Progress on the Study of Self-Interaction of a Bunch in a Bend

R. Li, C. L. Bohn, J. J. Bisognano
Thomas Jefferson National Accelerator Facility, 12000 Jefferson Ave., Newport News, VA 23606

Abstract

When a short (mm-length) bunch with high (nC-regime) charge is transported through a magnetic bending system, self-interaction via coherent synchrotron radiation and space charge may cause emittance growth. Earlier we studied analytically the shielded transient self-interaction of a rigid-line bunch entering from a straight path to a circular orbit, and estimated the concomitant emittance degradation in parts of Jefferson Lab's infrared free-electron laser (IR-FEL). In this paper, we generalize our earlier results by calculating the curvature-induced steady-state longitudinal wakefield on particles with transverse offsets from the design orbit. Recent progress in developing a self-consistent simulation are also presented.

1 INTRODUCTION

For short bunches with high charge, the curvature effect on the bunch self-interaction as it traverses through a magnetic bend, by way of coherent synchrotron radiation (CSR) and the space-charge force, may cause serious emittance degradation. In earlier papers we explored analytically the shielded transient self-interaction of a rigid-line bunch entering from a straight path to a circular orbit [1], and estimated the concomitant emittance degradation in parts of the Jefferson Lab IRFEL [2]. In this paper, we generalize our earlier results by calculating the steady-state longitudinal wakefield on particles with transverse offset from the design orbit. We also report our recent progress in developing a self-consistent simulation, including both transverse and longitudinal beam dynamics, that incorporates a viable algorithm for calculating the CSR force by way of a macroparticle model.

2 OFF-AXIS LONGITUDINAL WAKEFIELD

We begin by studying the free-space steady-state longitudinal wakefield generated by a rigid-line bunch on particles with transverse offsets. Consider a line bunch orbiting an arc of radius ρ with angular frequency ω_0, carrying along an electromagnetic field moving with the same angular frequency as the bunch. For the field emitted by a "source" electron S' on the bunch with space-time coordinate (ρ, θ', t') to reach an off-axis "observer" electron S at (r, θ, t), causality requires

$$\rho\Delta\theta = \rho\Delta\phi + \beta R \quad (R = \sqrt{\rho^2 + r^2 - 2r\rho\cos\Delta\theta}), \tag{1}$$

with $\beta = \omega_0\rho/c$, $\Delta\theta = \theta - \theta'$. For the bunch center at angle $\omega_0 t$, we define $\phi = \theta - \omega_0 t$, $\phi' = \theta' - \omega_0 t'$; consequently the angular distance between S and S' in the bunch frame is $\Delta\phi = \phi - \phi'$. Let $\mathbf{n}$ be the unit vector from S' to S, $\mathbf{e}'_\theta$ be the longitudinal unit vector at S', and $\mathbf{e}_\theta$ be the longitudinal vector at S. We then define $\psi = \angle(\mathbf{n}, \mathbf{e}_\theta)$, and $\Theta = \angle(\mathbf{n}, \mathbf{e}'_\theta)$, which is the angle of the radiation flux vector from S' to S with respect to the center of the radiation cone emitted from S'. It can be shown that Θ and ψ are related to $\Delta\theta$ by

$$\Theta = \begin{cases} \sin^{-1}[(r\cos\Delta\theta - \rho)/R] & (r \geq \rho) \\ \sin^{-1}[(\rho - r\cos\Delta\theta)/R] & (r < \rho) \end{cases}, \tag{2}$$

$$\psi = \sin^{-1}[(\rho - r\cos\Delta\theta)/R], \tag{3}$$

which implies $\psi = \Delta\theta + \Theta$ for $r \geq \rho$ and $\psi = \Delta\theta - \Theta$ for $r < \rho$. The longitudinal force on S from S' is $E_{\theta 0} = \sqrt{P(\Theta)}\sin\psi$, where $P(\Theta)$ is the radiation power at S from S' [3]. The functional dependence of $E_{\theta 0}$ on $\Delta\phi$ can be obtained by combining Eqs. (1), (2) and (3). For $r > \rho$, the radiation from S' to S is strongest when $\Theta = 0$, or when $\Delta\theta = \Delta\theta_m$ with $\cos\Delta\theta_m = \rho/r$. For a given transverse offset $x \equiv r - \rho$, one can replace Eq. (1) with $\Delta\theta = \Delta\theta_m$ and get the longitudinal displacement of S' which gives the strongest radiation on S:

$$\Delta\phi_m \simeq -0.94(x/\rho)^{3/2} \quad (x/\rho \gg \gamma^{-2}). \tag{4}$$

For $r < \rho$, S is in the inner side of the design orbit. In this case Eq. (2) shows that the bigger the magnitude of $|x|/\rho$, the further S is away from the center of the radiation cone and the smaller field it experiences. The maximum $P(\Theta)$ occurs at minimum Θ when $d\Theta/d\Delta\theta|_{\Delta\theta_m=0}$, or $\cos\theta_m = 1 - |x|/\rho$; however, this corresponds to $\psi_m = 0$, and consequently $E_{\theta 0} = 0$. The maximum of $E_{\theta 0}$ can be determined from the interplay of $P(\Theta)$ and $\sin\psi$. In addition, it is interesting to note that from Eq. (1) one has

$$d^2\Delta\phi/d\Delta\theta^2|_{\Delta\theta=\Delta\theta_m} = 0, \tag{5}$$

which corresponds to the steep rise of the step-function-like behavior of $\Delta\theta(\Delta\phi)$.

The longitudinal field E_θ on S from the whole bunch is the superposition of single-particle fields $E_{\theta 0}$ generated from sources all over the bunch. For a Gaussian line-bunch, the wakefields versus $s \equiv \rho\phi$ for typical Jefferson Lab IRFEL parameters are plotted in Fig. 1. From this plot, one finds that in comparison with

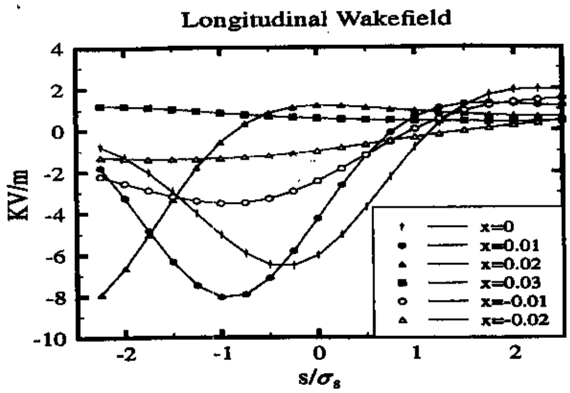

Figure 1: Free-space steady-state longitudinal wakefield generated by a Gaussian line-bunch with $\rho = 1$ m, $\sigma_s = 1$ mm and $I_{\text{peak}} = 36$ A, measured at various transverse offsets.

the on-axis wakefields, wakefields for positive offsets have negative peaks bigger in magnitude and shifted toward the tail of the bunch. Although wakefields with negative offsets are also shifted toward the tail of the bunch, they are smaller in magnitude compared to the on-axis wakefields. Notice in Fig. 1 the negative peaks for $x > 0$ appear at $s = \rho\Delta\phi_m$ as predicted by Eq. (4). This is because for $x > 0$, the field emitted by $\phi' = 0$, where the source particles have the maximum density, is the strongest at $\phi = \Delta\phi_m$ given by Eq. (4). All other features of E_θ in Fig. 1 can also be understood from the functional dependence of $E_{\theta 0}$ on $\Delta\phi$.

3 STATUS OF CSR SIMULATION

Currently we are developing a two-dimensional (2D) self-consistent simulation that includes transverse-longitudinal self-interaction for a bunch transported through an optical system involving bends. To suppress numerical noise, a bunch is simulated using a distribution of 2D disc-like macroparticles with macroparticle number and size chosen to ensure the continuity of the bunch-charge distribution. After a design orbit is established for a given lattice, the equations of motion are solved for the transverse and longitudinal offsets from the design orbit for the macroparticle centroids using the leap-frog method. The core of the simulation is to calculate correctly the force on each macroparticle due to CSR and space charge. For Gaussian-round 2D macroparticles, the force on a test macroparticle centered at (x, y, t) is:

$$\mathbf{F} = \sum_i^N \int d^2\mathbf{r}' \, \frac{n_m^{(i)}}{R^2} \left\{ -\left(\boldsymbol{\beta}_t \cdot \frac{d\boldsymbol{\beta}_0^{(i)}(t')}{cdt'} R\right)\mathbf{n} + (1 - \boldsymbol{\beta}_t \cdot \boldsymbol{\beta}_0^{(i)}) \left(1 + \frac{R}{\sigma_{ms}} \frac{\mathbf{r}' - \mathbf{r}_0^{(i)}(t')}{\sigma_{ms}} \cdot \boldsymbol{\beta}_0^{(i)}\right)\mathbf{n} + (\mathbf{n} \cdot \boldsymbol{\beta}_t - 1)\left(\frac{R}{\sigma_{ms}} \frac{\mathbf{r}' - \mathbf{r}_0^{(i)}(t')}{\sigma_{ms}} \cdot \boldsymbol{\beta}_0^{(i)}(t')\right)\boldsymbol{\beta}_0^{(i)} + (\mathbf{n} \cdot \boldsymbol{\beta}_t)\boldsymbol{\beta}_0^{(i)} + (\mathbf{n} \cdot \boldsymbol{\beta}_t - 1) R \frac{d\boldsymbol{\beta}_0^{(i)}(t')}{cdt'} \right\}, \quad (6)$$

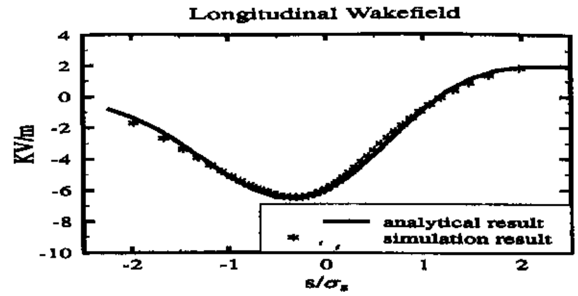

Figure 2: The free-space steady-state longitudinal wakefield on macro-particles obtained from simulation, compared with the analytical result. Here 40 macro-particles are used with the ratio of the macro rms size to the bunch rms size being 0.4.

where $n_m^{(i)} = n_m(x' - x_0^{(i)}(t'), y' - y_0^{(i)}(t'))$ is the charge-density function for the ith macroparticle, with $\mathbf{r}_0^{(i)}(t') = (x_0^{(i)}(t'), y_0^{(i)}(t'))$ the vector for its centroid when the field is emitted. In addition, σ_{ms} is the rms size of a macroparticle, N is the total number of macroparticles, $\mathbf{R} = (x-x', y-y')$, $R = |R|$, $\mathbf{n} = \mathbf{R}/R$, $\boldsymbol{\beta}_0^{(i)} = d\mathbf{r}_0^{(i)}(t')/cdt'$, with $t' = t - R/c$, and $\boldsymbol{\beta}_t = \mathbf{v}/c$ refers to the test macroparticle on which the force is exerted.

The free-space steady-state longitudinal wakefield generated by CSR for a rigid-line bunch in a bend is well known and can be used to benchmark the above macro-particle algorithm. For this purpose, we used one-dimensional (1D) Gaussian macroparticles with their centers subject to an ordered Gaussian distribution, and let them move with the design angular frequency in the bend to simulate a rigid-line bunch. The longitudinal wakefields as a sum of fields generated by the macroparticles were then computed using the 1D version of Eq. (6), the results of which are shown in Fig. 2. Good agreement between numerical and analytical result is achieved when the macroparticles are well overlapped to ensure the continuity of the whole bunch distribution. The general self-consistent 2D simulation for any given lattice is currently in progress, with the goal of providing prediction of CSR effect on beam emittance for the CSR experiment planned for Jefferson Lab IRFEL during early 1998 [5].

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