A MODEL FOR TWO-PION PHOTOPRODUCTION AMPLITUDES *

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We present a brief discussion of the general form of the amplitude that describes the two-pion photoproduction process. We outline an effective Lagrangian method that we are using to calculate this amplitude, and comment briefly on a few aspects of the calculation.

1 Introduction

Experiments in which two pions are photoproduced are expected to play a major role in our quest for the baryons that have been predicted to exist in various versions of the quark model, but for which there is little evidence in the present analyses of the experimental data [1]. The high precisions expected from Jefferson Laboratory, and from other facilities around the globe, as well as the various polarization measurements, should allow us to probe all but the very weakest of these apparently weakly coupled states. On the other hand, a systematic analysis of two-pion photoproduction processes that is similar to those which exist for single pion photoproduction, has not, as far as we know, yet been attempted.

In the past, this process has been analyzed by assuming that the final state arises mainly from a quasi-two-body process such as $\gamma N \to N\rho \to N\pi\pi$ or $\gamma N \to \Delta\pi \to N\pi\pi$ [2]. Application of kinematic cuts consistent with this assumption then yielded some set of results. The recent calculation of Oset *et al.* [3] should highlight the dangers of such a procedure, however. In their calculation, they attempt to fit the recent Mainz data [4] by including a number of resonant and non-resonant terms in an effective Lagrangian approach. They find that the dominant contribution to the cross section comes not from either of the two processes described above, but from the interference of one '$\Delta\pi$' resonant contribution with one of their non-resonant terms.

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In this case, performing kinematic cuts on the data for the purpose of analyzing them in terms of the quasi-two-body channels $\Delta\pi$ and $N\rho$ would yield meaningless results. It is also not at all intuitive that such an interference term should dominate the cross section, in *any* energy range.

With this said, it should be quite clear that a systematic theoretical treatment of the two-pion photoproduction (and electroproduction) process is essential as the starting point for a comprehensive analysis of the high quality data expected. In addition, we believe that it is crucial that future analyses move away from 'parametrizing the non-resonant background' to 'understanding the apparently non-resonant background', as valuable information may be hidden in this background.

To this end, we have undertaken to construct a model for two-pion photoproduction that is as general as possible. An outline of our procedure is the subject of this article: we can not attempt to describe all of the details of the calculation in these few pages, nor can we present any final results, as this is very much work in progress.

In the next section of this note, after a brief discussion of the kinematics of the reaction, we describe the most general form that the amplitude for this process must take. This amplitude requires a certain number of form factors, and we next describe how we calculate the contributions to these form factors, using the effective Lagrangian approach. After a brief discussion of the merits and disadvantages of this approach, we conclude with some comments on how our analysis may lead to new information on the structure of hadrons, both baryons and mesons.

2 Amplitude

2.1 Kinematics

Before we present the amplitude for the process of interest, we first describe the kinematics of the process. The kinematics are shown schematically in figure 1. k is the momentum of the photon, p_1 is that of the target nucleon, p_2 is that of the scattered nucleon, and q_1 and q_2 are the pion momenta. Momentum conservation gives

$$k + p_1 = p_2 + q_1 + q_2. \qquad (1)$$

Thus, when we construct the amplitude for the process using all the four-vectors at our disposal, we can eliminate one of these from consideration.

The total center-of-mass energy of the process is $\sqrt{s}$, where $s = (k + p_1)^2$. We may define a variable t as the square of the momentum transfered to the nucleon. Thus, $t = (p_2 - p_1)^2$, which can be related to the scattering angle of the nucleon in the c-o-m frame.

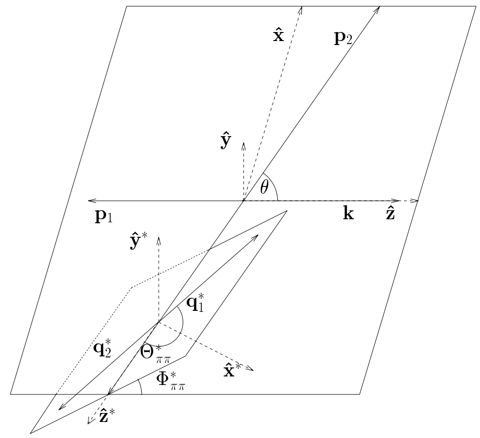

Figure 1. Kinematics of the two-pion photoproduction process. In the figure, the axes $\hat{y} = \hat{y}^*$, $\hat{z}^* = \frac{\vec{q}_1+\vec{q}_2}{|\vec{q}_1+\vec{q}_2|}$, and $\hat{x}^* = \hat{z}^* \times \hat{y}^*$.

The differential cross section for this process is described in terms of 5 kinematic variables. These may be, for instance, two Lorentz invariants and three angles. One obvious choice for one of the invariants is s. The choice of the other four quantities can be fairly arbitrary, and will depend on what information is being presented. One choice is the scattering angle of the nucleon, θ, or equivalently, t. For the other three variables, we can choose, for example, $s_{\pi\pi} \equiv (q_1 + q_2)^2$ and $d\Omega^*_{\pi\pi} \equiv d\Theta^*_{\pi\pi} d\Phi^*_{\pi\pi}$, as illustrated in the figure. Another equally valid choice would be $s_{N\pi_1} \equiv (p_2 + q_1)^2$ and $d\Omega^*_{N\pi_1}$, where the solid angle is defined in the rest frame of the nucleon-pion pair.

2.2 General Amplitude

Our starting point is the identification of the most general form for the transition amplitude for this process. While the requirements of Lorentz covariance and gauge invariance delimit the form of the amplitude, we find that there is nevertheless quite a bit of freedom in the form chosen. The most general form is

$$i\mathcal{M} = \overline{U_f(p_2)} \varepsilon_\mu \mathcal{O}^\mu U_f(p_1) \tag{2}$$

where

$$\begin{aligned} \mathcal{O}^\mu &= a_1 p_1^\mu + a_2 p_2^\mu + a_3 q_1^\mu + a_4 \gamma^\mu + \not{k}\left(a_5 p_1^\mu + a_6 p_2^\mu + a_7 q_1^\mu + a_8 \gamma^\mu\right) \\ &+ \not{q}_1\left(a_9 p_1^\mu + a_{10} p_2^\mu + a_{11} q_1^\mu + a_{12}\gamma^\mu\right) + \not{q}_1\not{k}\left(a_{13} p_1^\mu + a_{14} p_2^\mu + a_{15} q_1^\mu + a_{16}\gamma^\mu\right) \end{aligned} \tag{3}$$

Note that we have no terms in $\not{p}_1$ nor $\not{p}_2$, as the initial and final nucleons each satisfy

$$\not{p} U(p) = m U(p). \tag{4}$$

The form factors a_i are all functions of the kinematic variables s, $s_{\pi\pi}$, θ, θ^* and ϕ^*, or whatever combination of kinematic variables is chosen. Their exact dependence on each of these variables will be determined by the specific model constructed.

Gauge invariance of the amplitude requires that $k_\mu \mathcal{O}^\mu = 0$, which leads to the four relations

$$a_1 k \cdot p_1 + a_2 k \cdot p_2 + a_3 q_1 \cdot k = 0, \tag{5}$$

$$a_4 + a_5 k \cdot p_1 + a_6 k \cdot p_2 + a_7 q_1 \cdot k = 0, \tag{6}$$

$$a_9 k \cdot p_1 + a_{10} k \cdot p_2 + a_{11} q_1 \cdot k = 0, \tag{7}$$

$$a_{12} + a_{13} k \cdot p_1 + a_{14} k \cdot p_2 + a_{15} q_1 \cdot k = 0. \tag{8}$$

Note that there is no condition on either of the form factors a_8 or a_{16}.

From these equations, we can eliminate four of the form factors, leaving us with twelve independent form factors, or Lorentz-Dirac structures, to describe the amplitude. One choice would be to eliminate a_1, a_4, a_9, a_{12}, giving

$$\begin{aligned} \varepsilon_\mu \mathcal{O}^\mu &= \left\{ \frac{1}{p_1 \cdot k} \left[(a_2 + a_{10} \not{q}_1) p_{2\mu} p_{1\nu} + (a_3 + a_{11} \not{q}_1) q_{1\mu} p_{1\nu} \right] + (a_5 + a_{13} \not{q}_1) p_{1\nu} \gamma_\mu \right. \\ &+ \left. (a_6 + a_{14} \not{q}_1) p_{2\mu} \gamma_\nu + (a_7 + a_{15} \not{q}_1) q_{1\mu} \gamma_\nu - \frac{1}{2} (a_8 + a_{16} \not{q}_1) \gamma_\mu \gamma_\nu \right\} F^{\mu\nu}, \end{aligned}$$

where $F^{\mu\nu} = \varepsilon^\mu k^\nu - \varepsilon^\nu k^\mu$. Another choice is a_1, a_5, a_9, a_{13}, giving

$$\begin{aligned} \varepsilon_\mu \mathcal{O}^\mu &= \left\{ \frac{1}{p_1 \cdot k} \left[(a_2 + \not{k} a_6 + \not{q}_1 a_{10} + \not{q}_1 \not{k} a_{14}) p_{2\mu} p_{1\nu} + (a_4 + \not{q}_1 a_{12}) p_{1\nu} \gamma_\mu \right. \right. \\ &+ \left. \left. (a_3 + \not{k} a_7 + \not{q}_1 a_{11} + \not{q}_1 \not{k} a_{15}) q_{1\mu} p_{1\nu} \right] - \frac{1}{2} (a_8 + a_{16} \not{q}_1) \gamma_\mu \gamma_\nu \right\} F^{\mu\nu}. \end{aligned}$$

In terms of the a_i we can generate CGLN-type [5] amplitudes from the structures given. We write the amplitude as [6]

$$i\mathcal{M} = \psi_f^\dagger \mathcal{F} \psi_i, \tag{9}$$

with

$$\begin{aligned} \mathcal{F} &= \vec{\varepsilon} \cdot \vec{q_1} F_1 + \vec{\varepsilon} \cdot \vec{p_2} F_2 + \vec{\varepsilon} \cdot \vec{q_1} \vec{\sigma} \cdot \vec{p_2} \vec{\sigma} \cdot \vec{q_1} F_3 + \vec{\varepsilon} \cdot \vec{p_2} \vec{\sigma} \cdot \vec{p_2} \vec{\sigma} \cdot \vec{q_1} F_4 \\ &+ \vec{\varepsilon} \cdot \vec{q_1} \vec{\sigma} \cdot \vec{p_2} \vec{\sigma} \cdot \vec{k} F_5 + \vec{\varepsilon} \cdot \vec{p_2} \vec{\sigma} \cdot \vec{p_2} \vec{\sigma} \cdot \vec{k} F_6 + \vec{\varepsilon} \cdot \vec{q_1} \vec{\sigma} \cdot \vec{q_1} \vec{\sigma} \cdot \vec{k} F_7 + \vec{\varepsilon} \cdot \vec{p_2} \vec{\sigma} \cdot \vec{q_1} \vec{\sigma} \cdot \vec{k} F_8 \\ &+ \vec{\sigma} \cdot \vec{q_1} \vec{\sigma} \cdot \vec{\varepsilon} F_9 + \vec{\sigma} \cdot \vec{p_2} \vec{\sigma} \cdot \vec{\varepsilon} F_{10} + \vec{\sigma} \cdot (\vec{\varepsilon} \times \vec{k}) F_{11} + \vec{\sigma} \cdot \vec{p_2} \vec{\sigma} \cdot \vec{q_1} \vec{\sigma} \cdot (\vec{\varepsilon} \times \vec{k}) F_{12}, \end{aligned} \tag{10}$$

or

$$\begin{aligned} \mathcal{F} &= \vec{\varepsilon} \cdot \vec{q_1} F'_1 + \vec{\varepsilon} \cdot \vec{p_2} F'_2 + \vec{\varepsilon} \cdot \vec{q_1} \vec{\sigma} \cdot \vec{p_2} \times \vec{q_1} F'_3 + \vec{\varepsilon} \cdot \vec{p_2} \vec{\sigma} \cdot \vec{p_2} \times \vec{q_1} F'_4 \\ &+ \vec{\varepsilon} \cdot \vec{q_1} \vec{\sigma} \cdot \vec{p_2} \times \vec{k} F'_5 + \vec{\varepsilon} \cdot \vec{p_2} \vec{\sigma} \cdot \vec{p_2} \times \vec{k} F'_6 + \vec{\varepsilon} \cdot \vec{q_1} \vec{\sigma} \cdot \vec{q_1} \times \vec{k} F'_7 + \vec{\varepsilon} \cdot \vec{p_2} \vec{\sigma} \cdot \vec{q_1} \times \vec{k} F'_8 \\ &+ \vec{\sigma} \cdot \vec{q_1} \times \vec{\varepsilon} F'_9 + \vec{\sigma} \cdot \vec{p_2} \times \vec{\varepsilon} F'_{10} + \vec{\sigma} \cdot (\vec{\varepsilon} \times \vec{k}) F'_{11} + \vec{\sigma} \cdot \vec{p_2} \times \vec{q_1} \vec{\sigma} \cdot (\vec{\varepsilon} \times \vec{k}) F'_{12}. \end{aligned} \tag{11}$$

Neither of these may be the optimal forms, as we are yet to explore which choice of structures will lead to the simplest representation of helicity amplitudes, differential cross sections, etc. In these last two expressions, the F_i and F_i' are linear combinations of the a_i.

We close this section with a short comment. The form we have written is the most general that can be written for this amplitude. All quantities of interest can now be calculated in terms of the form factors, the a_i. Thus, up to this point, everything has been completely model independent. All that is now left to be done is to construct a model for the a_i. A variety of approaches are possible, but we confine ourselves to a brief discussion of the model we have chosen for this work.

3 Effective Lagrangian

The approach we use to calculate the a_i introduced in the last section is that of the effective Lagrangian. In this approach, we treat all particles as essentially point-like; any structure in these particles will be accounted for by the introduction of appropriate form factors. Next, a set of vertices involving these point-like particles is defined, and a set of 'Feynman' diagrams describing the process of interest is drawn. From each of these diagrams, the contribution to each of the a_i is extracted, and the a_i are thus built from a number of such diagrams. We note that we perform the calculation only at tree level, as it would become rapidly intractable if loops are included, as will be apparent from what follows.

To date, in our calculation, we have included nucleons and Δ's of spin 1/2 and 3/2, as well as vector mesons and, of course, pions. As it is our aim to make our calculation as general as possible from the outset, we do not identify, for instance, any particular spin 1/2 resonance, but simply include a generic set of terms that would be valid for any spin 1/2 resonance.

For spin 1/2 resonances, as well as for the ground state nucleon, the wave function is a Dirac spinor $u(p)$ satisfying

$$\not{p}u(p) = mu(p), \tag{12}$$

where m is the mass of the resonance. The corresponding propagator is

$$\frac{\not{p} + m}{p^2 - m^2 + im\Gamma}, \tag{13}$$

where Γ is the total width of the state. For the ground-state nucleon, the $+im\Gamma$ in the propagator is replaced by the usual $+i\epsilon$.

In the case of the spin-3/2 baryons, the wave function is the Rarita-Schwinger field u_μ, with

$$\not{p}u_\mu(p) = mu_\mu(p). \tag{14}$$

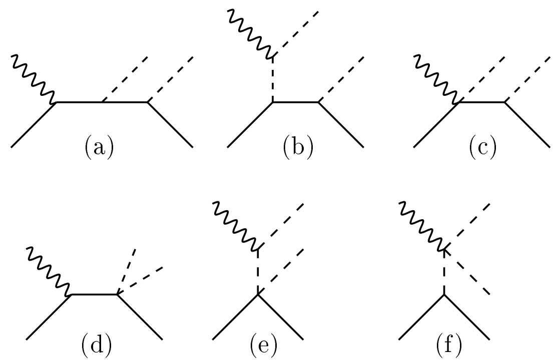

Figure 2. Born diagrams: diagrams containing only ground-state nucleons and pions.

u_μ also satifies the auxiliary conditions

$$p_\mu u^\mu(p) = 0, \quad \gamma_\mu u^\mu(p) = 0, \tag{15}$$

and the appropriate propagator is

$$\Theta_{\mu\nu} = \frac{\not{p} + m}{p^2 - m^2 + im\Gamma}\left(g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{p_\mu\gamma_\nu - p_\nu\gamma_\mu}{3m} - \frac{2p_\mu p_\nu}{3m^2}\right). \tag{16}$$

In writing this form for the propagator, we are ignoring the so-called off-shell contributions, at least for the time being. The off-shell contributions may also be included at the vertices, as has been done, for example, by Benmerrouche *et. al* [7].

In addition to the pions and the photon, the treatment of which is presumed to be well known, the only other particles that we have in our calculation to date are the vector mesons, which are described by the polarization vector ε_μ. This satisfies

$$\varepsilon_\mu(p)p^\mu = 0, \tag{17}$$

and the corresponding propagator is

$$\Theta^{\mu\nu} = \frac{g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}}{p^2 - m^2 + im\Gamma}. \tag{18}$$

Once these fields have been defined, we next proceed to construct effective Lagrangians for them, concentrating on the interaction vertices. With these vertices, one can construct a number of diagrams that contribute to the amplitude, and hence to each of the a_i. Space does not permit us to show all of the vertices that enter into this calculation, so we spare the reader the torture of several tens of vertex diagrams. Instead, we show some of the diagrams that we draw for the process of interest. In all of the diagrams, solid lines represent the ground state nucleons, thick solid lines are baryon resonances, wavy lines are photons, dashed lines are pions, and curly lines are vector mesons.

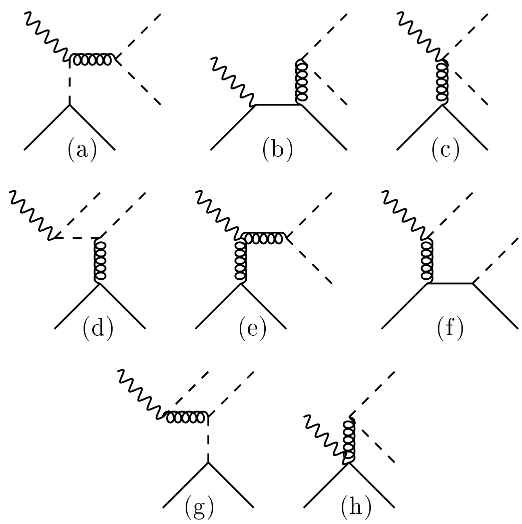

Figure 3. Diagrams containing vector mesons.

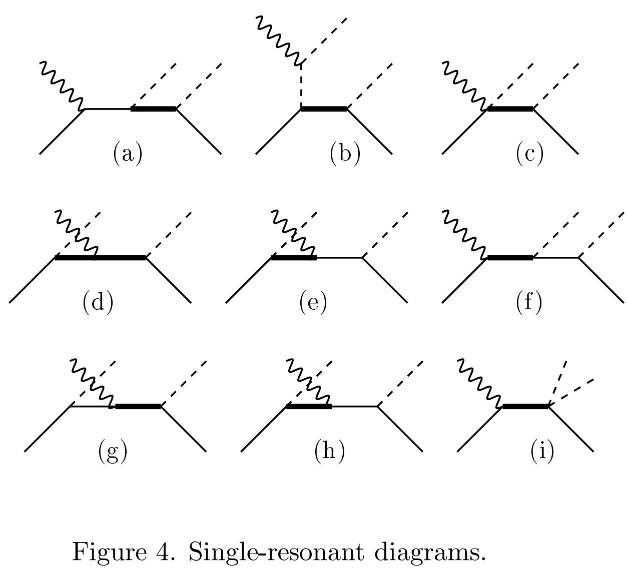

Figure 4. Single-resonant diagrams.

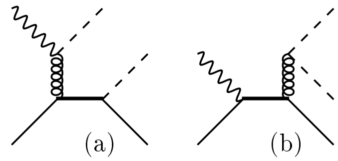

Figure 5. Diagrams containing a single resonance with a vector meson.

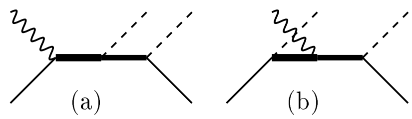

Figure 6. Double-resonant diagrams.

The first set of diagrams shown in figure 2 are the so-called Born terms. By this, we mean diagrams that contain neither baryon nor meson resonances. We point out that each diagram shown actually corresponds to a set of diagrams, as we do not show all of the permutations that arise. For example, figure 2(a) represents 6 different diagrams, the other 5 arising from attaching the photon and pion lines to the nucleon line in all possible ways. In figure 2, diagram (f) arises from the so-called chiral anomaly vertex, which will be the object of study of at least one experiment at Jefferson Lab [8]. Figure 3 shows the diagrams that contain vector mesons in addition to pions and nucleons. Among these diagrams, figure 3(a) provides an amplitude that has the same form as the chiral anomaly diagram of figure 2.

Figure 4 shows the diagrams that contain a single baryonic resonance. In these diagrams, the resonance may be *any* baryon resonance of any allowed spin, isospin and parity. A set of diagrams like these should therefore be included for *every* resonance that one wishes to consider. Note that figure 4(d) leads to amplitudes that are dependent on the magnetic moment(s) of the resonance. Figure 5 shows the diagrams that contain both a baryon resonance and a vector meson, and we again emphasize that a set of diagrams like these should be included for every resonance that is considered. In fact, it may turn out that the contributions of some of these diagrams are very small. However, it is precisely some of these small terms that must be understood if we are to fully comprehend the information provided by the experimental data.

Figure 6 shows the diagrams that contain two different baryonic resonances. The diagrams of figure 6 depend on some 'new' couplings, in general. For instance, figure 6(a) is proportional to the set of couplings $g_{N_1^* N_2^* \pi}$, while figure 6(b) depends on the couplings $g_{N_1^* N_2^* \gamma}$. Thus, contributions like these can provide new information on hadronic structure. As with the diagrams containing a single baryon resonance, these diagrams can contain any combination of two baryon resonances.

4 Conclusion

As of this writing, we have not yet compared our calculation with experimental results, partly because there are several questions that must be addressed before such a comparison is made. One immediate question is that of the validity of the effective Lagrangian approach for particles with size and structure. We propose to accomodate the structure (and size) of these particles by replacing the many coupling constants with form factors.

A second, very important question is that of unitarity in the amplitudes we calculate. This problem could be solved by including the appropriate loops in the calculation, but the inclusion of loop diagrams would make this calculation intractable (there would be far too many possible diagrams for a meaningful analysis to be done). In some sense, the inclusion of form factors at the vertices would partially address this problem. Perhaps a better approach would be to use these amplitudes as input to a coupled-channel treatment.

One obvious possible criticism of our approach is the need for a very large number of diagrams. However, we believe that this is inherent to the process that we are studying, and the information that we seek from this process. We are trying to understand the baryon spectrum, in the hope of learning more about non-perturbative QCD. Thus, we must know the baryon masses and couplings, and there are many couplings for each baryon. These are precisely the parameters that enter into this (or for that matter, *any*) calculation of this process. Thus, comparison of this (or any) calculation with data will allow extraction of these parameters. We note that this is essentially what is done in analyses of pion-nucleon elastic scattering data, from which we have essentially all of the information on the baryon spectrum that is presented in the Particle Data Book [9]. We note in passing that the number of possible diagrams increases dramatically if loop diagrams are included.

One novel 'result' of this calculation is the fact that the two-pion photoproduction process can provide new ways of testing our understanding of hadron structure, by way of the 'new' couplings $g_{N_1^* N_2^* \pi}$ and $g_{N_1^* N_2^* \gamma}$ discussed near the end of the previous section. The value in this new information is obviously dependent on the reliability of the extraction of these numbers from the data. By the same token, we may also be able to probe 'new' aspects of meson structure in this process. For example, figure 3(e) may contain contributions from the dipole and quadrupole moments of the vector meson.

Clearly, cross section data alone will not be enough to delimit the possible choices of parameters that can provide 'good' fits to the available and expected data. There is an essential need for high quality polarization data. This will be crucial if we are to exploit the many opportunities lurking in the shadows of this process. As a corallory we may also need to define new polarization observables for the two-pion process. By this, we mean that polarization observables must be defined for the process $\gamma N \rightarrow N\pi\pi$, *not* for the assumed sub-processes like $\gamma N \rightarrow \Delta\pi$ and $\gamma N \rightarrow N\rho$. We are currently examining this problem.

We note that it should be reasonably straightforward, if tedious, to extend the method we have presented to the study of two-pion electroproduction. However, we make no further comments on this here, as we would like to understand as much as possible about photoproduction before extending the model to study electroproduction.

We conclude by pointing out that two-pion photoproduction can provide information related to many issues in hadron phenomenology: the question of missing baryons; exotic states; the chiral anomaly; and new details of baryon and meson structure, to name just a few. However, the complexity of the theoretical treatment required means that extraction of any meaningful information will require a Herculean effort. We believe that what we have presented herein is one small step in that effort.

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