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# Optical Modeling of the Jefferson Laboratory IR Demo FEL

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## ABSTRACT

The Thomas Jefferson National Accelerator Facility (Jefferson Lab) is in the process of building a 1 kW free-electron laser operating at 3 microns. The details of the accelerator driver are given in other papers in these proceedings. The optical cavity consists of a near-concentric resonator with transmissive outcoupling. Though several free-electron lasers have used similar designs, they have not had to confront the high average-power loading present in this laser. It is useful to know the limits of this type of optical cavity design. The optical system of the laser has been modeled using the commercial code GLAD® by using a Beer's-law region to mimic the FEL interaction. The effects of mirror heating have been calculated and compared with analytical treatments. The magnitude of the distortion for several materials and wavelengths has been estimated. The model developed here allows one to quickly determine whether the mirror substrates and coatings are adequate for operation at a given optical power level once the absorption of the coatings, substrate, and transmission are known. Results of calculations of the maximum power level expected using several different sets of mirrors will be presented. Measurements of the distortion in calcium fluoride from absorption of carbon dioxide laser light are planned to benchmark the simulations. Multimode simulations using the code ELIXER have been carried out to characterize the saturated optical mode quality. The results will be presented.

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## 1. INTRODUCTION

Thomas Jefferson National Accelerator Facility (Jefferson Lab) has almost completed construction of a free-electron laser operating at 3 to 6.6 μm with power output exceeding one kilowatt. The details of the accelerator design are presented in another paper at this conference [1]. Schedule and cost considerations led us to choose a near-concentric resonator geometry. Such a design has the advantages of existing designs, decoupling of mirror steering from cavity length, excellent output coupling efficiency, and a very large magnification of the optical mode from the cavity mode waist to the mirror. Here we define the magnification $M \equiv (1+\left(L/2z_R\right)^2)$ where $L$ is the cavity length and $z_R$ is the Rayleigh range. Designers of most near-concentric cavities are not usually worried about mirror heating and distortion so the issue of how such a design scales with power had to be addressed. The intent of this paper is to study the scaling of a near-concentric design and establish limits on its performance.

The wiggler and accelerator parameters are given in reference [1]. Simulations using ELIXER [2] indicate a small signal gain of >60% and saturated output power well in excess of the one kilowatt design value. The losses are small compared to the small signal gain so the laser should saturate strongly. Reference [3] discusses the origin of the requirements for the optical resonator. The design values chosen shown in table 1 indicate several areas of concern. Note the rather tight tolerances on the mirror tilt and radius of curvature. This is a consequence of being very close to the concentric limit. Another potential drawback is the fact that mirror heating reduces the spot size on the mirror, enhancing the mirror distortion. This positive feedback is discussed is section 3. Finally the cavity is nearly degenerate, i.e. the phase advance from the wiggler exit through the optical cavity back to the wiggler entrance is nearly the same for all transverse modes. This might lead to the buildup of multiple transverse modes and a subsequent deterioration in the mode quality. These potential problems in the near-concentric design will be addressed in this paper.

Very little expansion in the mirror surfaces due to mirror heating can be tolerated because of the sensitivity of the cavity to the radii of curvature. This is the subject of the next section. One can reduce the mirror heating by increasing the magnification but this leads to even more sensitive constraints on the radii of curvature and the tilt tolerances. Experience from other near-concentric systems convinced us that a magnification M much greater than 100 (for example in the Boeing resonator [4]) requires active feedback on the mirror tilt. The alternative solution of using a Rayleigh range much less than the wiggler length has also been proposed by Small et al. [5] and will not be studied here.

## 2. MIRROR DISTORTION–AN ANALYTIC APPROACH

First let us lay out the assumptions used in the analytical model.

- Assume that only the output coupler is distorted by mirror heating. This is justified by the ability to use an opaque substrate with excellent thermal properties such as copper.
- The optical cavity is constructed with cylindrical symmetry and every effort is made to ensure that the boundary conditions are also cylindrically symmetric. Since the optical mode is also cylindrically symmetric, the heating of the mirror should lead only to a change in the effective radius of curvature of the mirror as well as producing spherical aberrations.
- Finally, spherical aberrations much less than one fifth of a wave of the optical mode will not significantly alter the FEL performance [6].

Let us first consider the effects of the change in the mirror radius of curvature. Since only one mirror is distorted, the waist in the resonator will change in both size and position. Equation (10) in reference [3] gives the change in $z_R$ when both mirrors change their radii of curvature $R_c$ by the same amount. Since only one mirror changes the change in the Rayleigh range is only half as much:

$$\frac{\Delta z_R}{z_R} = \frac{M}{4}\frac{\Delta R_c}{R_c} \qquad (1)$$

The change in the waist position can also be calculated and expressed in terms of the magnification $M$, the cavity length, and the relative change in the radius of curvature,

$$\Delta z = -\frac{L}{4g}\frac{\Delta R_c}{R_c} = \frac{LM}{4(M-2)}\frac{\Delta R_c}{R_c} \qquad (2)$$

Note that a change in the Rayleigh range increases the intensity on the mirrors leading to a larger change in the Rayleigh range. The full effect on the cavity mode must be calculated self-consistently and can be much larger than the first order effect. As noted in reference [3], the gain in this laser will start to decrease rapidly if the Rayleigh range increases to over ~80 cm. The radius of curvature for the output coupler can therefore change by up to 4% for $M$=100 (producing a factor of two change in $z_R$). This leads to a waist position shift of 8 cm. This small a change in waist position does not appreciably change the mode coupling to the electron beam.

It is possible to solve analytically for the steady state temperature in a mirror heated volumetrically along the center by a Gaussian mode, whose outer edge is clamped uniformly at some temperature. One first substitutes a Gaussian distribution in the dimensionless variable $\rho = r / a$ where $a$ is the mirror radius into the two-dimensional steady-state diffusion equation. The power absorbed is just the bulk absorption coefficient time the intensity inside the mirror. When this is done, one finds the following equation:

$$\nabla^2 T(\rho) = \frac{2a^2 \alpha P_l}{M z_R \lambda k_{th}} e^{-(b\rho)^2} \tag{3}$$

where $T$ is the temperature profile, $k_{th}$ is the thermal conductivity of the mirror, $b = \sqrt{2}(a/w_m)$, $w_m$ is the mode waist on the mirror and $\alpha$ is the bulk absorption. The solution to this equation is given by the expression

$$T(\rho) = -\left(\frac{\alpha P_l}{4\pi k_{th}}\right)\left[\ln \rho^2 + \lim_{t\to 0}\left(\Gamma(t,(b\rho)^2) - \Gamma(t,b^2)\right)\right] \tag{4}$$

where $\Gamma(t,x)$ is an incomplete Gamma function. The mirror distortion due to this temperature rise should, to first order, be proportional to the temperature rise. If this is the case, the mirror distortion should be $\delta z(r) \equiv \alpha_e (h/2) T(r/a)$ where $\alpha_e$ is the thermal expansion coefficient of the mirror substrate and h is the thickness of the mirror. The incomplete Gamma functions do not allow the physics of the distortion to be easily calculated. Though approximations for this solution are available [7], it is more useful to expand the solution in Zernike polynomials [8]. The scale for the polynomials is chosen to be twice the optical mode radius rather than the mirror radius. This incorporates the local nature of the mirror distortion while providing a much more accurate approximation of the effect over the optical mode. We then have:

$$\delta z \cong \frac{P_l}{8\pi F}\left[c_0 + c_1(2\chi^2 - 1) + c_3(6\chi^4 - 6\chi^2 + 1) + c_3(20\chi^6 - 30\chi^4 + 12\chi^2 - 1)\right] \qquad (5)$$

$$c_1 = -1.17,\ c_2 = 0.44,\ c_3 = -0.17$$

where F is a figure of merit for the mirror material, defined by $F = k_{th}/(h\alpha_e\alpha)$, and the $c_i$ are the Zernike coefficients (as a function of $\chi = r/(2w_m)$) of the expansion of the mirror distortion multiplied by ($\gamma + \ln b^2$) where $\gamma$ is the Euler-Mascheroni constant [7].

The fit to the mirror distortion calculated from ANSYS [9] for a Rayleigh range of 37 cm is shown in Figure 1 along with the optical intensity. The Zernike polynomial fit to the distortion is quite good over the range of the optical mode.

From the Zernike polynomial expansion above and equation (1) we can show that the change in the Rayleigh range due to the mirror heating is

$$\frac{\Delta z_R}{z_R} \cong \frac{1.17 P_l}{32 F\lambda}\frac{M}{\sqrt{M-1}}. \qquad (6)$$

Note the simplicity of this equation. For a given mirror, wavelength, allowable Rayleigh range variation, and allowable magnification one can calculate the maximum power. One can also assume the power required and determine the specification for the mirror.

One can also compute the spherical aberration. The maximum aberration amplitude normalized to the optical wavelength occurs at the mirror center and has an amplitude $\Delta z(0)/\lambda = 0.61 P_l/8\pi F\lambda$. The ratio of this normalized aberration to the relative change in $z_R$ is

$$\frac{\delta z(0)}{\lambda} = 0.664\frac{\sqrt{M-1}}{M}\frac{\Delta z_R}{z_R} \qquad (7)$$

Equation (7) shows clearly that, for very large values of *M* the aberration is negligible even for large changes in the Rayleigh range.

The power absorbed by a given mirror may be dominated by either the bulk or the surface absorption. The temperature distribution induced by heating the surface with a Gaussian mode can also be calculated by solving the heat diffusion equation in steady state. The problem in this case is three dimensional. The solution has been calculated in reference [10]. The solution has the following properties.

1) The integral of the temperature rise along the axial coordinate $z$ yields the function given in equation (4) as expected.

2) Calculation of the mirror distortion due to surface heating using ANSYS indicates that the net distortion from a surface heating is the same as if the mirror were heated uniformly along the axial coordinate.

The latter point is a bit surprising and must be confirmed by experimental measurements or by theoretical analysis. The figure of merit for the mirror can be modified to take surface heating into account by averaging the power absorption over the mirror. The change in the Rayleigh range is then given by

$$\frac{\Delta z_R}{z_R} \cong \frac{1.17 P_l}{32 F' \lambda} \frac{M}{\sqrt{M-1}} \text{ where } F' = \frac{k_{th}}{(h\alpha_B + \alpha_s(1+1/t_c))\alpha_e} \quad (8)$$

where $t_c$ is the mirror transmission and equation (7) still applies as long as equation (8) is used to calculate the relative change in the Rayleigh range.

## 3. MIRROR DISTORTION–NUMERICAL ANALYSIS

### 3.1 Analytic calculations

If we assume the resonator parameters in Table 2 we can calculate the effect of the mirror distortion using the equations in section 3. The figure of merit for 6.35 mm thick calcium fluoride, sapphire, and magnesium fluoride mirrors with 0.1% surface absorption is shown in table 2. It is obvious from Table 2 that $CaF_2$ is not an adequate choice. Sapphire is superior both in terms of the effects of mirror heating and radiation damage. $MgF_2$, though acceptable before iteration, is also unacceptable. One may also reverse the reasoning, using the fact that a 60% initial increase in the Rayleigh range will lead to an iterated increase of 100% for a magnification of 101, and show that the calcium fluoride mirror is acceptable up to 350 Watts, the magnesium fluoride mirror is good up to 700 Watts and the Sapphire mirror is good up to 2740 Watts.

### 3.2 Simulations

The analysis so far does not include the time dependent distortion when the mirror is first heated. It is possible that the aberration and radius of curvature changes during startup may be larger than those shown so far. This can be modeled self-consistently using the simulation computer code GLAD [11]. Ultimately one would like to simulate the FEL interaction accurately as well using some code such as FELEX [12]. GLAD correctly models the propagation of the optical mode through an arbitrary series of mirrors, lenses, or apertures. Although the code does not include a FEL gain model, a surrogate gain can be applied over a super-Gaussian region using Beer's law to mimic the FEL interaction [10]. The saturation power of the Beer's law formula was arbitrarily set to a value that produced 1 kW of outcoupled power.

The outcoupler was assumed to transmit 13% of the incident power and absorb 0.1%/cm through the 0.635 cm of its thickness. The case run is equivalent to a figure of merit of 8.1 x $10^6$ in table 2. As shown above the results can be scaled to other wavelengths, powers, and coating and bulk absorption according to $P_l / F\lambda$.

Thermal effects can be modeled in GLAD by the use of an impulse function approach. That is, a delta function heat source in the center of a cylindrically symmetric plate will spread as a function of time ($t$) according to a Gaussian heat distribution (the impulse function) satisfying the thermal diffusion equation. The details of this are given in reference [10].

A scan of the intensity and phase at the outcoupler and the mode shape entering the wiggler for t=6.15 sec are shown in Figure 3. The peak temperature rise in the center of the mirror is 4.4 °C, 20% more than predicted by the analytic approach. There is a corresponding increase in the peak distortion. Nonetheless the mode comes to a stable operating point with modes that are close to Gaussian. The outcoupled mode is essentially diffraction limited with a Strehl of > 0.9. The primary effect of the distortion is a slight change in the radius of curvature in the outcoupled wave. This will be compensated in the real system by moving a collimating mirror by 5 cm as the FEL operating point stabilizes in power. The model confirms predictions that the system comes to a stable operating point over a real time period of tens of seconds. Negative effects on the overall performance of the FEL are negligible at this level of distortion.

### 3.3 Multimode simulations

Using the code ELIXER [2] we studied the behavior of the optical cavity with the gain medium more accurately modeled. Up to six optical modes were included in the simulation. The ratio of the $TEM_{00}$ optical mode power to the total power from a simulation of the laser at the peak small signal gain is shown in figure 3. The lowest order Gaussian mode has only 95% of the power during the laser buildup but this fraction grows to 99.6% at saturation. In all simulations to date we have found that the saturated mode is greater than 99% $TEM_{00}$.

## 4. CONCLUSIONS AND FUTURE WORK

We have derived an analytic solution to the change in the cavity parameters in a near-concentric resonator with mirror heating. The dominant effect is found to be a change in the Rayleigh range. Spherical aberration is negligible for cavities with large magnification. We have also defined a figure-of-merit for comparing mirrors which allows one to calculate the limiting power for a given magnification, wavelength, and acceptable change in the Rayleigh range. It is found that sapphire is a good choice for the IR Demo free-electron laser. We have also found that the mode quality should be excellent despite the near-degeneracy of the transverse modes.

In order to benchmark our theory, we have fabricated a test stand to directly study the distortion induced by mirror heating. A 12 Watt carbon dioxide laser is used to heat a calcium fluoride mirror. The surface of the mirror is analyzed using interferometric techniques. The spot size of the laser on the mirror may be varied to study the effect of different Rayleigh ranges and wavelengths and the surface can be carbonized to enhance surface heating or left clean to emphasize bulk heating. This setup will not only benchmark the theory but will also test the mirror holder's ability to provide cylindrically symmetric boundary conditions. All components of the test stand are operational and data will be taken and analyzed in September. We also plan to carry out more multimode simulation at different wavelengths to see whether the good mode quality is preserved.

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Table 1. Optical resonator design for high-power, kilowatt operation.

| **Parameter** | **Requirement** |
|---|---|
| Resonator type | Stable near-concentric |
| Center wavelength ($\lambda$) | 3–4 μm |
| Cavity length ($L$) | 8.0105m |
| Rayleigh range ($z_R$) | 40 cm |
| Output coupling | 13% |
| Length stability | 0.5 μm rms |
| Controls and diagnostics | Remotely controlled and read out. |
| Non-output-coupling losses | <1% |
| Mirror tilt tolerance | <2.6 μrad rms |
| Mirror radius of curvature | 4.045±0.008 m |
| Mirror radius | 2.54 cm |

Table 2. Mirror distortion effects for three different mirror materials. Both the first order result and the result from enough iterations to form a stable solution are shown for the temperature rise, the increase in the Rayleigh range and the shift in the mode waist.

| | $CaF_2$ | sapphire | $MgF_2$ |
|---|---|---|---|
| Assumed bulk absorption at 3 μm | 0.05% | 0.1% | 0.55% |
| Figure of merit F assuming 0.1% surface absorption and 13% transmission. | $5.7x\ 10^5$ | $4.5 \times 10^6$ | $1.15 \times 10^6$ |
| Temperature rise at mirror center °C | 48 | 17 | 30 |
| Iterated temperature rise °C | 60 | 18 | 36 |
| Increase in Rayleigh range | 140% | 25% | 80% |
| Iterated change in Rayleigh range | 320% | 31% | 153% |
| Shift in mode waist (cm) | 20 | 2.3 | 9 |
| Iterated shift in mode waist (cm) | 87 | 3.0 | 23 |
| Max. aberration (waves) | 0.14 | 0.016 | 0.05 |

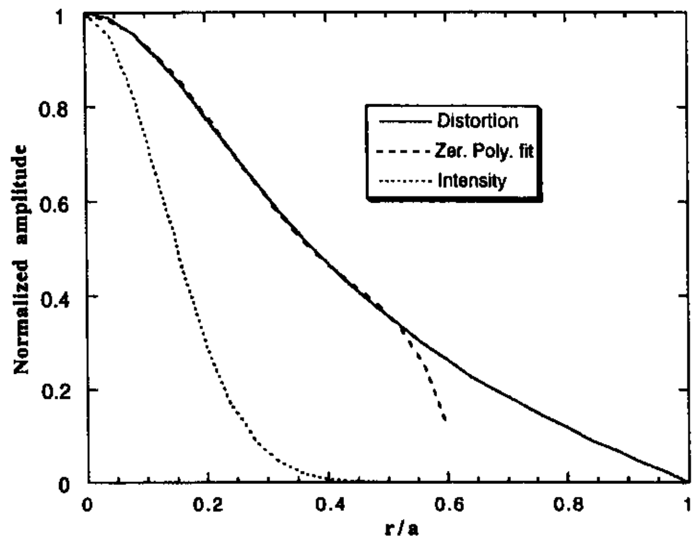


Figure 1. The normalized mirror distortion calculated by ANSYS is plotted along with the intensity of the optical mode and a Zernike polynomial fit including the first three circle polynomials.

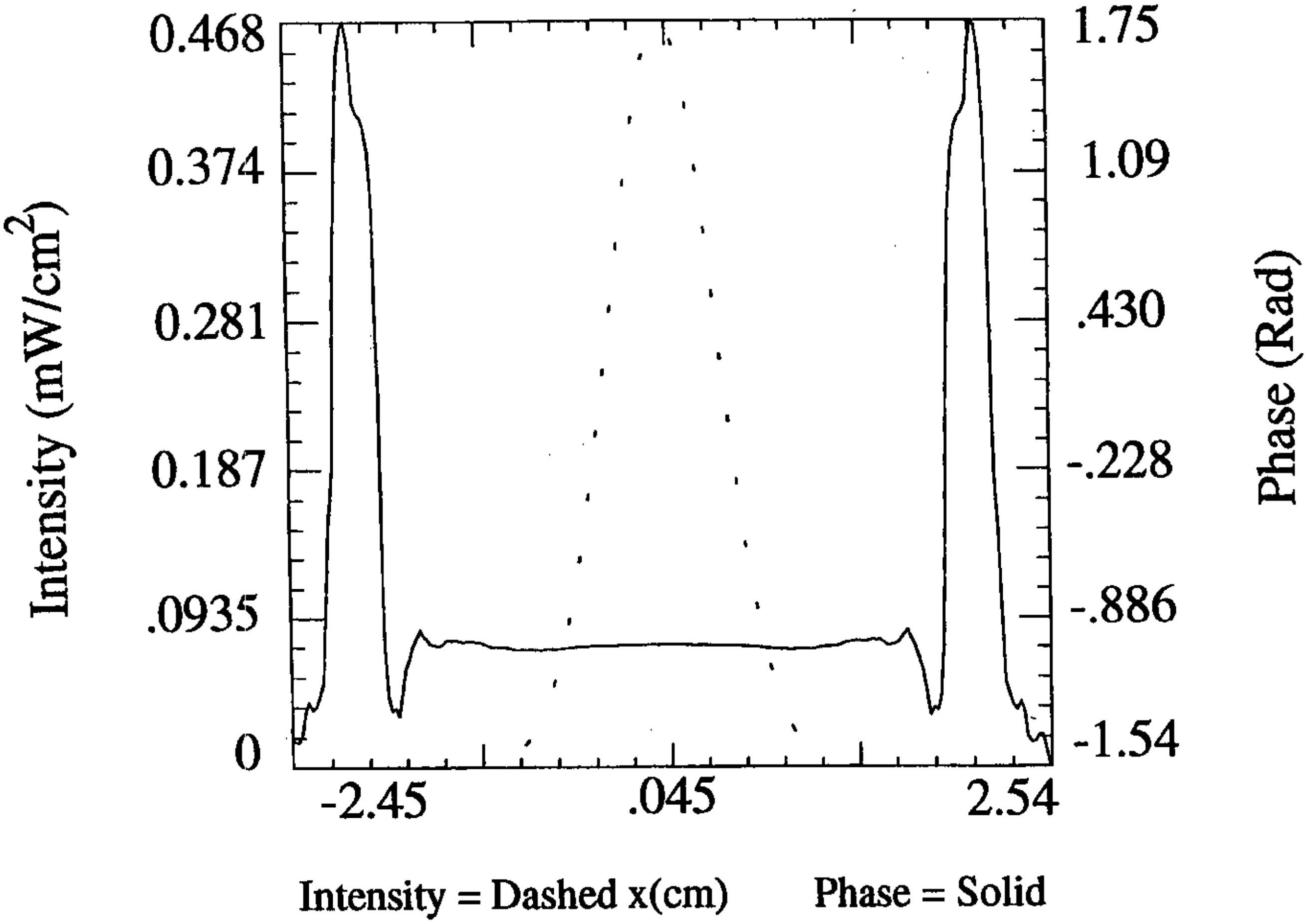


Figure 2. Intensity and phase of the beam at the output coupler at saturation.

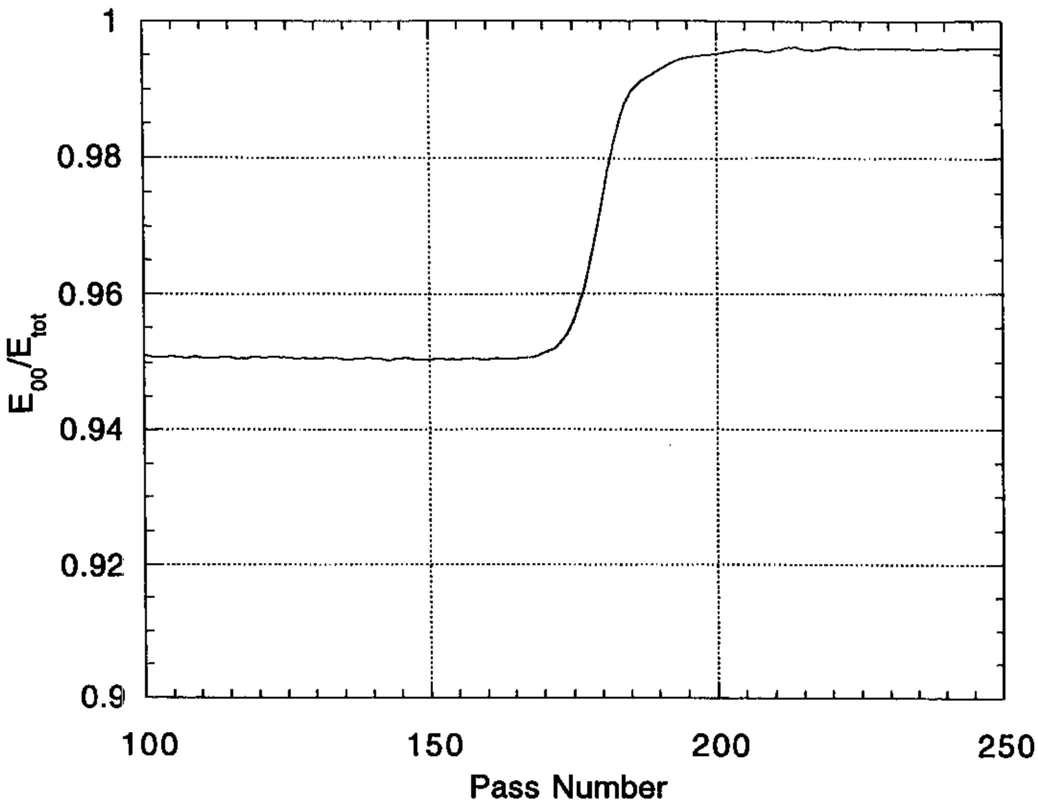


Figure 3. Ratio of the energy in the $TEM_{00}$ mode to the total energy during laser turn-on as modeled in ELIXER.