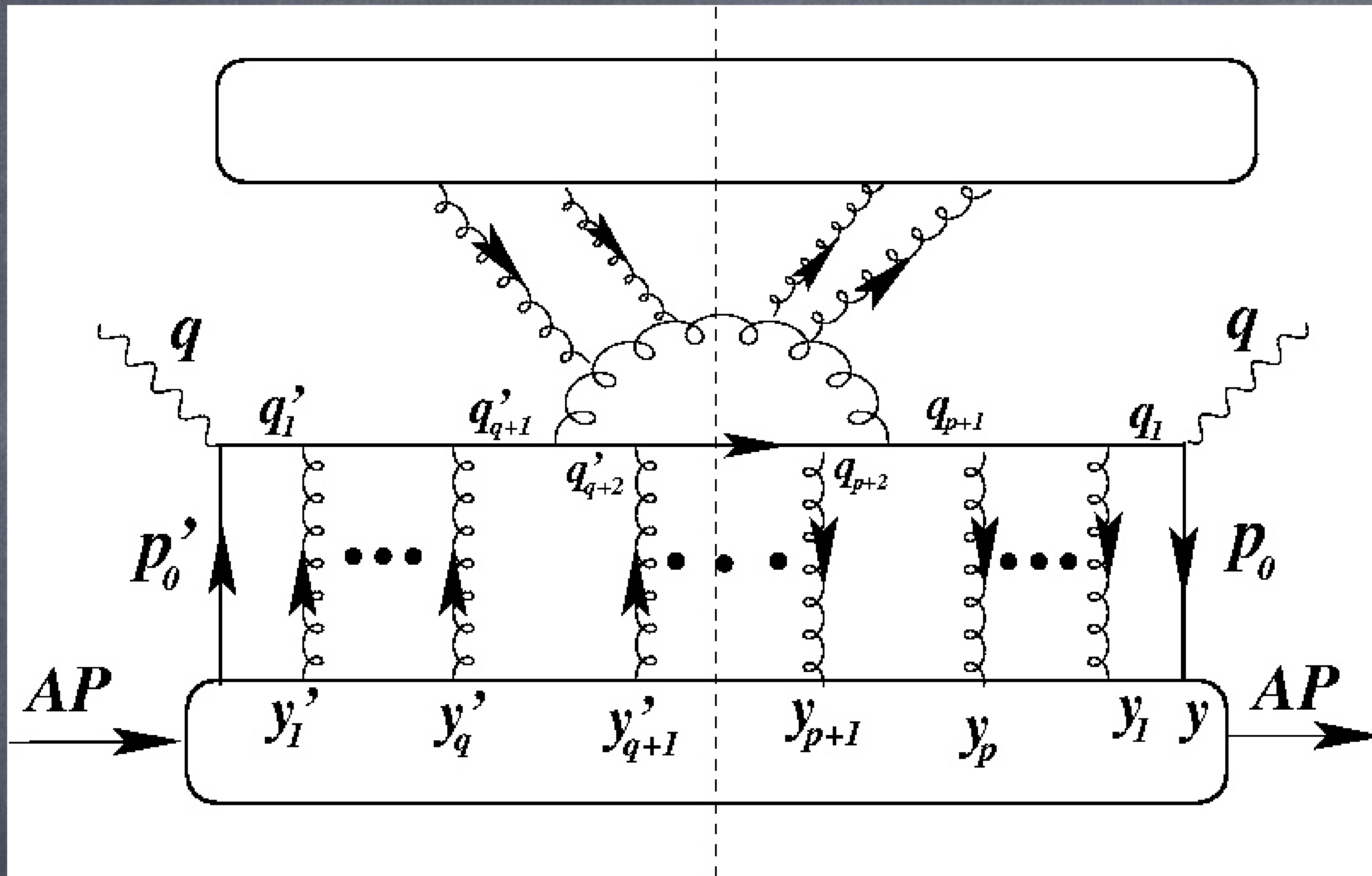


$$\int \frac{d\,l_\perp^2}{l_\perp^2} \int_{\frac{p_h}{p}}^{1} \frac{d\,y}{y} P(y)\, M(\vec{r}, y, l_\perp)\, D\left(\frac{p_h}{p\,y}\right)$$

Gluon radiation is sensitive to all these transport coefficients

And a bunch of off diagonal and higher order transport coefficients

The single gluon emission kernel

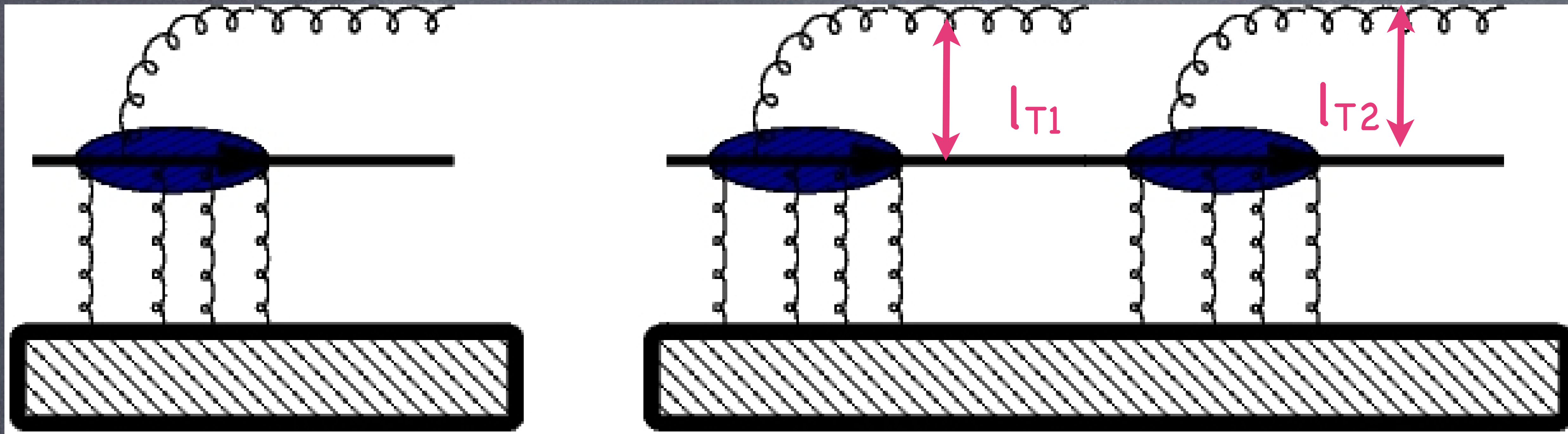


Calculate 1 gluon emission with quark & gluon N-scattering with transverse broadening and elastic loss built in

Finally solved analytically, in large Q^2 limit.

A. Majumder Phys. Rev. D 85, 014023 (2012)

Need to repeat the kernel



What is the relation between subsequent radiations ?

In the large Q^2 we can argue that there should be ordering of l_T. $if\ \hat{q}L < Q^2$

$$\text{then} \quad \frac{dQ^2}{Q^2}\left[1 + c_1\frac{\hat{q}L}{Q^2}\right] \leq \frac{dQ^2}{Q^2}[1 + c_1]$$

However, at lower Q^2, possible anti-ordering

Coherence effects and broadening in medium-induced QCD radiation off a massive q q antenna

Néstor Armesto, Hao Ma, Yacine Mehtar-Tani, Carlos A. Salgado, Konrad Tywoniuk

JHEP 1201 (2012) 109

Analytical calculations always have approximations

$$\frac{\partial D_q^h(z,\mu^2)}{\partial \log(\mu^2)} = \frac{\alpha_s}{2\pi}\int_z^1 \frac{dy}{y} P_{q\to i}(y) D_i^h\left(\frac{z}{y},\mu^2\right)$$

Thus you need a grid in z, q⁻ , and ζ

+

$$\frac{\partial D_q^{h^2}(z,M^2,q^-)|_{\zeta_i}^{\zeta_f}}{\partial \log(M^2)} = \frac{\alpha_s}{2\pi}\int_z^1 \frac{dy}{y}\frac{\tilde{P}_{q\to i}(y)}{M^2}\int_{\zeta_i}^{\zeta_f} d\zeta \frac{2\pi\alpha_s}{N_c} \times \rho_g(\zeta)\left[2 - 2\cos\left\{\frac{M^2(\zeta-\zeta_i)}{2q^- y(1-y)}\right\}\right] \times D_q^{h^1}\left(\frac{z}{y}, M^2, q^- y\right)\Big|_{\zeta}^{\zeta_f}$$

Really hard numerically, so far grid in z, q^-, and in z,ζ

To go beyond this would require a MC Evt. Gen.

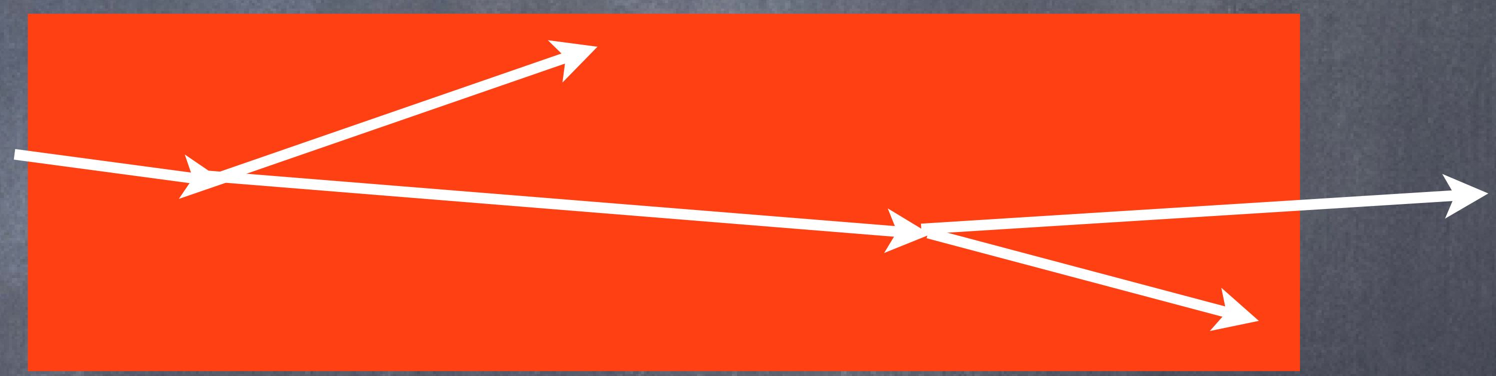
Majumder 2009

A DGLAP formalism requires an upper scale and a lower scale

Upper scale is p_T^2 , same as in vacuum

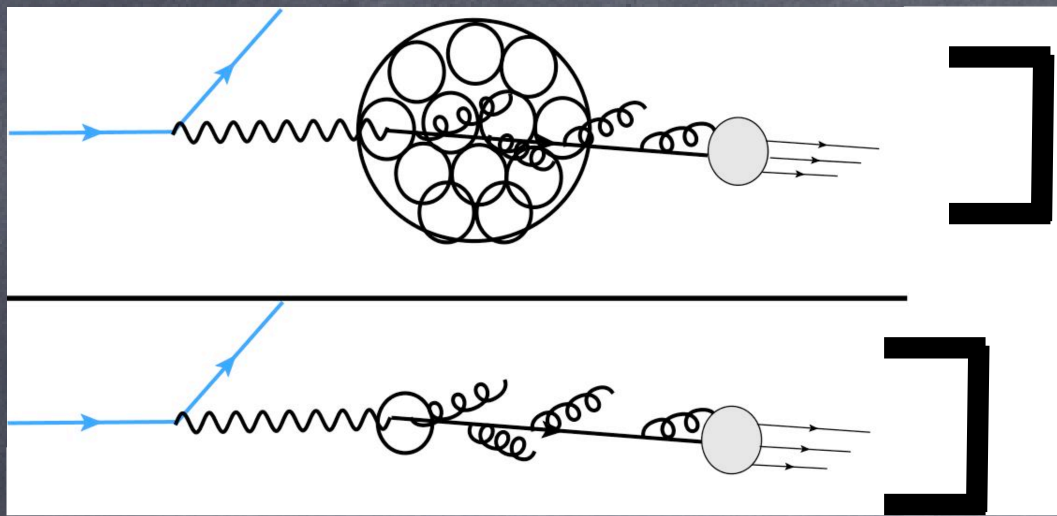
Lower scale: virtuality of parton on exit

Natural choice
$Q^2_{min} = E/L$



Realistically, should be done for each path
In reality: average kernel over many paths
and calculate a mean distance based on the maximum length that the jet can travel in the representative brick

Can explain suppressed yield of hadrons in DIS



$$\left[\;\right] \Big/ \left[\;\right] = \frac{N_A^h}{A \times N_p^h}$$

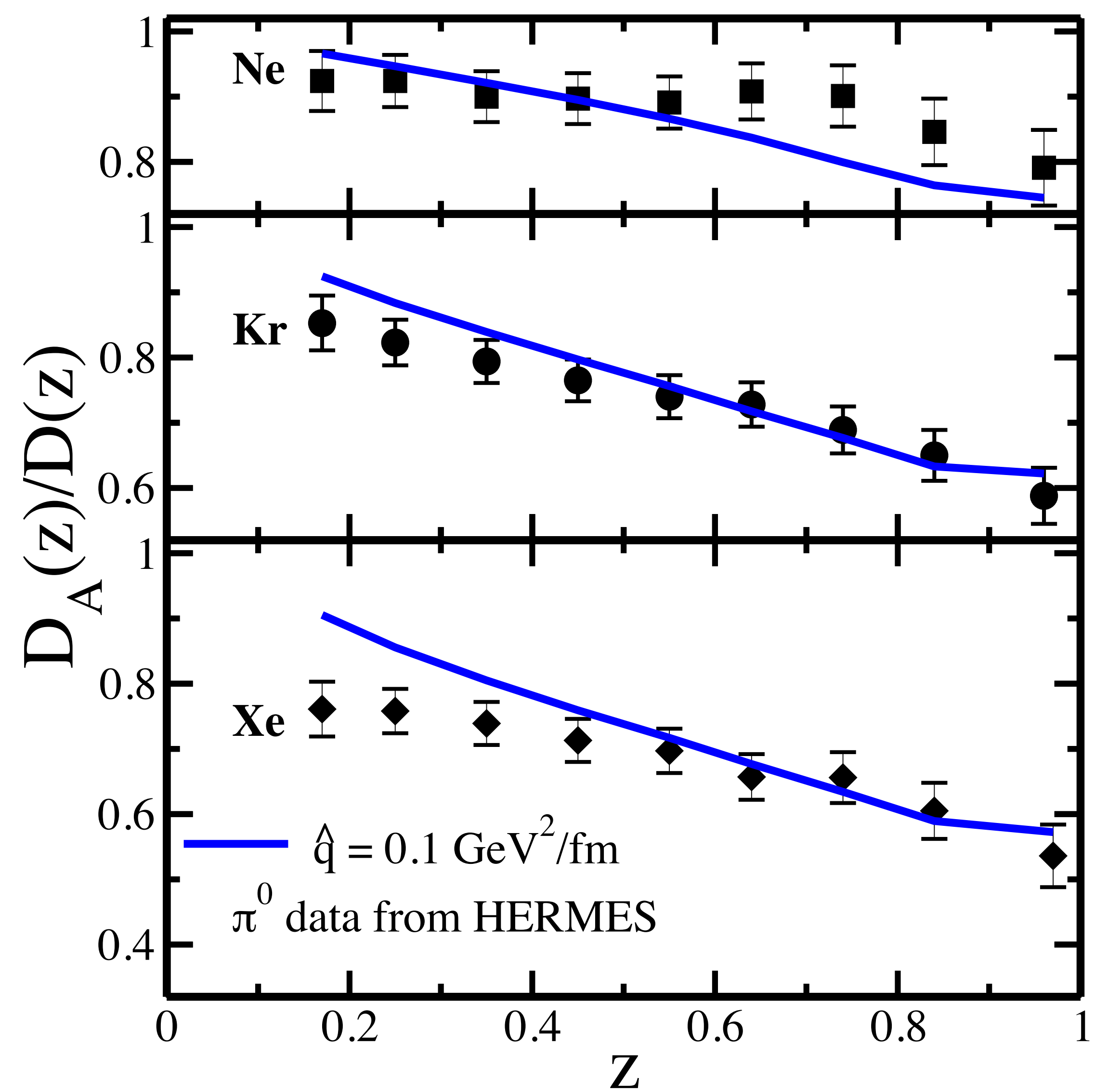
Data from HERMES at DESY

Three different nuclei

one $\hat{q}$ = 0.1GeV²/fm

Fit one data point in Ne
everything else is prediction

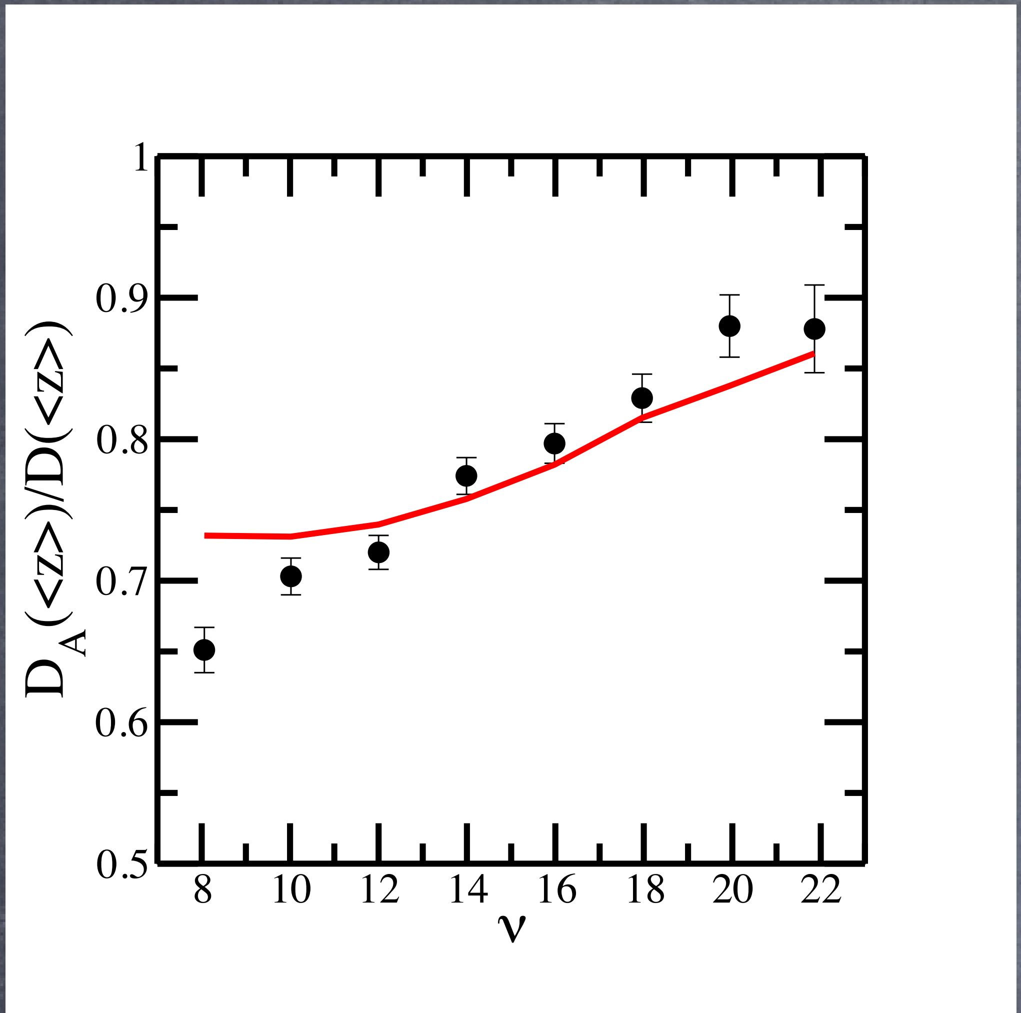
$Q^2 = 3\text{GeV}^2$, ν = 16-20 GeV

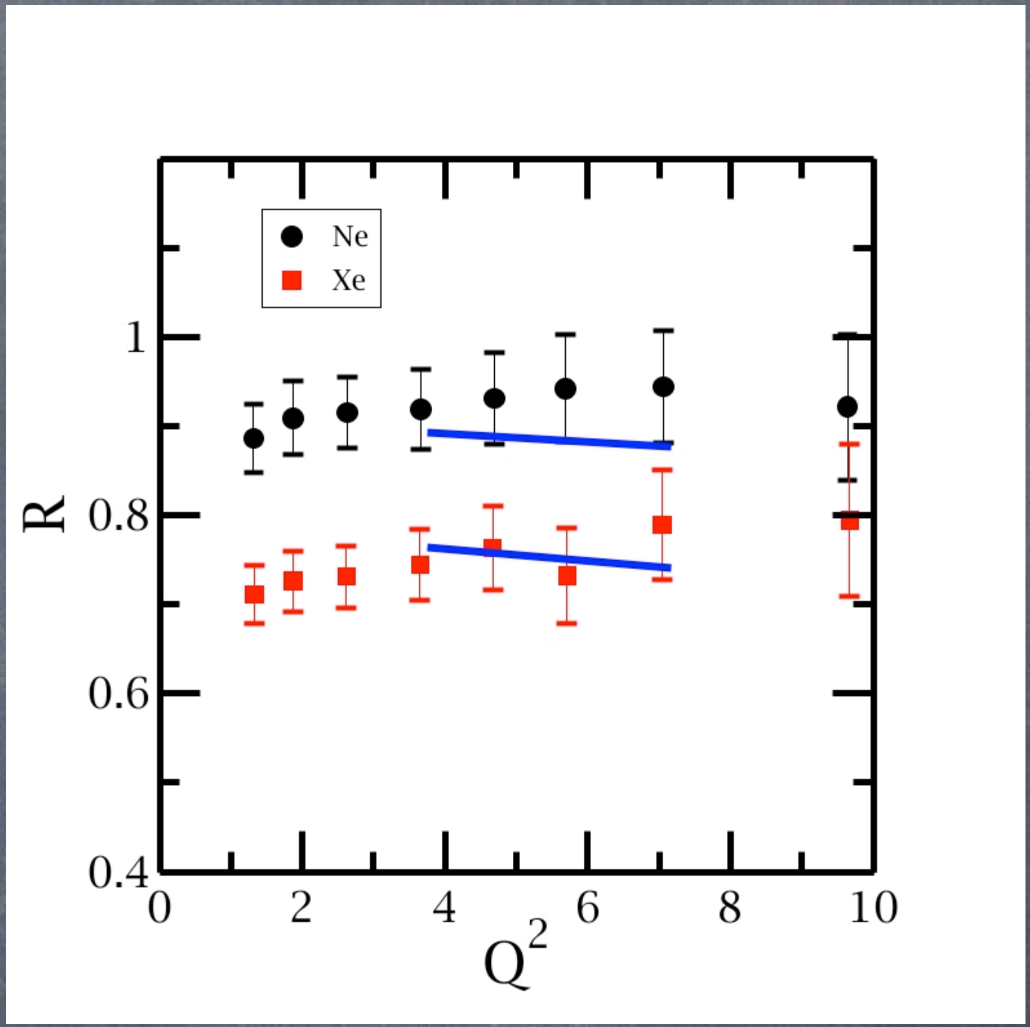


$z = E_h/E_\gamma$

A. Majumder
2009

The ν and Q^2 dependence





Many approximations made!

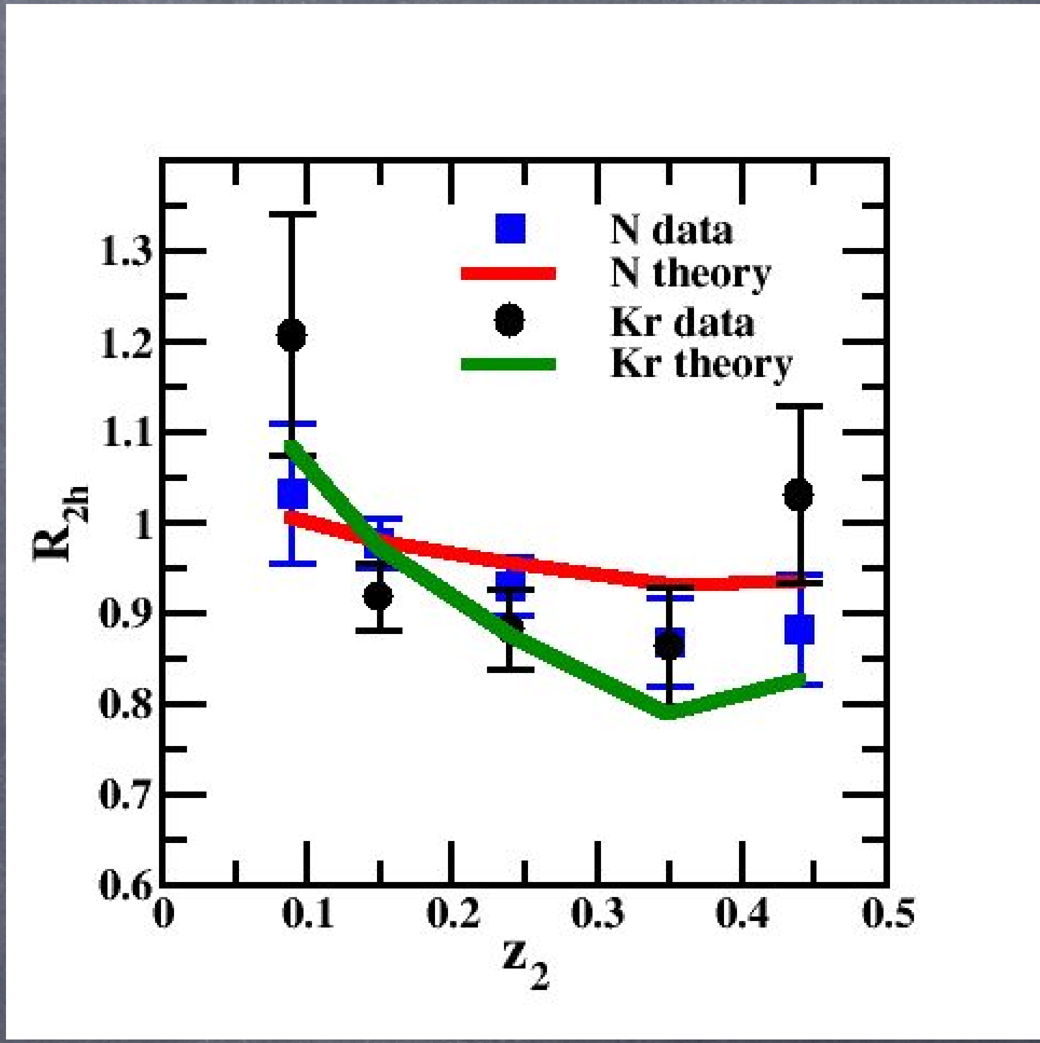
$$\tilde{D}(z, Q^2, \nu)\Big|_{\zeta}^{\zeta_f} \rightarrow \tilde{D}(z, Q^2, \nu)\Big|_{\zeta_i}^{\zeta_f}$$

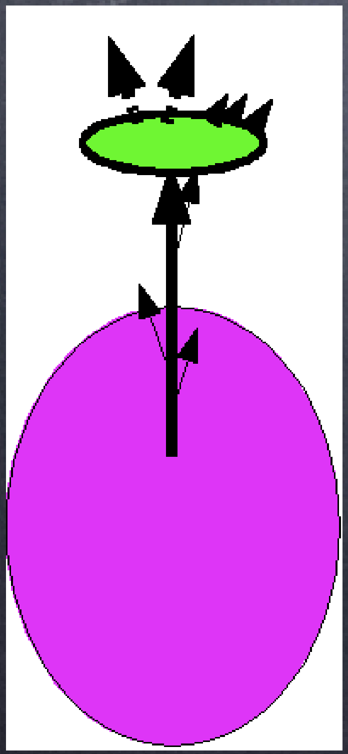
Dihadrons, yet another test of the formalism

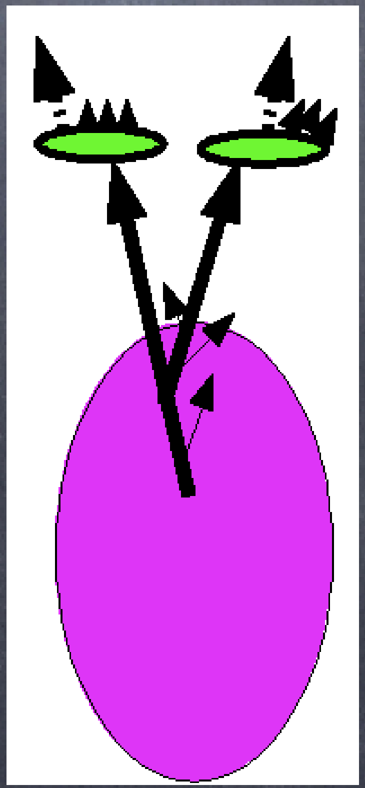
Works in DIS with no additional parameters

Works in HIC with no additional parameters

Requires the same non-pert. input a dihadron fragmentation func.





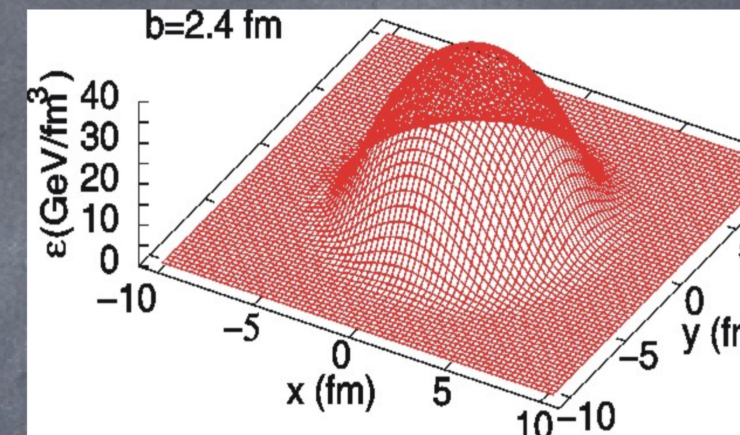


A. Majumder, E. Wang and X.-N. Wang, Phys. Rev. Lett. 99, 152301 (2007)

Medium in HIC described by viscous fluid dynamics

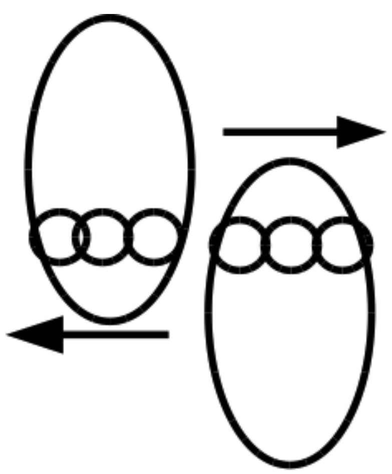
Medium evolves hydro-dynamically as the jet moves through it

Fit the q for the initial T in the hydro in central coll.

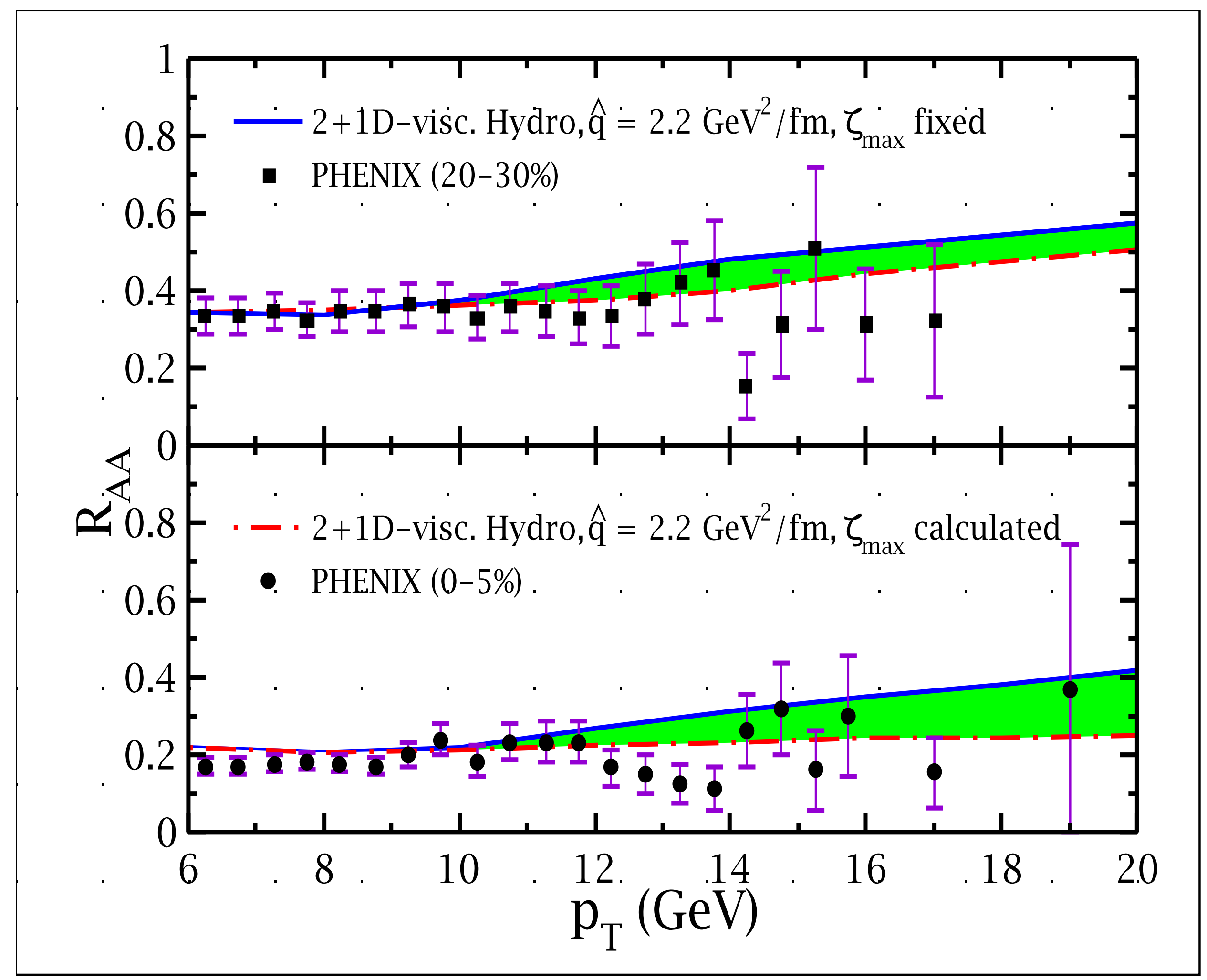


$$\hat{q}(\vec{r},t) = \hat{q}_0 \frac{s(\vec{r},t)}{s_0}$$

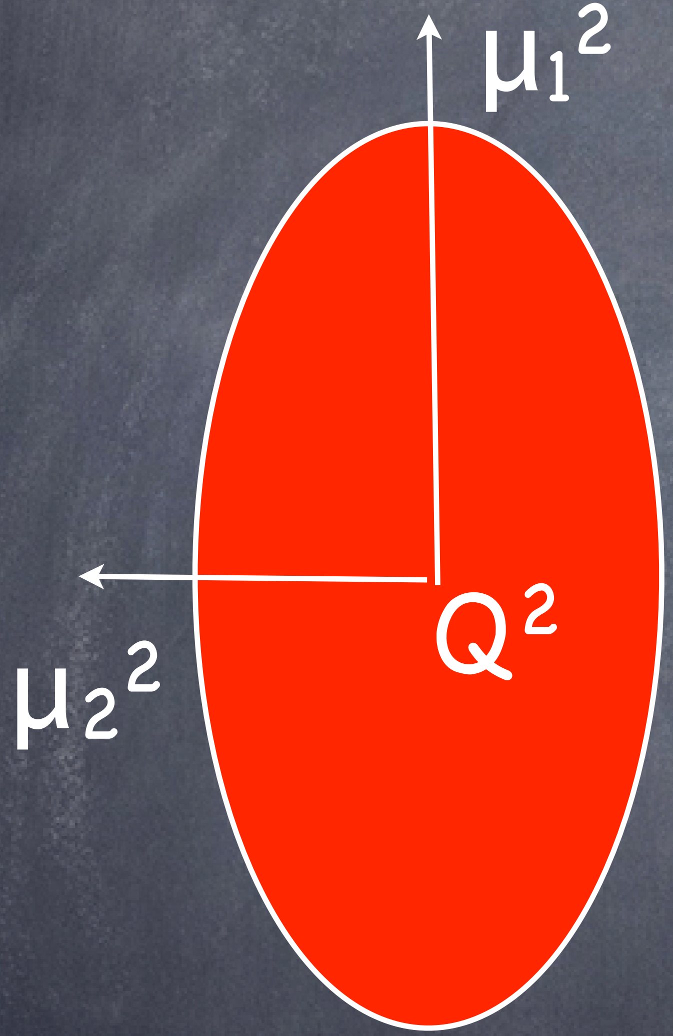
$$s_0 = s(T_0)$$

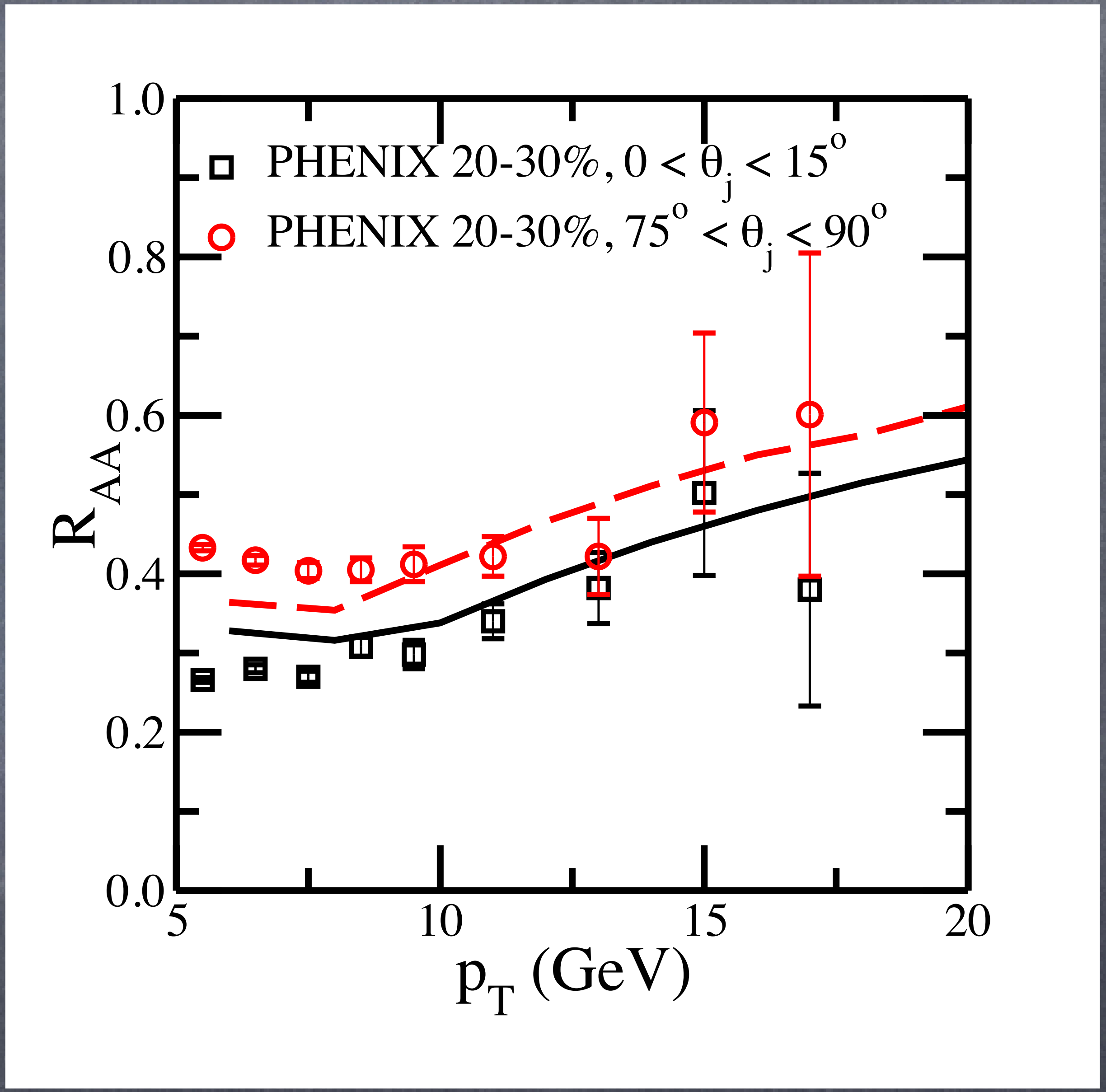


$$R_{AA} \sim \frac{\frac{dN_{AA}}{dp_T dy}}{N_{bin} \frac{dN_{pp}}{dp_T dy}}$$



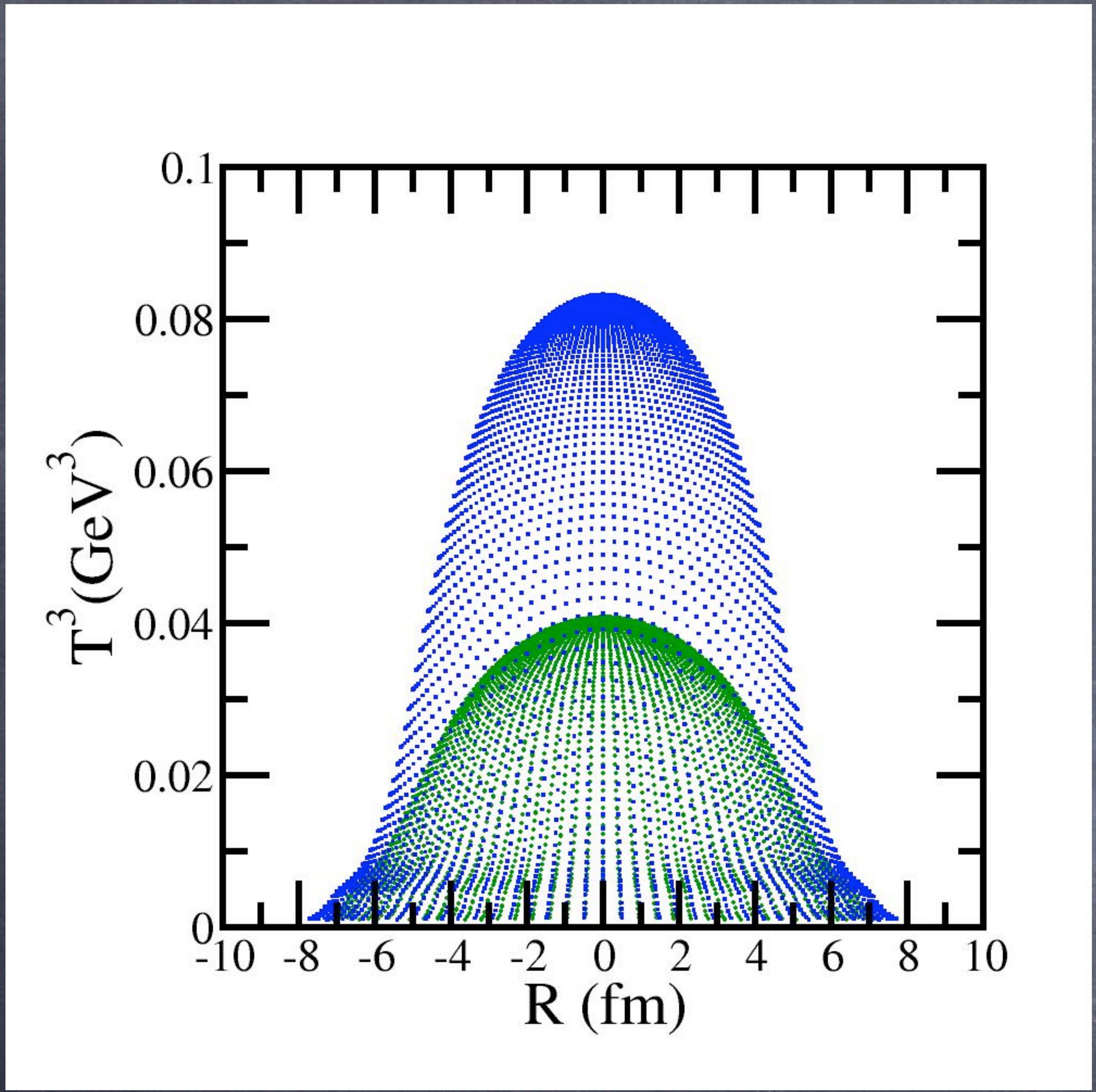
versus reaction plane

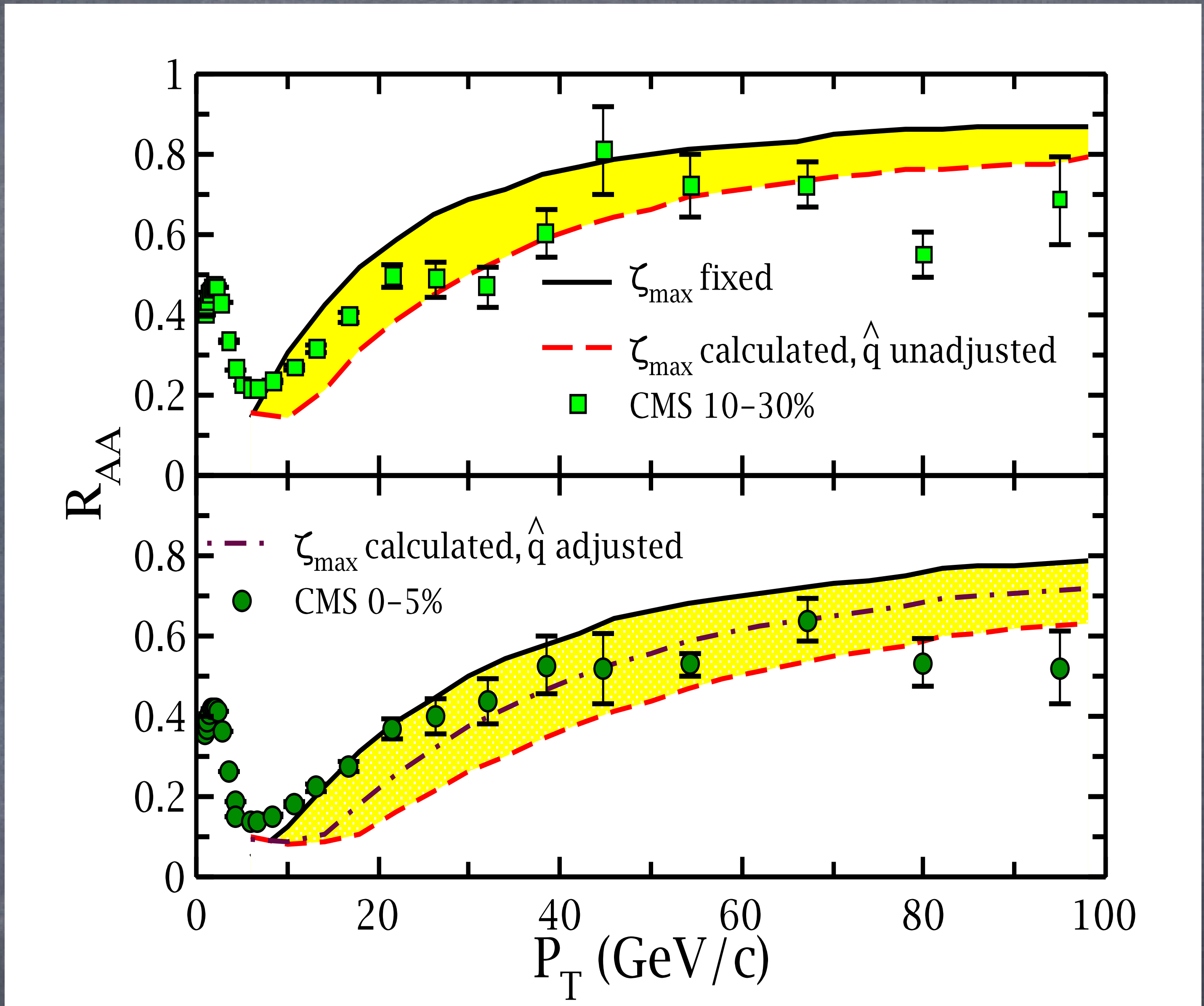




A. Majumder and C. Shen, Phys. Rev. Lett. 109, 202301 (2012)

Versus reaction plane, versus energy



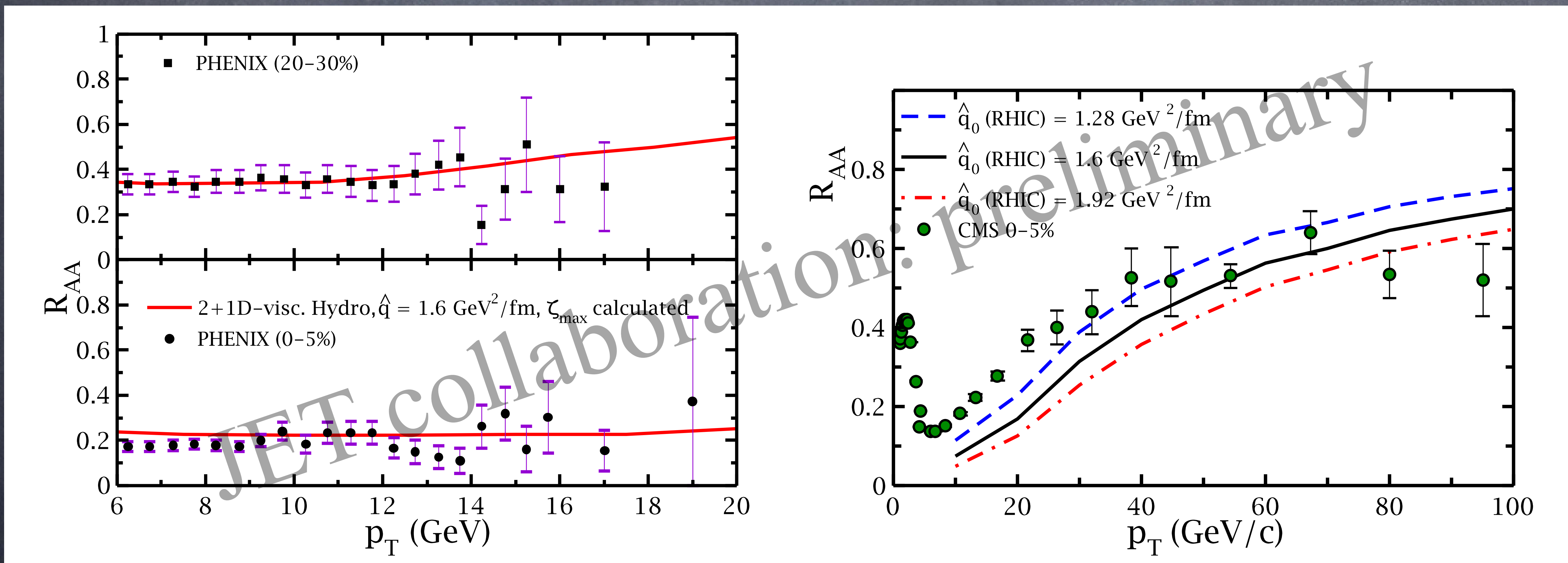


Reasonable agreement with data

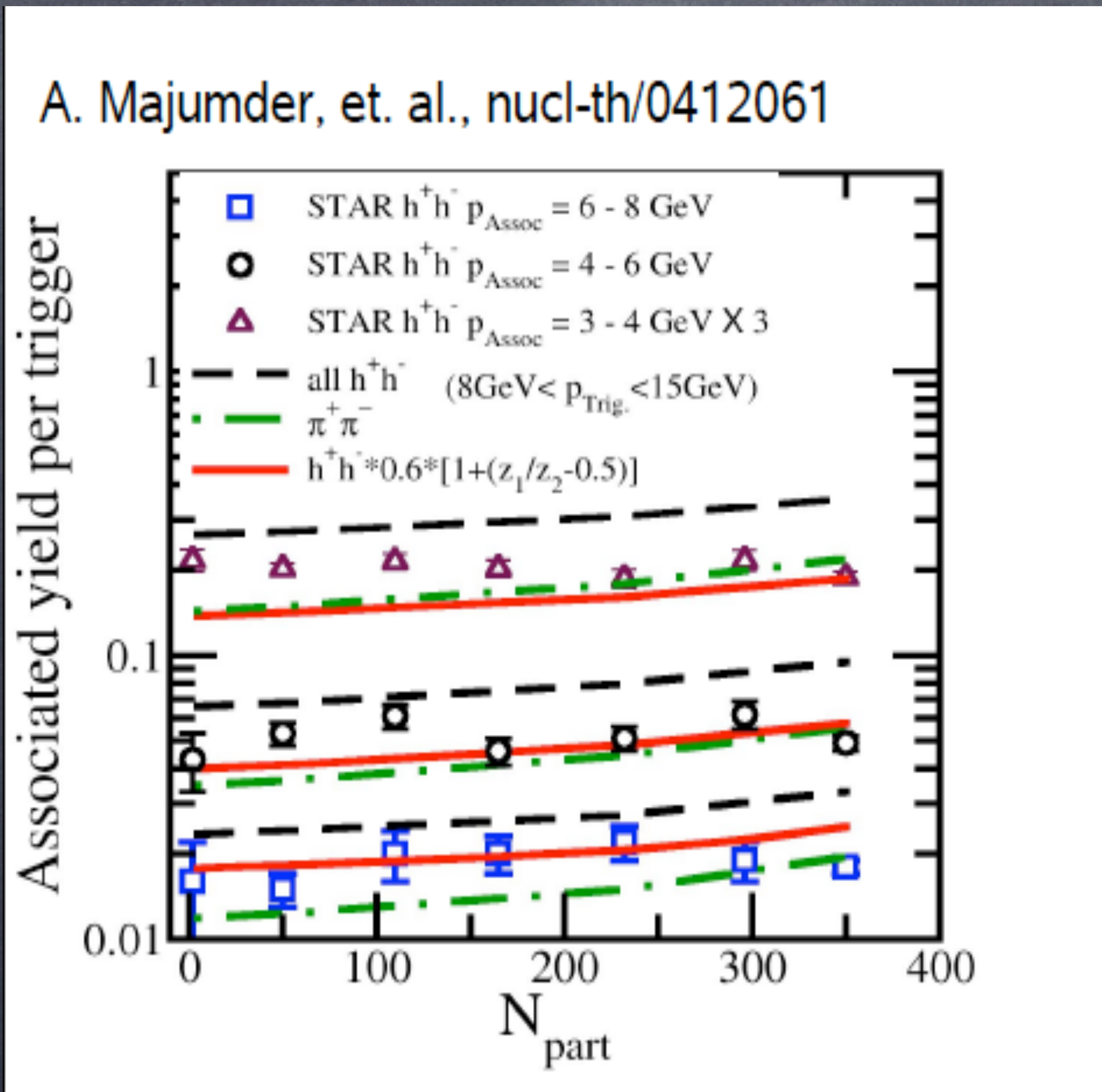
Several improvements can be made from this point

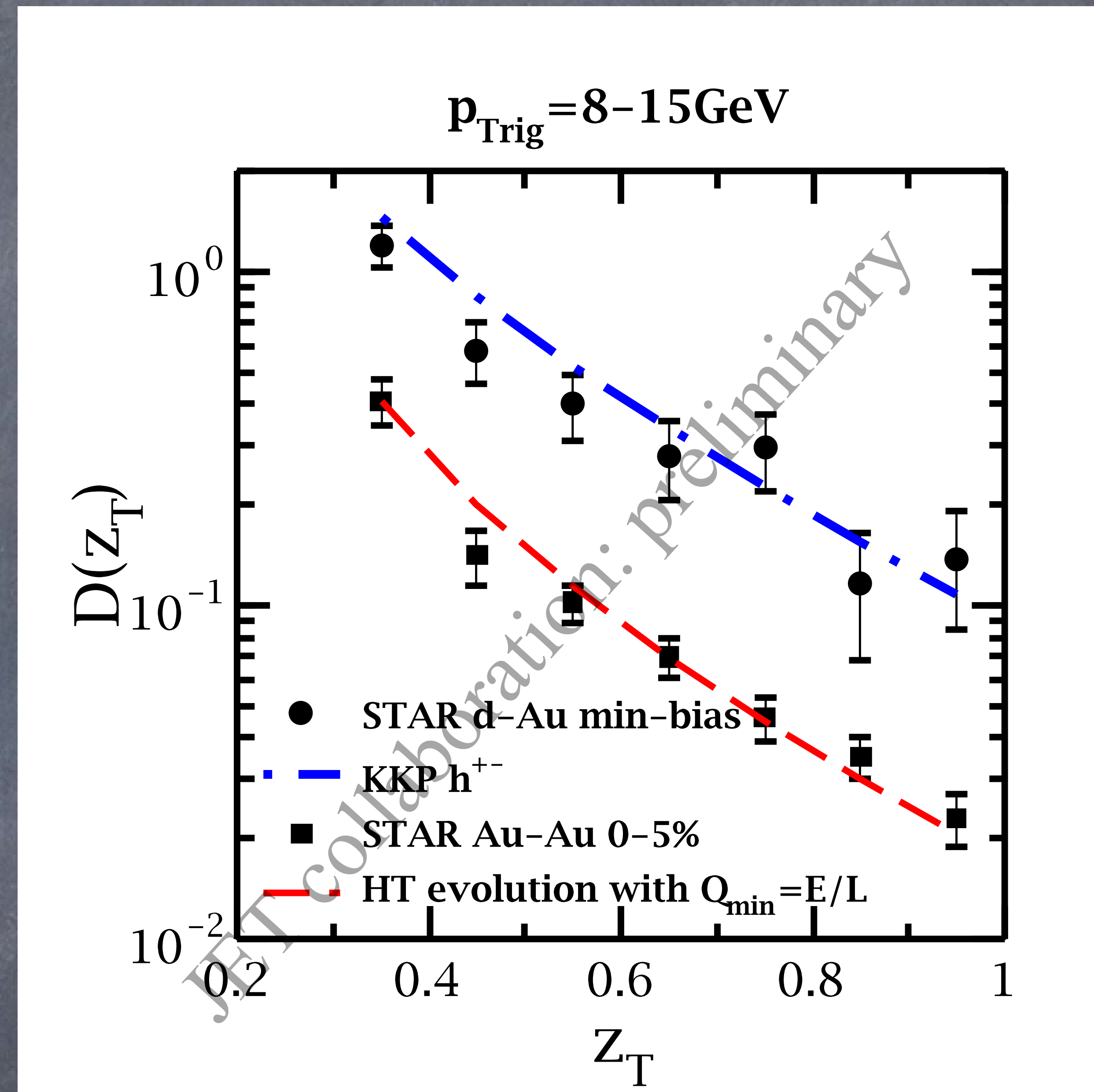
A. Majumder and C. Shen, Phys. Rev. Lett. 109, 202301 (2012)

Better hydro, better jet quenching: T_{AB} for jets matched with hydro initial conditions



Completely consistent predictions for Dihadrons





These are parameter free calculations

The near side involves a new non-perturbative object the dihadron fragmentation function

JET Collaboration

From factorized analytical approaches to event generators

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Looks at full jet, so less sensitive to fragmentation

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Usual background subtraction, includes jet medium interaction as part of jet

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Looks at full jet, so less sensitive to fragmentation

In reality, have to include a detailed model of fragmentation so not as well defined as few particle observables

Always an issue with separating the jet from the medium

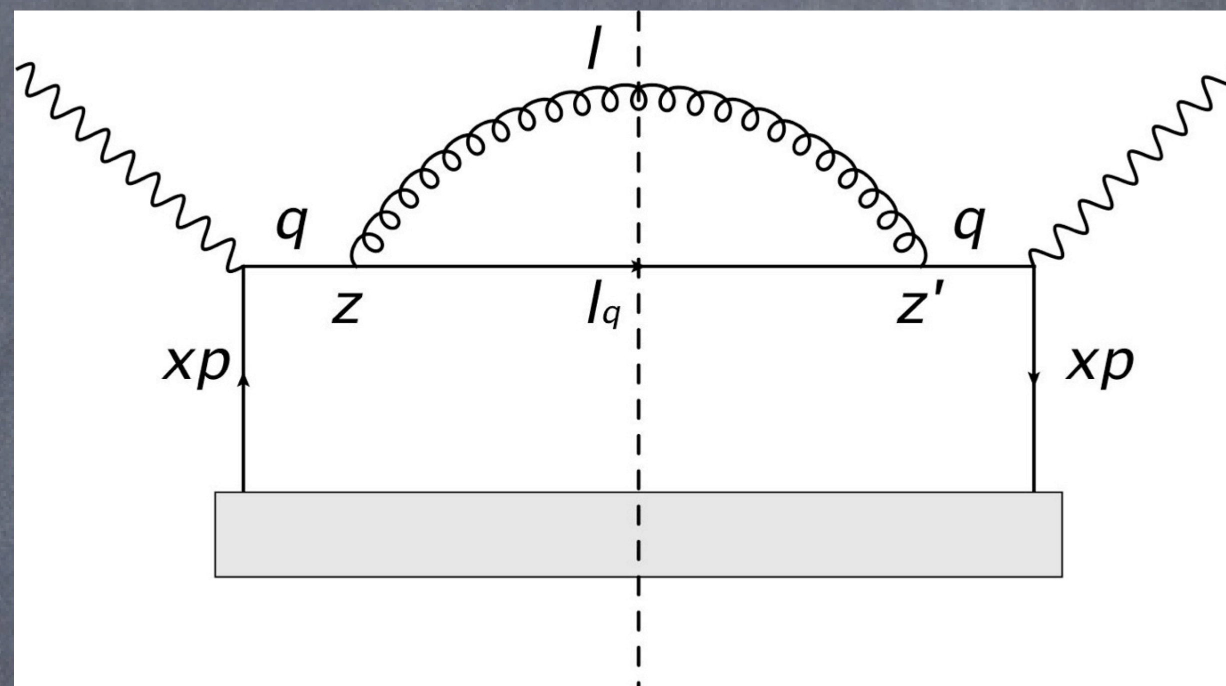
Usual background subtraction, includes jet medium interaction as part of jet

Rigorously calculating this requires more non-perturbative transport coefficients

Main problem: Introducing distance into a DGLAP shower

No space-time in the usual Monte-Carlo showers

$$\bar{z} = \frac{z + z'}{2}$$



$$\delta z = z - z'$$

z and z′ position of emission in amplitude and c.c.

$$\int_0^\infty d^4\bar{z} \exp\left[i(\delta q)\bar{z}\right]$$

$$\int d^4\delta z \exp\left[i\delta z(l + l_q - q)\right]$$

δq is the uncertainty in q,

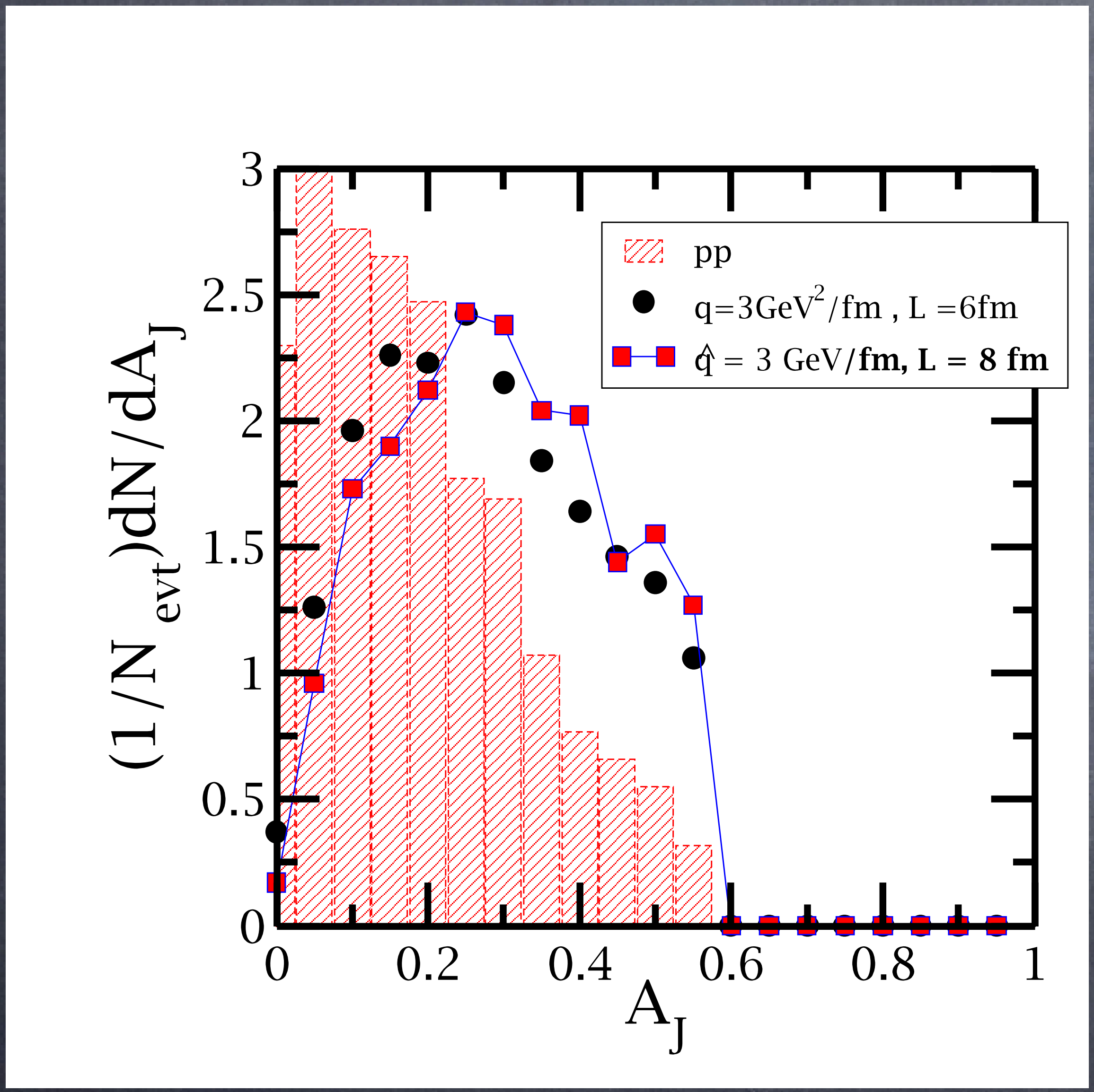
We obtain a Wigner transform like formalism

with δq^+ and z^-

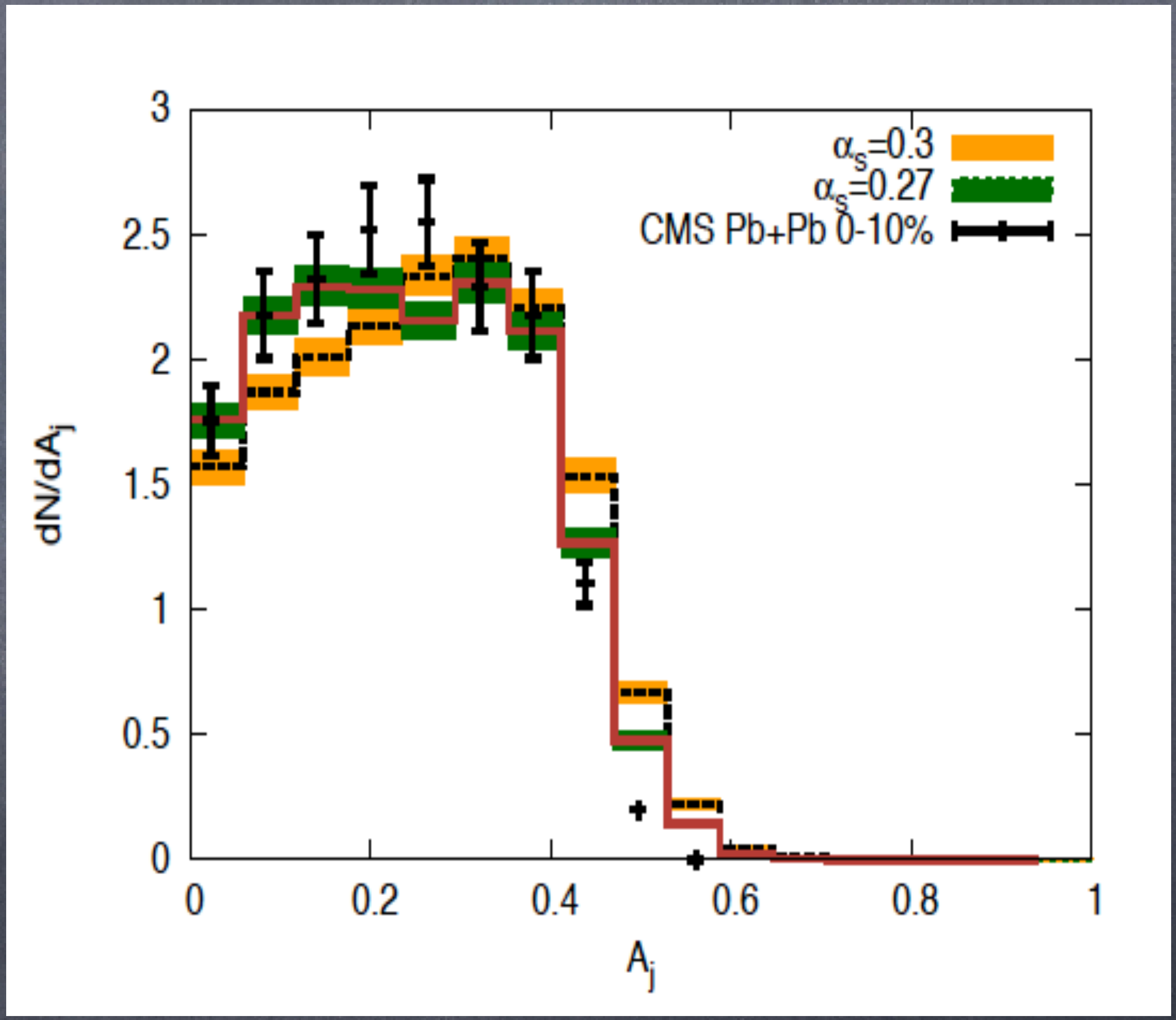
A. Majumder, Phys. Rev. C 88, 014909 (2013)

Observables 1. A_J

If you ignore R_{AA} this is not hard



Higher Twist in box

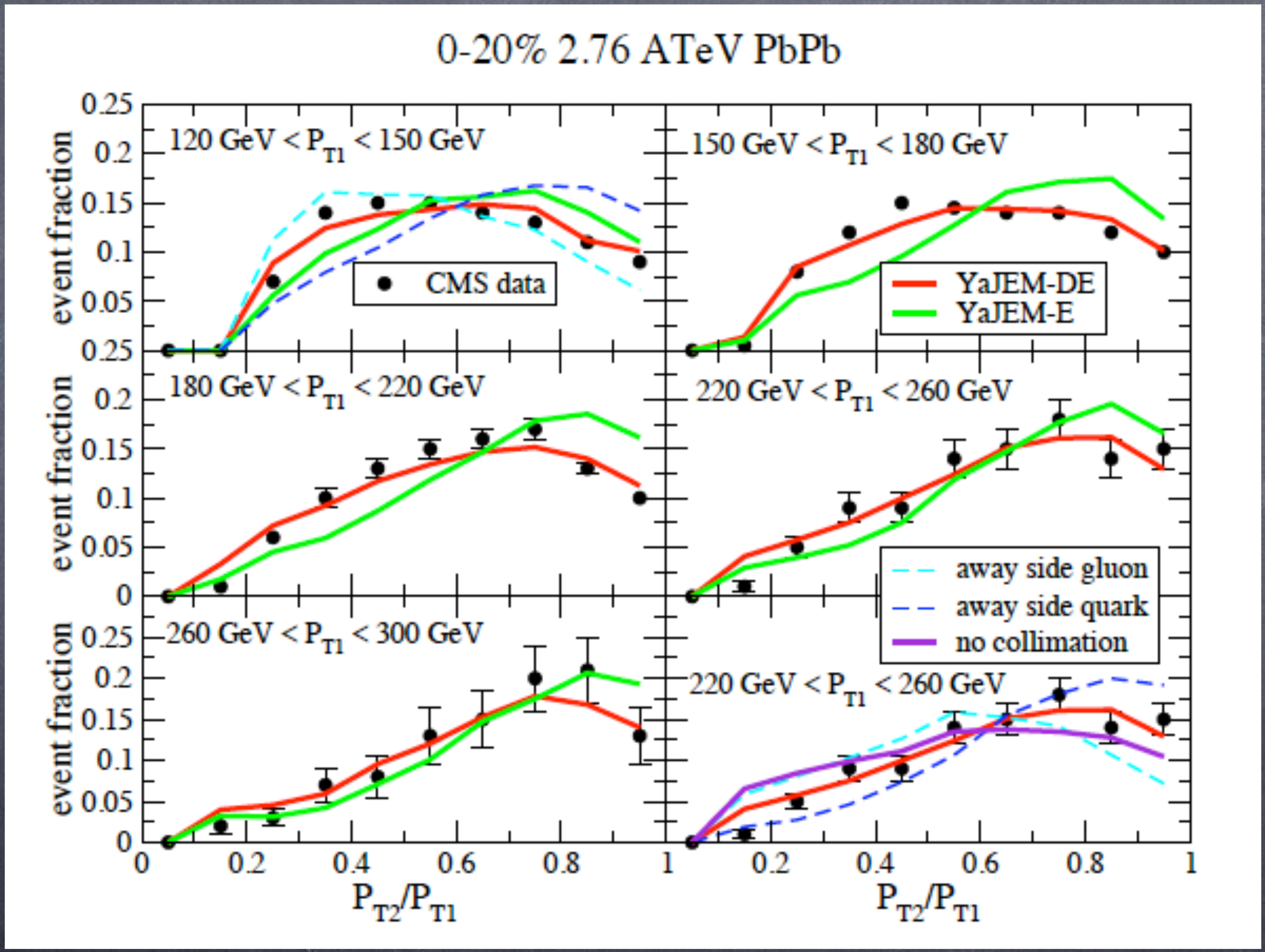


MARTINI without R_{AA}

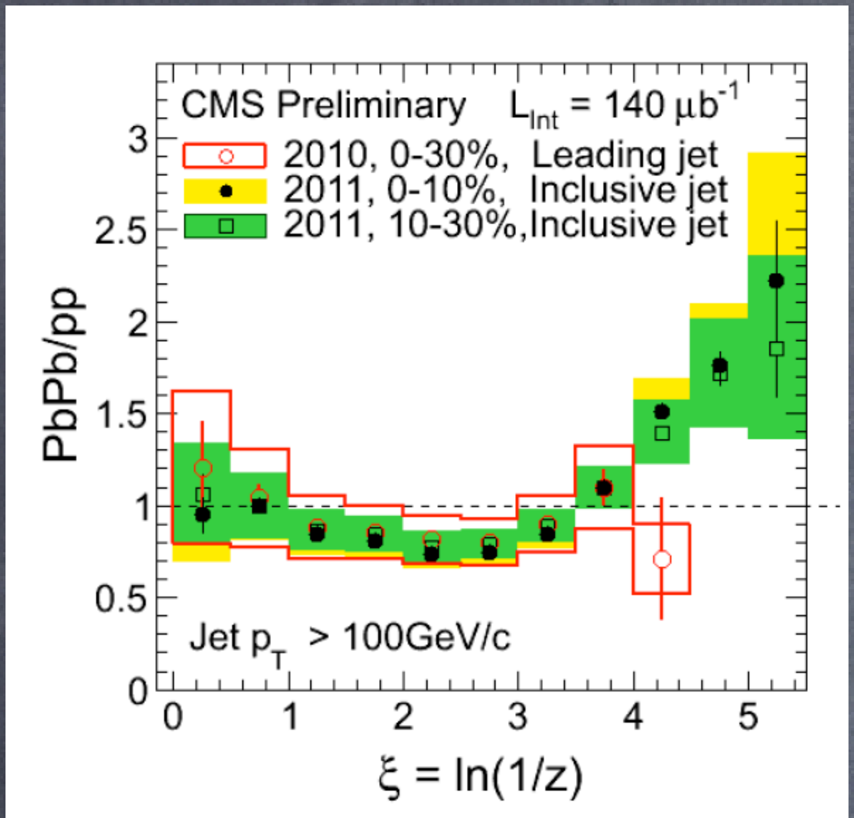
Observables 1. A_J

If you ignore R_{AA} this is not hard

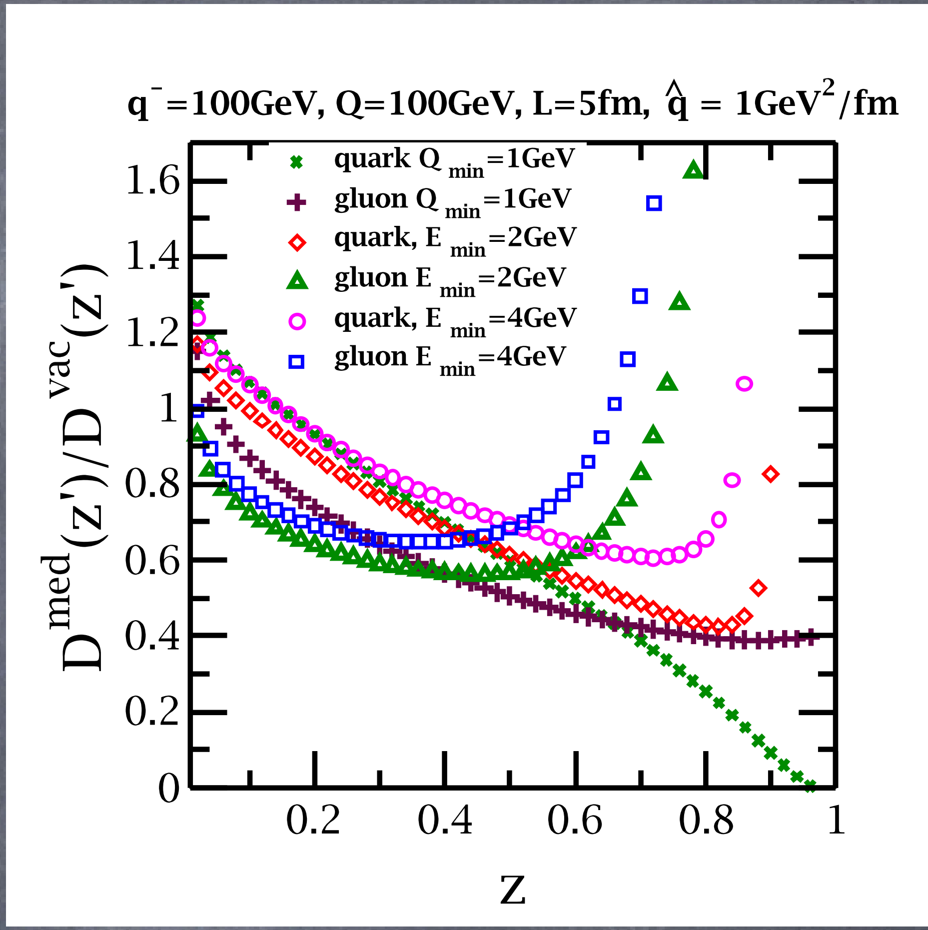
T. Renk, Phys. Rev. C 86, 061901 (2012)



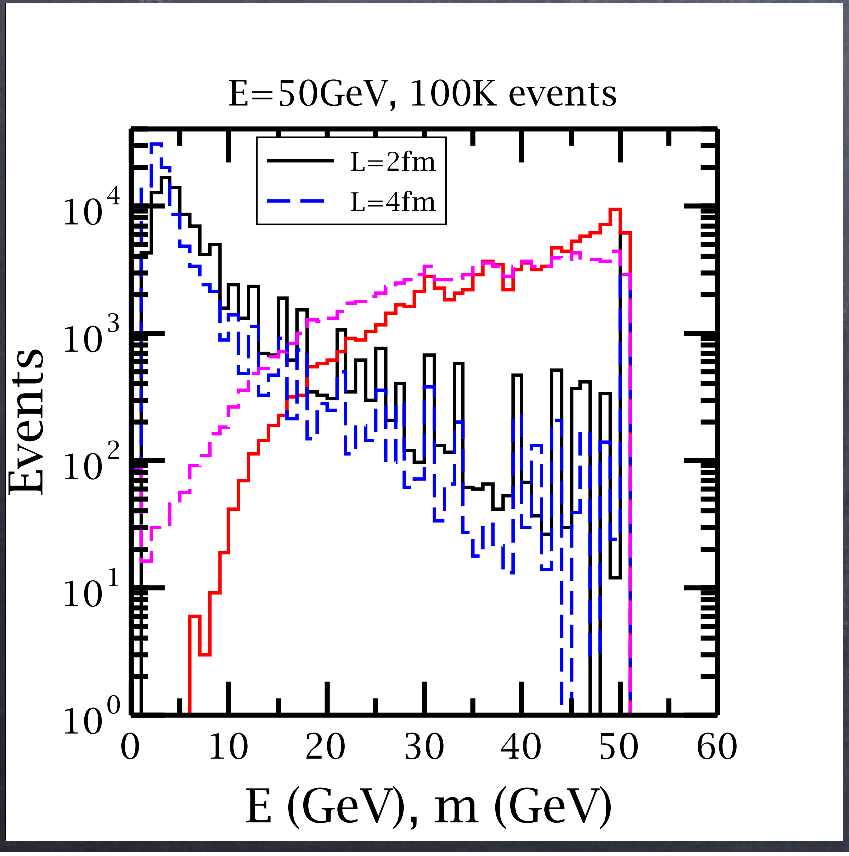
Observable 2: Fragmentation function!



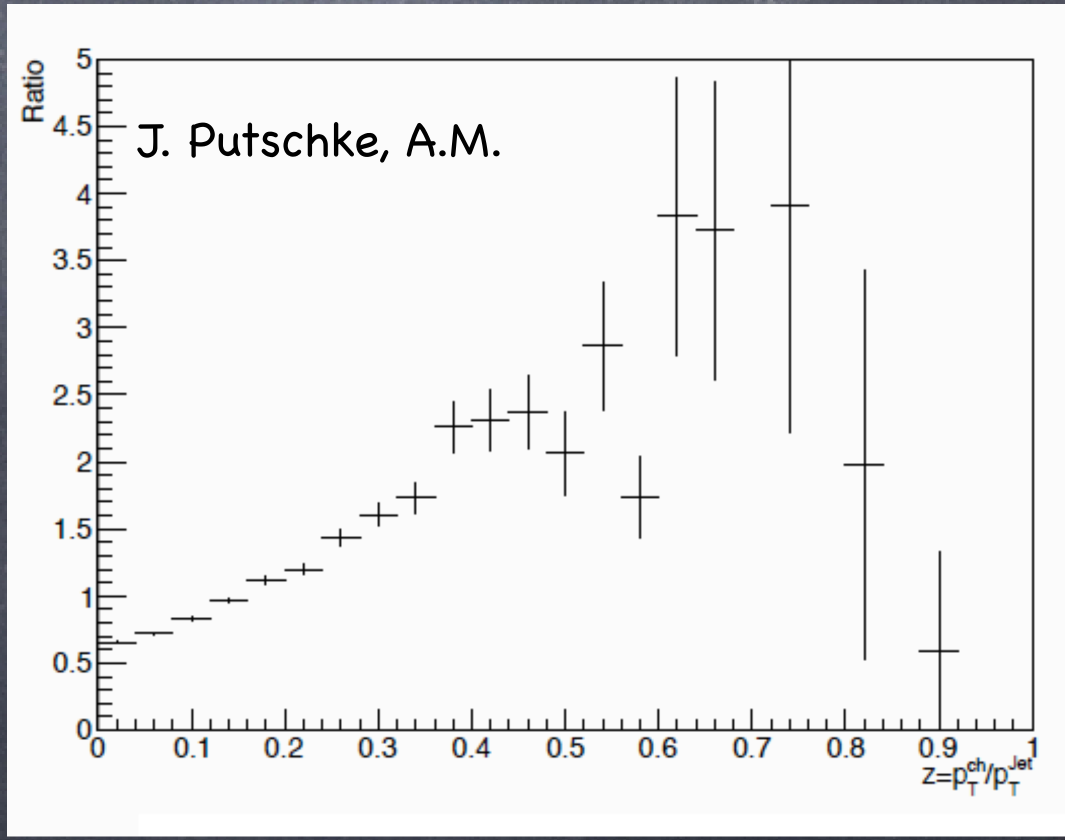
lost energy ->



loss of virtuality

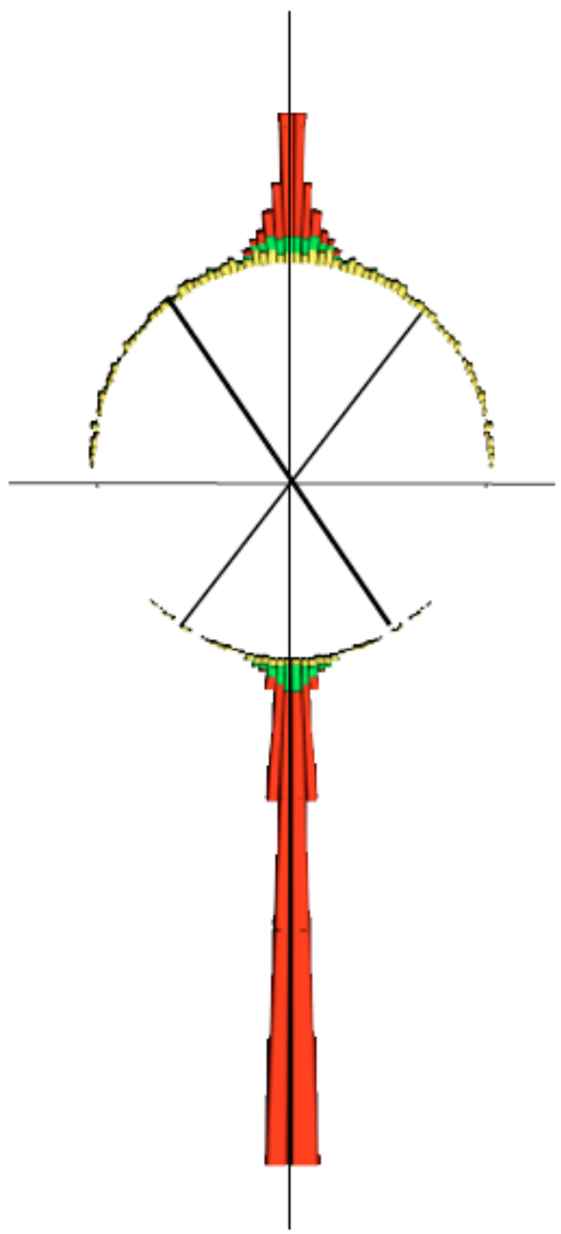


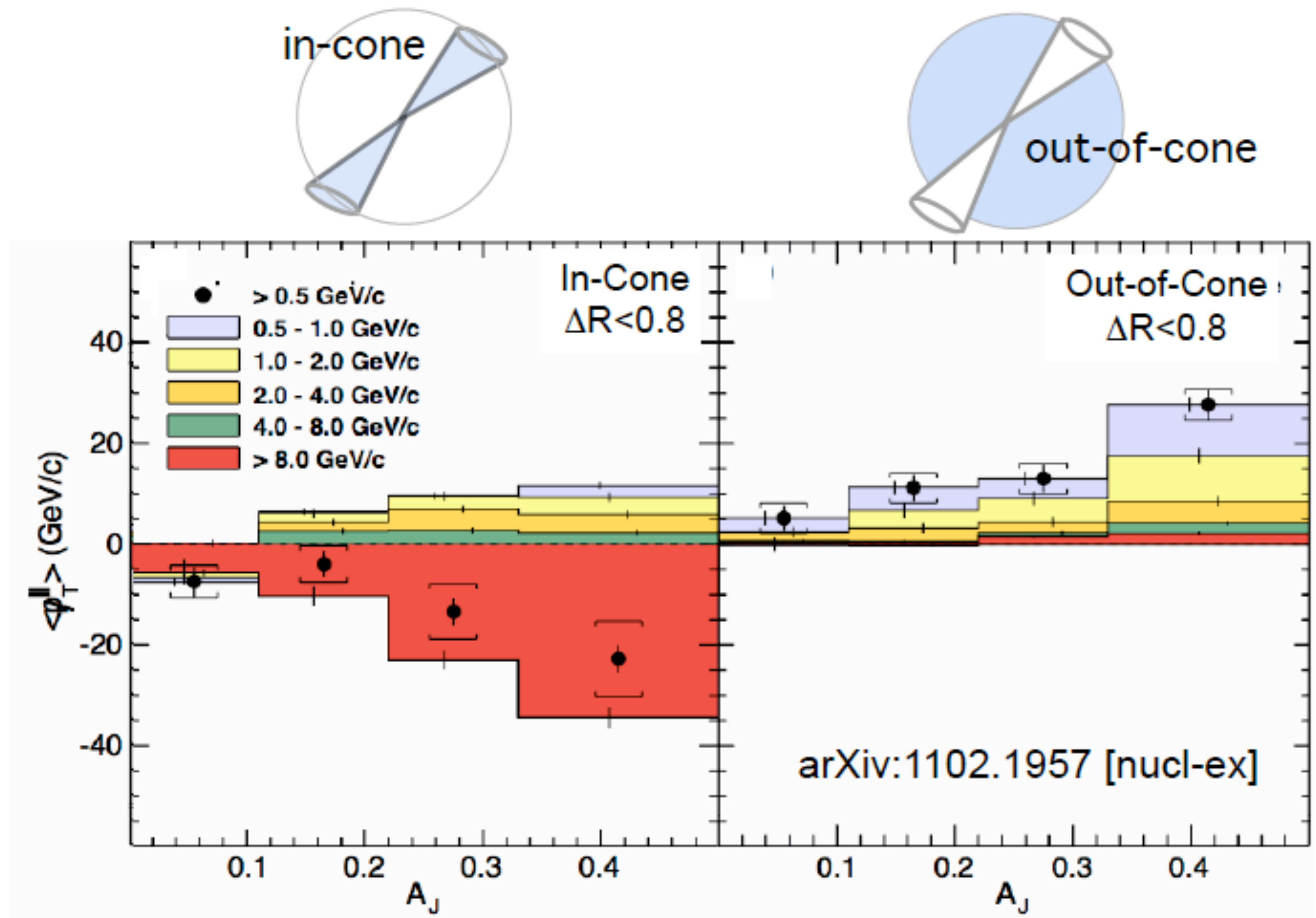
ratio of fragmentation functions with different virtuality



Observable 3. Appearance of lost Energy

PRC 84 (2011) 024906

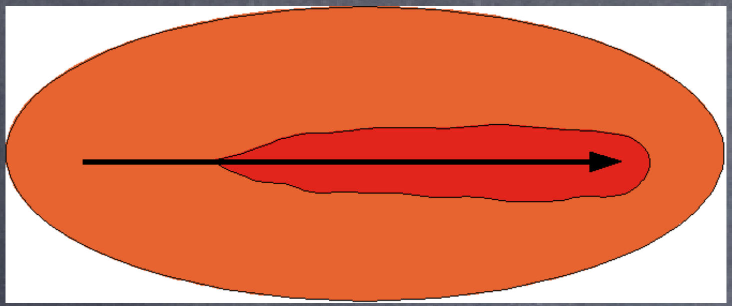




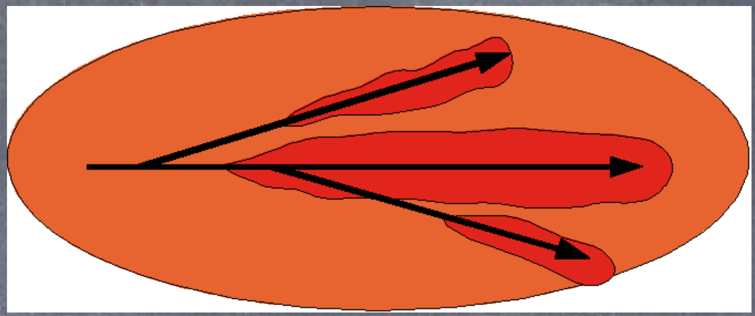
Momentum balance in the event is carried by low momentum particles at large angles to jets

$$\not{p}_{\mathrm{T}}^{\parallel} = \sum_{\text{Tracks}} -p_{\mathrm{T}}^{\text{Track}} \cos\left(\phi_{\text{Track}} - \phi_{\text{Leading Jet}}\right)$$

To understand this need to know how jets deposit energy into a medium

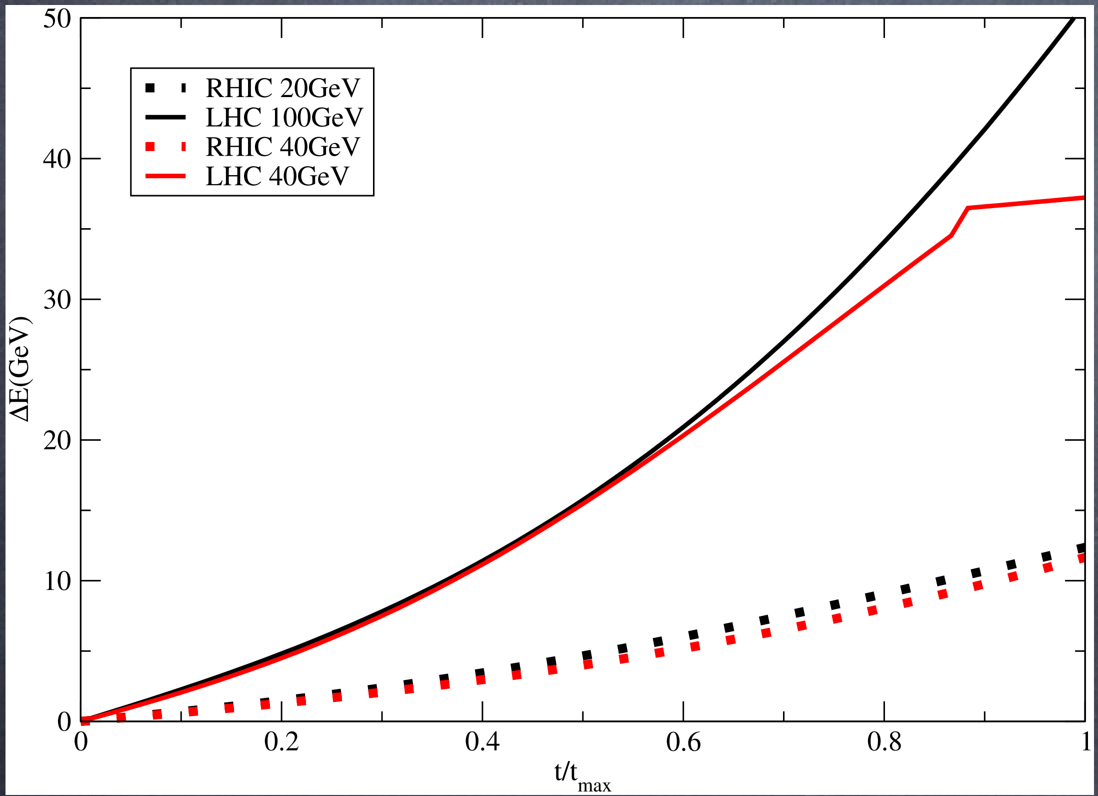


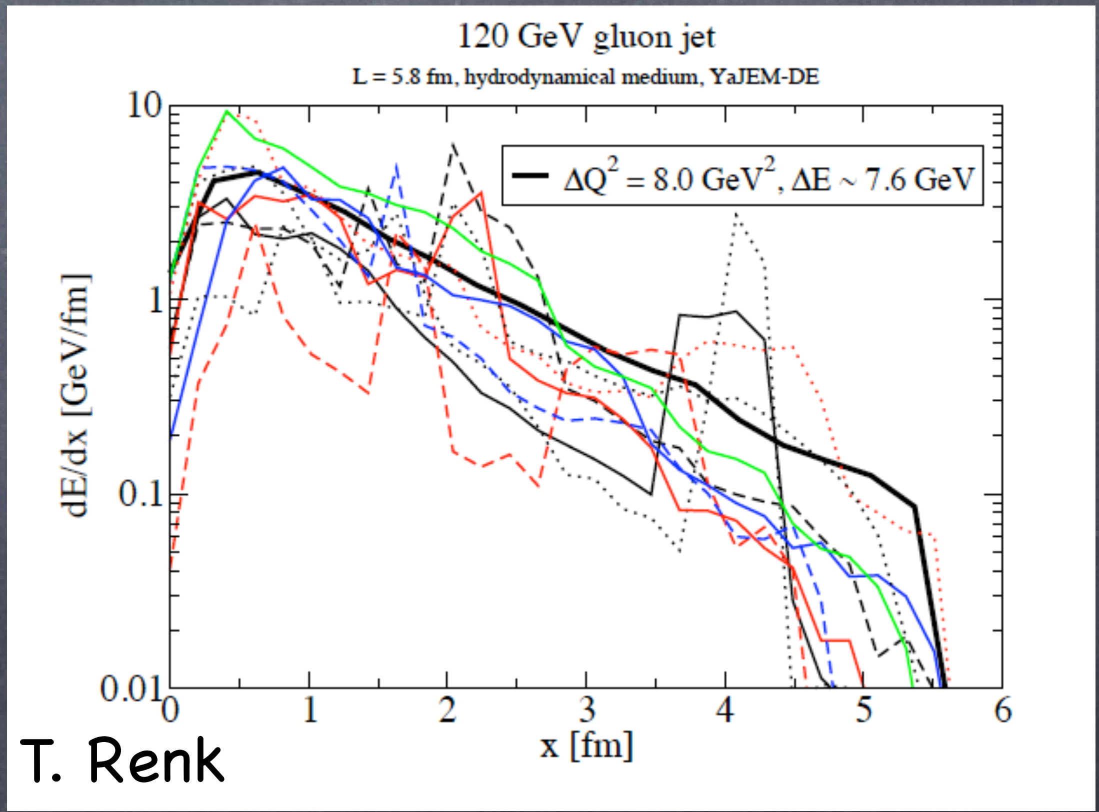
G.-Y. Qin, A. Majumder, H. Song and U. Heinz, Phys. Rev. Lett. 103, 152303 (2009)



Rate of energy deposition greater at LHC
large part of the jet escapes the medium

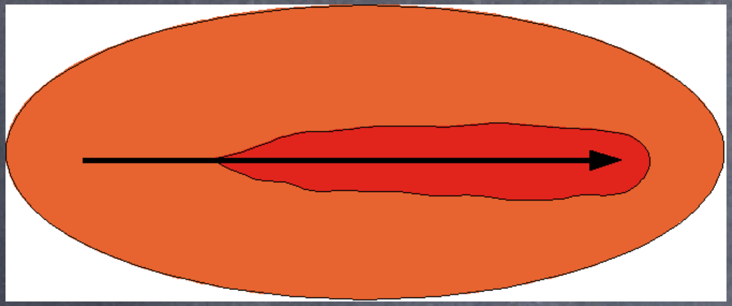
Medium dissipates in time,
so early energy loss is important



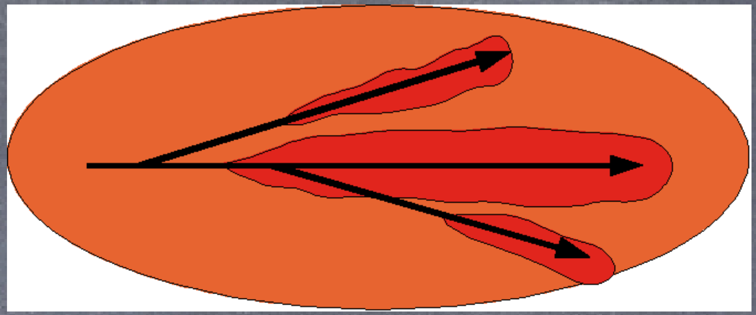


How will all of this look like at an EIC??

To understand this need to know how jets deposit energy into a medium

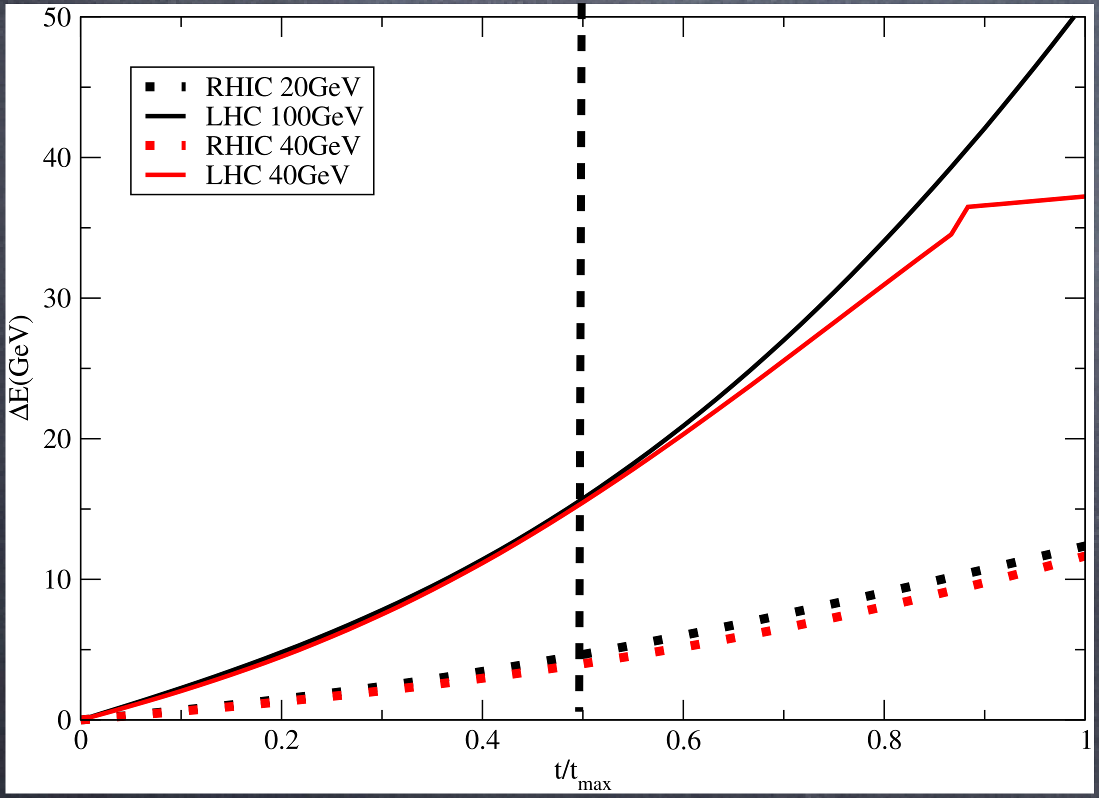


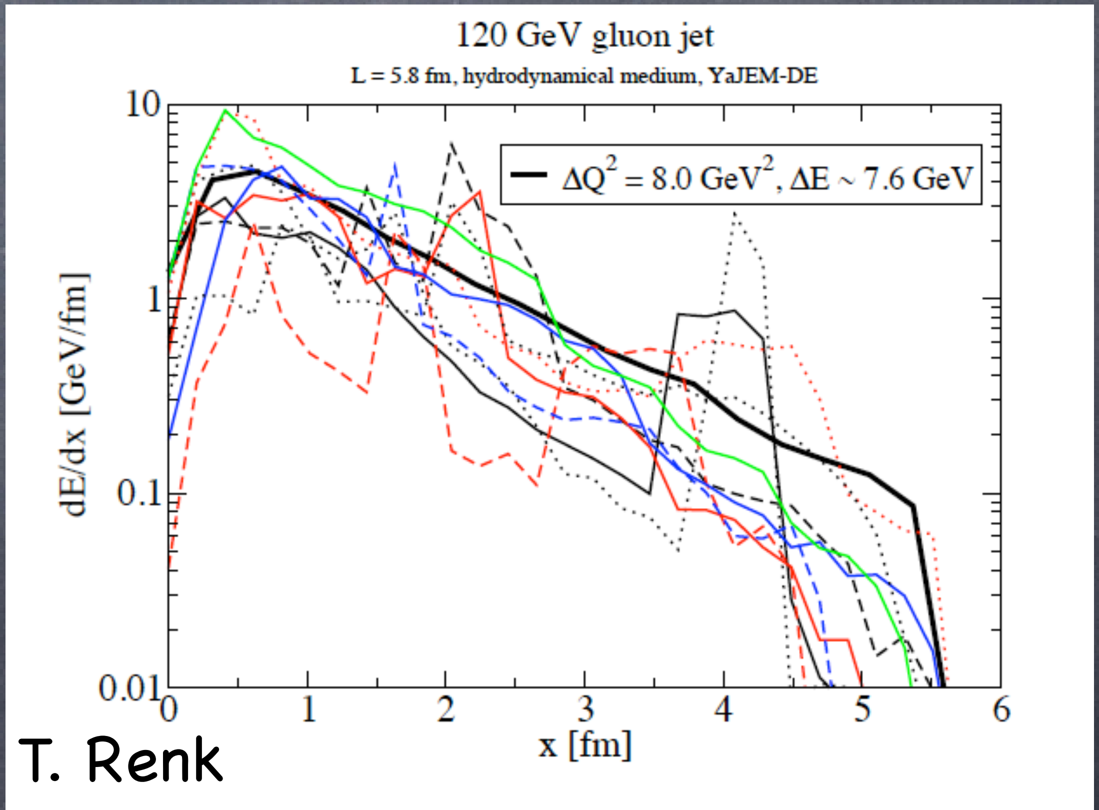
G.-Y. Qin, A. Majumder, H. Song and U. Heinz, Phys. Rev. Lett. 103, 152303 (2009)



Rate of energy deposition greater at LHC large part of the jet escapes the medium

Medium dissipates in time, so early energy loss is important

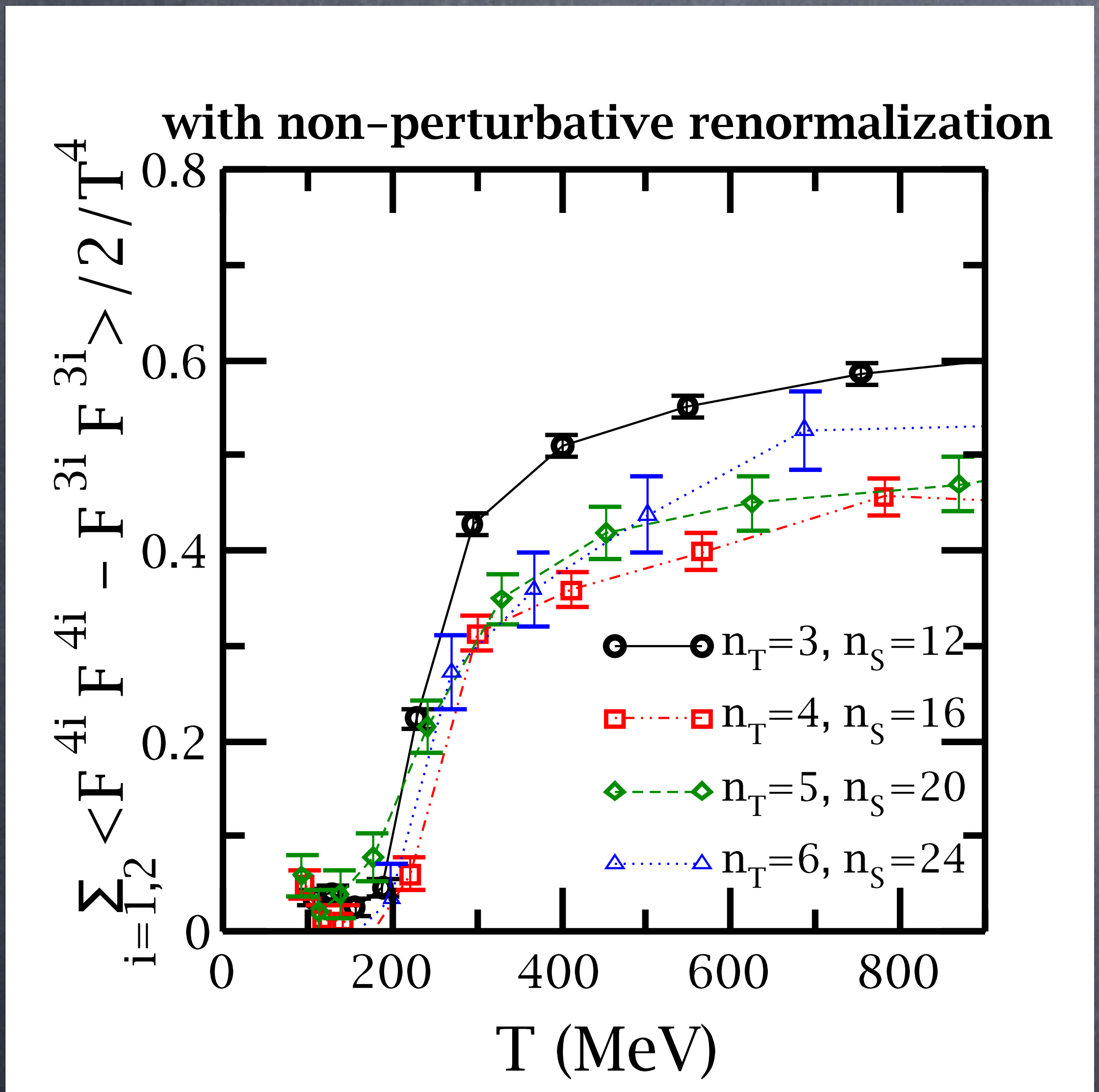




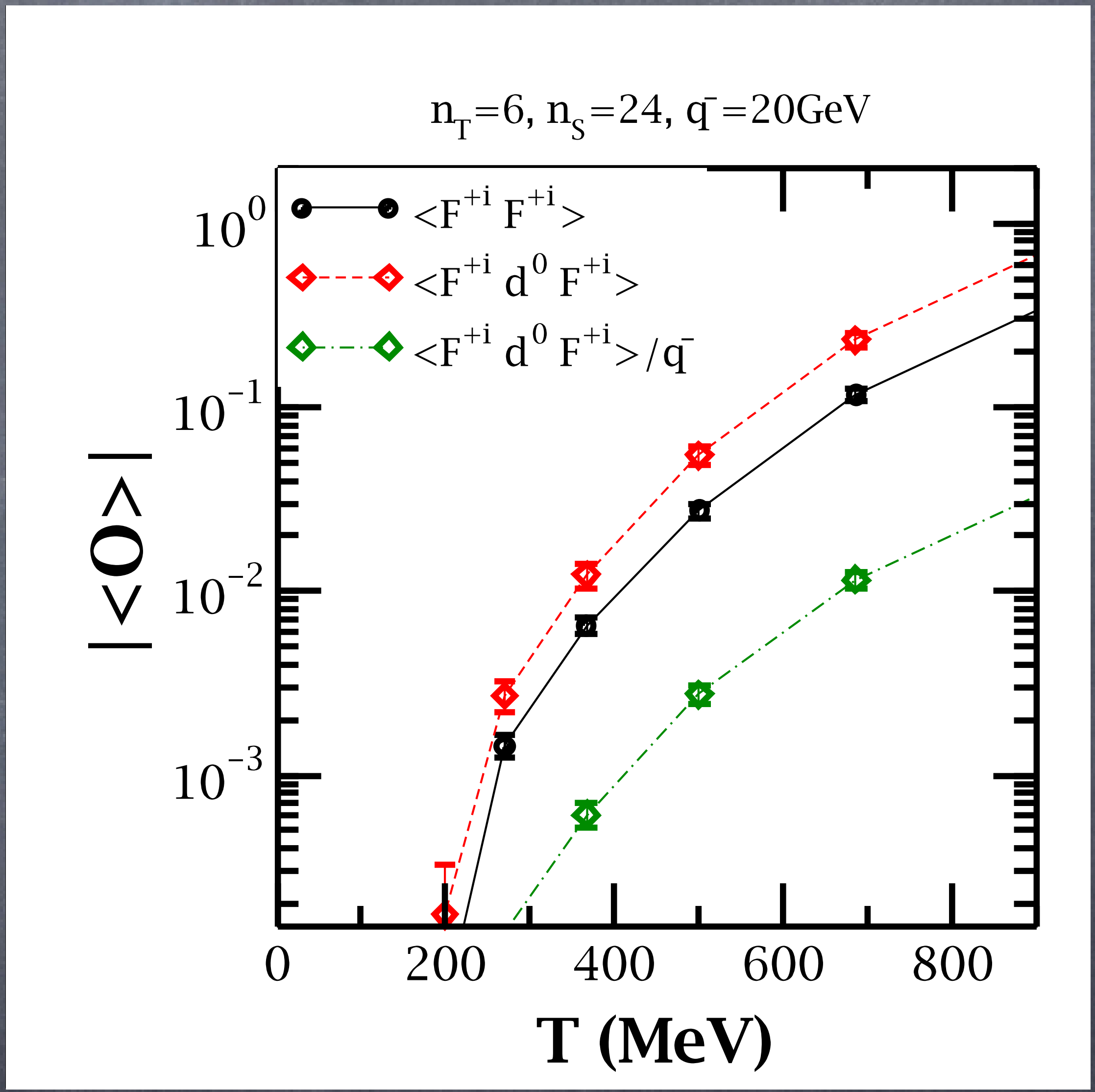
How will all of this look like at an EIC??

Getting ahead of the experiment

Calculating q̂ on the lattice



A Majumder, Phys Rev C 87 034905

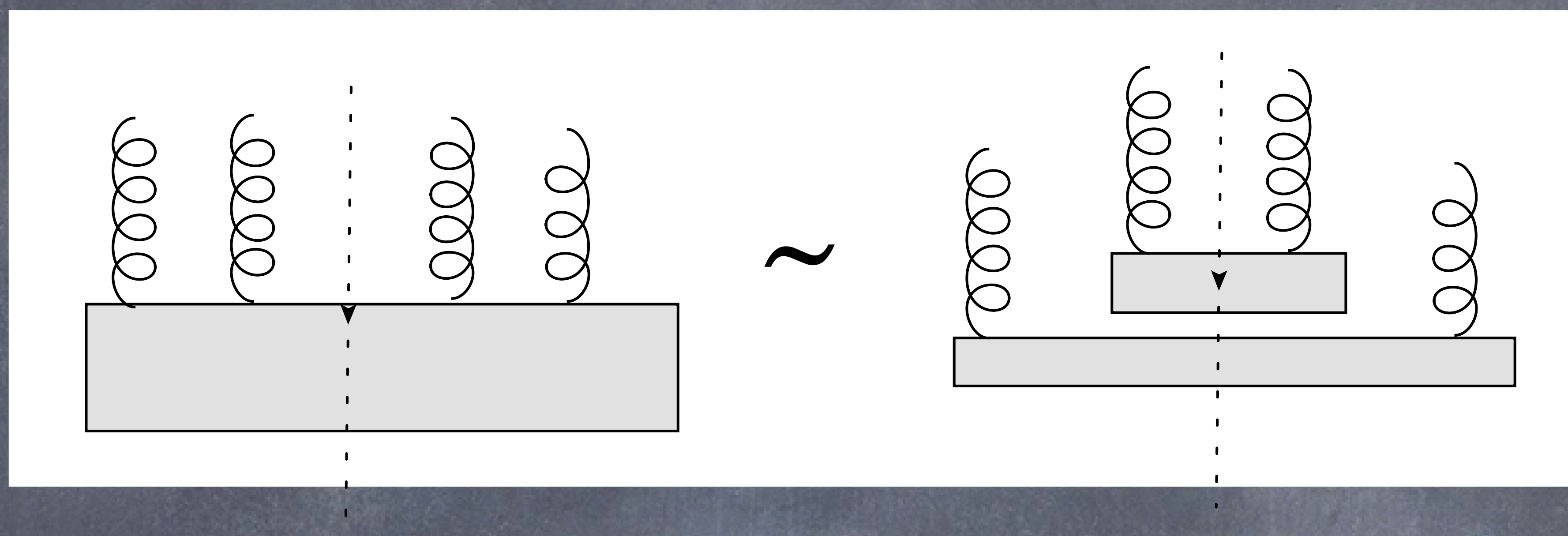


Future calculations will have T dependent q̂ input from lattice
Difficult inside a nucleus!

Conclusions

- Factorization paradigm allows direct comparison between cold and hot matter using JETs
- At few particle level, very good agreement with theory for hard jets
- precision study at EIC will yield more info on transport coefficients
- New physics probed by full jets, not yet completely under control
- May lead to new insights at EIC

Take the extreme limit of a nucleus, A -> inf. and nucleons are very small compared to nucleus



All four gluons from one nucleon: prop. to L

Two in one nucleon, two in another: prop. to L^2

2 n gluon expection ---> n 2 gluon expectation