Complete Leading Order Analysis in Chiral Perturbation Theory of the Decays $K_L \to \gamma\gamma$ and $K_L \to \ell^+\ell^-\gamma$

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Abstract

The decays $K_L \to \gamma\gamma$ and $K_L \to \ell^+\ell^-\gamma$ are studied at the leading order p^6 in Chiral Perturbation Theory. One-loop contributions stemming from the odd intrinsic parity $\mid \Delta S \mid = 1$ effective Lagrangian of order p^4 are included and shown to be of possible relevance. They affect the decay $K_L \to \gamma\gamma$ adding to the usual pole terms a piece free of counterterm uncertainties. In the case of the $K_L \to \ell^+\ell^-\gamma$ decays the dependence of the form factor on the dilepton invariant mass requires a counterterm. The form factor may receive a sizeable contribution from chiral logarithms. Including considerations from the $K_L \to \pi^+\pi^-\gamma$ direct emission amplitude, we obtain two consistent scenarios. In one scenario the long distance contributions from the one-loop terms are important, while in the other they are marginal. In both cases the counterterm is shown to be significant.

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The radiative decays of the long lived neutral K meson, K_L, have been the subject of numerous theoretical analyses, in Chiral Perturbation Theory (χPT) [1]. Among these decays, the decay $K_L \to \gamma\gamma$ and the Dalitz decays $K_L \to \ell^+\ell^-\gamma$ still show open issues which we address in this paper. Both types of decays proceed at order p^6 in χPT. The pole-type terms given by π^0 and the η_8 intermediate states [2–4] are individually of leading order p^4, but their added contribution cancels at this order due to the Gell-Mann-Okubo relation [3] and is in effect of order p^6. It is, therefore, important to perform a complete analysis of the decays at this order. We show that the one-loop terms of order p^6 stemming from the odd-intrinsic parity terms of the $\mid \Delta S \mid = 1$ non-leptonic effective Hamiltonian of order p^4 and not previously evaluated, affect both types of decays in a potentially significant way. We find, in particular, a contribution to the amplitude of the two-gamma decay proportional to $M_K^2 + M_\pi^2$ that can be a substantial addition to the pole-type term. In the case of the Dalitz decays, the experimentally observed non-trivial form factor in terms of the dilepton invariant mass is usually described using the VMD model of Bergström, Massó and Singer [5], which gives a satisfactory parameterization. The work of Ko [6] provides a thorough study of VMD in radiative decays. Recently, other models like the factorization model (FM) and the weak deformation model (WDM) have been considered as well [7]. Our calculation includes long distance pieces given by the chiral loops, and a short distance piece due to a counterterm needed to renormalize the loops is determined by fitting to the data. For our purpose, the experimental situation is good. The two-gamma mode is experimentally very precisely known: BR($K_L \to \gamma\gamma$)= $(5.92 \pm 0.15) \times 10^{-4}$ [8]. The branching ratios of the Dalitz decays are on the other hand known within 10%: $BR(K_L \to e^+e^-\gamma) = (9.2 \pm 0.5 \pm 0.5) \times 10^{-6}$ (NA31 [9] and BNL-E845 [10] collaborations), and $BR(K_L \to \mu^+\mu^-\gamma) = (3.23 \pm 0.23 \pm 0.19) \times 10^{-7}$ (E799 collaboration [11]). The rate of the $\mu^+\mu^-$ mode is about 30% larger than it would be for the case of a point-like form factor. The form factor also shows up clearly in dilepton invariant mass distribution of both modes. Using this data and some input from the $K_L \to \pi^+\pi^-\gamma$ decay [12], we are able to two phenomenologically acceptable scenarios.

We work in the standard framework of χPT. The octet of pseudoscalar mesons is parameterized by the SU(3) matrix

$$U = \exp\,(i\,\frac{\Pi}{F_0}),$$
$$\frac{1}{\sqrt{2}}\Pi = \begin{pmatrix} \sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\,\eta_8 & \pi^+ & K^+ \\ \pi^- & -\sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\,\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\,\eta_8 \end{pmatrix}, \tag{1}$$

where $F_0 \sim F_\pi = 93$ MeV is the pion decay constant in the chiral limit. For the processes we consider the only gauge couplings we need are those to the EM field. We thus denote:

$$L_\mu \equiv i\,U^\dagger\,D_\mu\,U$$
$$D_\mu U = \partial_\mu U - i A_\mu\,[\hat{Q},\,U], \tag{2}$$

where $\hat{Q}$ is the quark charge matrix.

We need the strong interaction effective chiral Lagrangian up to order p^4 [13], including, of course, EM couplings and the Wess-Zumino (WZ) Lagrangian. The non-leptonic $\mid \Delta S = 1 \mid$ effective chiral Lagrangian is also required up to order p^4 [14]. It contains an 8 and a 27 of SU(3); we advocate the $\Delta I = 1/2$ rule to disregard the marginal effects of the 27 piece. The octet piece of order p^2 is

$$\mathcal{L}^{(2)}_{|\Delta S|=1} = \frac{F_0^2}{4}\,C_8\,Tr(\lambda_6\,L_\mu\,L^\mu), \tag{3}$$

where $C_8 = 3.12 \times 10^{-7}$ is given by

$$C_8 = 4F_0^2 G_8,$$
$$G_8 = \sqrt{\frac{1}{2}} G_F s_1 c_1 c_3 g_8, \tag{4}$$

Here s_i, c_i are the sine and the cosine of CKM angles in the Kobayashi-Maskawa representation, and $g_8 \simeq 0.5$ is an effective low energy coupling. At order p^4 there is a long list of terms (forty eight octet terms when external sources of different kinds are included) [14]. The relevant terms to us are those that affect the virtual transitions $K_L \to \pi^0$, η_8, including terms that break SU(3) symmetry due to the quark masses, and terms that give the amplitudes $K_L \to \pi^+\pi^-\gamma$ and $K_L \to \pi^+\pi^-\gamma\gamma$. The first are contained in the terms bilinear in the meson fields of the $\mathcal{O}(p^4)$ Lagrangian [14], and the second are given by odd-intrinsic parity terms of which only two combinations of the operators O_i^8 in [14] appear, namely, $W_{29} \equiv O_{28}^8 + O_{29}^8$, and $W_{31} \equiv O_{30}^8 + O_{31}^8$. The piece of the Lagrangian $\mathcal{L}^{(4)}_{|\Delta S=1|}$ containing these two terms can be written as follows [12]:

$$\mathcal{L}^{(4)}_{29,\,31} = \frac{C_8}{64\,\pi^2}\left\{a_2\,\mathrm{Tr}(\Lambda\,[U^\dagger \tilde{F}_R^{\mu\nu} U,\; L_\mu L_\nu]) + a_4\,\mathrm{Tr}(\Lambda\,L_\nu)\,\mathrm{Tr}((\tilde{F}_L^{\mu\nu} - U^\dagger \tilde{F}_R^{\mu\nu} U)L_\nu)\right\} + \mathrm{h.c.} \tag{5}$$

where $\Lambda \equiv \frac{1}{2}(\lambda_6 - i\lambda_7)$. In our case, where the only external gauge field of relevance is the EM field, we have $F_R^{\mu\nu} = F_L^{\mu\nu} = F^{\mu\nu}\hat{Q}$, and $\tilde{F}^{\mu\nu} \equiv \epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ is its dual. Since the Lagrangians of order p^2 are of even intrinsic parity, they alone cannot generate odd intrinsic parity terms. Therefore, the low energy couplings a_2 and a_4 are renormalization-scale independent.

In the absence of CP violation K_L coincides with the CP-odd state $K_2 = \frac{1}{\sqrt{2}}(K^0 + \bar{K^0})$, and the pieces of $\mathcal{L}^{(4)}_{29,\,31}$ leading to the transitions $K_L \to \pi^+\pi^-\gamma$ and $K_L \to \pi^+\pi^-\gamma\gamma$ are explicitly:

$$\begin{aligned}
\mathcal{L}_{K_L\to\pi^+\pi^-\gamma} &= i\frac{e}{8\pi^2 F_0^3}\, C_8\, \epsilon^{\mu\nu\rho\sigma}\, [a_2\, K_L \partial_\mu\pi^+\partial_\nu\pi^- \\
&\quad + a_4\, \partial_\mu K_L\, (\pi^-\partial_\nu\pi^+ - \pi^+\partial_\nu\pi^-)]\, \partial_\rho A_\sigma \quad , \\
\mathcal{L}_{K_L\to\pi^+\pi^-\gamma\gamma} &= -\frac{e^2}{8\pi^2 F_0^3}\, C_8\, \epsilon^{\mu\nu\rho\sigma}\, [a_2\, K_L\, \partial_\mu A_\nu \\
&\quad - 2\, a_4\, \partial_\mu K_L\, A_\nu]\, \partial_\rho A_\sigma\, \pi^+\pi^- + \ldots
\end{aligned} \tag{6}$$

In the second Lagrangian we do not display terms that do not contribute in our calculation.

The SU(3) singlet meson η_1, being a massive state due to the $U_A(1)$ anomaly, can be integrated out or, equivalently, included explicitly in a meson pole dominance model [3, 4]. The η_1 contribution to the decays is naturally of order p^6. Since η_1 and η_8 mix with each other due to SU(3) breaking, we find it more convenient to include η_1 explicitly. We need to consider both, the terms that contribute to the virtual transition $K_L \to \eta_1$ as well as the WZ term, which leads to the standard amplitude for $\eta_1 \to \gamma\gamma$. The mixing angle between the η mesons and the decay constants as obtained from their two-gamma decays and the $\mathcal{O}(p^4)$ result for F_{η_8} [13] are:

$$\begin{aligned} \theta &= -21 \pm 5\,\mathrm{deg} \\ F_{\eta_8} &= 1.3\,F_\pi \\ F_{\eta_1} &= 1.1\,F_\pi \end{aligned} \tag{7}$$

The virtual weak amplitude $K_L \to \eta_1$ is not known, and is characterized in the following by a parameter κ.

In the limit of CP conservation, the $K_L \to \gamma\gamma^*$ amplitude has the most general form:

$$A(K_L \to \gamma\gamma^*) = F(t)\; \epsilon^{\mu\nu\rho\sigma}\; \epsilon_\mu k_\nu \epsilon^*_\rho k^*_\sigma. \tag{8}$$

Here γ^* is a virtual photon, and $t = k^{*2}$. The branching ratio for the two-gamma decay mode is

$$B(K_L \to \gamma\gamma) = \frac{M_K^3 \mid F(0) \mid^2}{64\pi\Gamma_{K_L}}, \tag{9}$$

and the dilepton distribution of the Dalitz decays is given by

$$\frac{d\Gamma_{K_L\to\ell^+\ell^-\gamma}}{dt} = \mid f(t) \mid^2 \Phi(t,\, m_\ell)\, \Gamma_{K_L\to\gamma\gamma}, \tag{10}$$

where $f(t) \equiv F(t)/F(0)$, m_ℓ is the mass of the lepton, and

$$\Phi(t,\, m_\ell) \equiv \frac{\alpha_{\mathrm{em}}}{t}\frac{2}{3\pi}\,(1 - \frac{t}{M_K^2})^3\,(1 + \frac{2m_\ell^2}{t})\sqrt{1 - \frac{4m_\ell^2}{t}}. \tag{11}$$

There are two different sets of diagrams contributing to $F(t)$. The first set are the pole terms shown in Figures 1 and 2 involving the virtual transitions $K_L \to \pi^0,\ \eta_8,\ \eta_1$ followed by the two-photon decay modes driven by the WZ term. The second set of diagrams, shown in Figure 2, requires the weak interaction transitions $K_L \to \pi^+\pi^-\gamma$ and $K_L \to \pi^+\pi^-\gamma\gamma$.

As mentioned before, the $\mathcal{O}(p^4)$ contributions due to the π^0 and η_8 poles cancel each other due to the Gell-Mann-Okubo (GMO) relation [3]. At $\mathcal{O}(p^6)$ one-loop corrections and counterterms must be considered. From this various effects result. One is due to the deviation from the GMO relation, which is taken into account by replacing the lowest order masses by the physical masses. Another one is due to SU(3) breaking in virtual amplitudes $K_L \to \eta_8$ with respect to $K_L \to \pi^0$, which we take into account by means of the parameter δ below. A final effect stems from the one-loop corrections to the WZ term. As shown in [15], there is a t independent piece that is taken into account by simply replacing the decay constants in the chiral limit by the physical decay constants, while the t dependent piece requires renormalization. Fortunately, the latter only appears in contributions to the $K_L \to \ell^+\ell^-\gamma$ decay of order p^8, and can therefore be altogether disregarded. The reason is that the t-dependent corrections to the WZ term have the same relative weight for the π^0 and η_8 as in the WZ term itself [15], and therefore the mechanism of cancellation by the GMO relation eliminates the order p^6 terms. Since the η_1 contribution is already of order p^6 at tree level, the one loop corrections are of order p^8 and, therefore, disregarded. From this, the pole piece of $F(t)$ can be expressed as follows [3, 4]:

$$
\begin{aligned}
F_1 &= -\frac{\alpha_{\rm em}}{2\pi F_\pi}\, C_8\, \tilde{F}_1 \\
\tilde{F}_1 &\equiv r_\pi + r_\eta \Theta_1 + r_{\eta'} \Theta_2,
\end{aligned}
\tag{12}
$$

where

$$
\begin{aligned}
r_P &\equiv 1/\left(1 - \frac{m_P^2}{M_{K_L}^2}\right), \\
\Theta_1 &= \frac{1}{3}\, \left((1+\delta)\, \cos\theta + 4\,\kappa\, \sin\theta\right) \left(\frac{F_\pi}{F_{\eta_8}}\, \cos\theta - 2\sqrt{2}\frac{F_\pi}{F_{\eta_1}}\, \sin\theta\right), \\
\Theta_2 &= \frac{1}{3}\, \left((1+\delta)\, \sin\theta - 4\,\kappa\, \cos\theta\right) \left(\frac{F_\pi}{F_{\eta_8}}\, \sin\theta + 2\sqrt{2}\, \frac{F_\pi}{F_{\eta_1}}\, \cos\theta\right).
\end{aligned}
\tag{13}
$$

We emphasize that F_1 is very sensitive to the parameteres κ and δ, and therefore, cannot be reliably predicted.

The one-loop diagrams of Figure 2 containing the vertices of order p^4 in (6) give the following contribution to $F(t)$:

$$
F_2(t) = \frac{\alpha_{\rm em} C_8}{192\pi^3 F_0^3}\, \left\{(a_2 + 2a_4)\, \left[16(M_\pi^2 + M_K^2) - D(t,\mu)\right] + CT(t,\mu)\right\} \quad , \tag{14}
$$

where CT denotes a counterterm, and the function $D(t,\mu)$ is given in dimensional regularization by

$$D(t,\mu) = t\,[\frac{10}{3} + 2\lambda - (\log\frac{M_K^2}{\mu^2} + \log\frac{M_\pi^2}{\mu^2})] \\ +4(F(M_\pi^2,t) + F(M_K^2,\mathrm{t})) \quad , \tag{15}$$

where

$$F(m^2,\ t) \equiv \left((1-\frac{y}{4})\sqrt{\frac{y-4}{y}}\log\frac{\sqrt{y}+\sqrt{y-4}}{-\sqrt{y}+\sqrt{y-4}} - 2\right)m^2, \\ y \equiv \frac{t}{m^2}, \quad \lambda \equiv \frac{1}{\varepsilon} + 1 + \log 4\pi - \gamma_E. \tag{16}$$

U-spin symmetry of $\mathcal{L}_{|\Delta S=1|}$ implies that the π^+ and the K^+ loops have the same sign.

Only the t-dependent piece needs renormalization provided by the counterterm $CT(\mu)$. The t-independent UV divergencies cancel when the two diagrams in Figure 2 are added. The t-dependent piece needs renormalization that is, as usual, accomplished by replacing in (14) $CT(t,\mu)$ plus the term proportional to λ by $C(\mu)\,t$. As shown later, this counterterm can be estimated by analyzing the data. Therefore, $F_2(0)$ is unaffected by counterterm uncertainties, and is non-vanishing and proportional to $M_K^2 + M_\pi^2$. This term is thus a new addition to the $K_L \to \gamma\gamma$ amplitude. Notice that one can write a finite counterterm for the $t = 0$ piece. Such a counterterm is included in F_1, and in the pole model it is given by the η_1 intermediate state as already discussed. The situation here resembles that of the strong interaction case at order p^4, where the term proportional to the low energy constant L_7 only provides a finite renormalization, and is in fact dominated by the η_1 pole [13]. The t independent pieces of $F(t)$ vanish in the chiral limit in accordance with the Veltman-Sutherland theorem.

The unknown parameters affecting the two-photon and Dalitz modes are $\tilde{F}_1$ and $(a_2 + 2a_4)$, while the counterterm $C(\mu)\,t$ only affects the Dalitz mode. When we neglect the t-dependence in F_2, (10) gives

$$\Gamma^{(0)}_{K_L\to\ell^+\ell^-\gamma} = \Gamma_{K_L\to\gamma\gamma}\int_{4m_\ell^2}^{M_K^2}\Phi(t,\ m_\ell)\,dt \tag{17}$$

and the ratios with the experimental values are [8]:

$$R_{e^+e^-\gamma} \equiv \frac{\Gamma^{Exp}_{K_L\to e^+e^-\gamma}}{\Gamma^{(0)}_{K_L\to e^+e^-\gamma}} = 0.97 \pm 0.05$$

$$R_{\mu^+\mu^-\gamma} \equiv \frac{\Gamma^{Exp}_{K_L\to \mu^+\mu^-\gamma}}{\Gamma^{(0)}_{K_L\to \mu^+\mu^-\gamma}} = 1.36 \pm 0.14 \tag{18}$$

Thus, (17) is an excellent approximation in the e^+e^- mode, where most of the rate is given by the low-t domain where the relative t-dependence is very small, while in the $\mu^+\mu^-$ case $t_{\min} = 4m_\mu^2$ is large enough for the t-dependence to be noticeable. Thus, on the basis of the total rate of the $\mu^+\mu^-$ Dalitz mode it is clear that there is a significant t-dependence in F_2, as was first experimentally noticed by the E799 Collaboration [11]. Denoting for the sake of convenience,

$$\begin{aligned} C_1 &\equiv (a_2 + 2a_4), \\ C_2 &\equiv \frac{1}{4}\,(a_2+2a_4)\left(\frac{10}{3} - (\log\frac{M_K^2}{\mu^2} + \log\frac{M_\pi^2}{\mu^2})\right) - \frac{C(\mu)}{4}\ , \end{aligned} \tag{19}$$

from the ratio $R_{\mu^+\mu^-\gamma}$ we find that they are approximately linearly related as

$$C_2 \simeq -\text{sign}(F(0))\,(3.8 \pm 1.3) + 1.1\,C_1. \tag{20}$$

With this relation we also find an excellent fit to both dilepton invariant mass distributions [9–11]. As expected, the Dalitz decays alone cannot pin-down C_1 and the counterterm. It is necessary to invoke further observables for this to be possible. The value of C_1 can also be estimated by means of two other processes, namely, $K_L \to \gamma\gamma$ and $K_L \to \pi^+\pi^-\gamma$. The amplitude of the first process is given in terms of $F(0) = \frac{\alpha_{\rm em} C_8}{2\pi F_\pi}\, L_1$ with

$$L_1 \equiv -\tilde{F}_1 + C_1\,\frac{M_K^2 + M_\pi^2}{6\pi^2\,F_\pi^2}. \tag{21}$$

From the $K_L \to \gamma\gamma$ width (9) this combination has the value ± 0.89. On the other hand, the second process permits a model dependent estimate of a different combination [12], namely,

$$L_2 \equiv C_1 - \tilde{F}_1. \tag{22}$$

While in (21) both terms are of the same chiral order, in (22) the first term is of lower order than the second. Indeed, C_1 gives the strength of the direct emission $M1$ amplitude of order p^4, while the second term appears in the correction of order p^6 to that amplitude [12]. There is, in addition, the theoretically well known internal bremsstrahlung amplitude of order p^2 that is CP violating and cannot interfere with the direct emission one because it is of electric type. Thus, the direct emission decay rate can be cleanly identified, and the combination in (22) is then estimated to be in the interval 0.3 to 0.9 [12]. The positive sign of the combination is favored by a factorization model [12], but the possibility of a negative sign is not altogether ruled out. For a similar negative interval the conclusions we draw here remain unchanged. Taking the estimated range at face value, we obtain C_1 and $\tilde{F}_1$ for the two possible signs of L_1 as shown in the table. A first consequence is that the amplitude for the two-gamma decay has a potentially important addition beyond the pole terms, depending on the scenario, defined by the sign of L_1, one considers. This is shown in the table by the ratio $F_2(0)/F_1$, which can be as large as 0.6.

The slope of the Dalitz decay form factor defined by

$$b \equiv \frac{1}{2}\,\frac{d}{dx} f^2(t)\,|_{x\to 0} = \frac{M_K^2}{24\,\pi^2\,F_\pi^2\,L_1}\,(\frac{4}{3}C_1 - C_2), \quad x \equiv \frac{t}{M_K^2} \tag{23}$$

is consistent with the experimental value obtained from the e^+e^- mode, $b_{\mathrm{Exp}} = 0.6 \pm 0.25$ [9, 10]. In fact, in the acceptable range of C_1 (20) gives $b = 0.45 \pm 0.2$. We notice that the chiral logarithm terms as well as the counterterm give positive contributions to the slope. The fraction of the slope due to the counterterm is shown in the last entry of the table. We see that in one case it is dominating, giving support to the VMD model [5], while in the other case there are important chiral logarithm terms as well.

Since the $\mu^+\mu^-$ mode shows more prominently the t-dependence, the results are mostly determined by that mode. In turn, for the e^+e^- mode we predict $R_{e^+e^-\gamma} = 1.025 \pm 0.010$. This ratio is consistent within the 5% error of the experimental result (18).

In conclusion, our leading order analysis shows two acceptable scenarios. In one of them there are sizeable long distance contributions from one-loop diagrams to both types of radiative decays considered, while in the other such contributions are small, implying that VMD models are an excellent picture. The question of which of the two scenarios is actually realized is clearly important.

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FIGURES

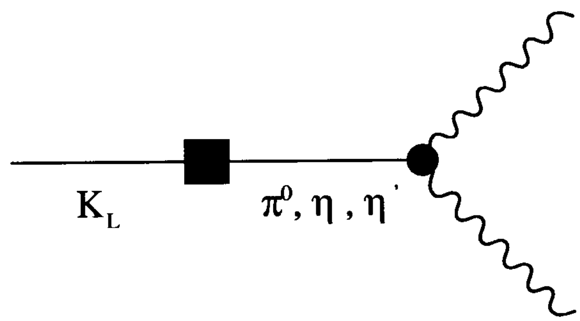

Figure 1: Pole diagrams. The square represents the insertion of the $\mid \Delta S = 1 \mid$ effective non-leptonic weak interaction and the dot is the WZ term.

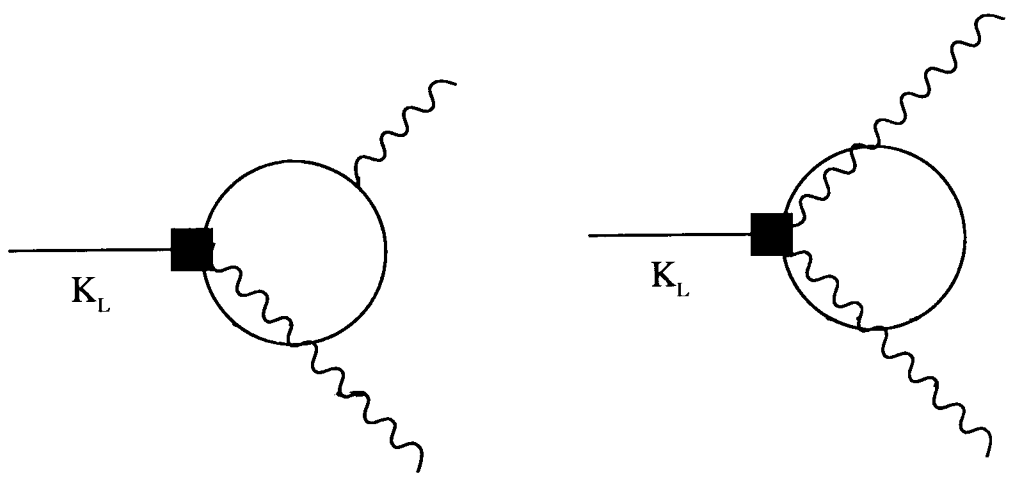

Figure 2: Order p^6 one-loop diagrams. The square represents the insertion of $\mathcal{L}^{(4)}_{29,\ 31}$. The mesons in the loop are π^+ and K^+.

TABLE

L_1	C_1	$\tilde{F}_1$	C_2	$F_2(0)/F_1$	b	$b_{\rm CT}/b$
0.89 $0.3 \leq L_2 \leq 0.9$	-0.6 ± 0.6	-1.2 ± 0.3	-4.5 ± 1.3	0.25 ± 0.25	0.5 ± 0.2	0.9 ± 0.1
-0.89 $0.3 \leq L_2 \leq 0.9$	3.1 ± 0.6	2.5 ± 0.3	7.2 ± 1.3	$0.65^{\pm}0.15$	0.4 ± 0.2	0.4 ± 0.1