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Light-Ray Evolution Equations and Leading-Twist Parton Helicity-Dependent Nonforward Distributions

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We discuss the calculation of the evolution kernels $\Delta W_\zeta(X,Z)$ for the leading-twist nonforward parton distributions $\mathcal{G}_\zeta(X,t)$ sensitive to parton helicities. We present our results for the kernels governing evolution of the relevant light-ray operators and describe a simple method allowing to obtain from them the components of the nonforward kernels $\Delta W_\zeta(X,Z)$.

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I. INTRODUCTION

Applications of perturbative QCD to deeply virtual Compton scattering [1–4] and hard exclusive electroproduction processes [5–7,4] require a generalization of usual parton distributions for the case when long-distance information is accumulated in nonforward matrix elements $\langle p-r|\mathcal{O}(0,z)|p\rangle|_{z^2=0}$ of quark and gluon light-cone operators. In refs. [2,6,4] it was shown that such matrix elements can be parametrized by two basic types of nonperturbative functions. The double distribution $F(x,y;t)$ specifies the light-cone "plus" fractions xp^+ and yr^+ of the initial hadron momentum p and the momentum transfer r carried by the initial parton. Since r^+ is proportional to p^+: $r^+ \equiv \zeta p^+$, it is possible to introduce the nonforward parton distribution $\mathcal{F}_\zeta(X;t)$ with $X = x + y\zeta$ being the total fraction of the initial hadron momentum taken by the initial parton[‡]. For processes mentioned above, the parameter $\zeta = 1 - (p'z)/(pz)$ characterizing the longitudinal momentum asymmetry ("skewedness") of the nonforward matrix element takes the values $0 < \zeta < 1$.

At leading twist, there are two light-ray quark operators $\bar\psi(0)\gamma_\mu E(0,z;A)\psi(z)$ and $\bar\psi(0)\gamma_\mu\gamma_5 E(0,z;A)\psi(z)$, where $E(0,z;A)$ is the standard path-ordered exponential which makes the operators gauge-invariant. In the forward case, the first operator is related to the spin-averaged distribution functions $f_a(x)$ while the second one corresponds to the spin-dependent distribution functions $\Delta f_a(x)$. The nonforward parton distributions related to the $\bar\psi(0)\gamma_\mu E(0,z;A)\psi(z)$ operators were studied in refs. [2,6,4]. In this paper, we will discuss flavor-singlet parton helicity-dependent nonforward distributions corresponding to quark operators $\bar\psi(0)\gamma_\mu\gamma_5 E(0,z;A)\psi(z)$ and the gluonic operator $G_{\mu\alpha}(0)E(0,z;A)\tilde G_{\alpha\nu}(z)$ mixing with each other under evolution.

II. NONFORWARD DISTRIBUTIONS

We define the nonforward quark distributions by writing the relevant matrix element as (cf. [1,4])

$$\begin{aligned}
&\langle\, p', s' \mid \bar\psi_a(0)\hat z\gamma_5 E(0,z;A)\psi_a(z) \mid p, s\,\rangle|_{z^2=0} \qquad (1)\\
&= \bar u(p',s')\hat z\gamma_5 u(p,s)\int_0^1 \left(e^{-iX(pz)}\mathcal{G}^a_\zeta(X;t) + e^{i(X-\zeta)(pz)}\mathcal{G}^{\bar a}_\zeta(X;t)\right)dX\\
&+ \frac{(rz)}{M}\,\bar u(p',s')\gamma_5 u(p,s)\int_0^1 \left(e^{-iX(pz)}\mathcal{P}^a_\zeta(X;t) + e^{i(X-\zeta)(pz)}\mathcal{P}^{\bar a}_\zeta(X;t)\right)dX,
\end{aligned}$$

where $t \equiv (p'-p)^2$, a denotes the quark flavor (here we consider only the flavor-diagonal distributions), M is the nucleon mass and s, s' specify the nucleon polarization. Throughout the paper, we use the "hat" convention $\hat z \equiv z^\mu\gamma_\mu$. In Eq.(1), we explicitly separated quark and antiquark contributions (cf. [4]). This definition corresponds to the parton picture in which the initial quark (or antiquark) takes the momentum Xp from the hadronic matrix element and "returns" into it the momentum $(X-\zeta)p$. Since the fraction $X-\zeta$ is positive for $X > \zeta$ and negative when $X < \zeta$, the nonforward distributions can be divided into two components. In the region $X \ge \zeta$, one can treat $\mathcal{G}^a_\zeta(X,t)$ as a generalization of the usual distribution function $\Delta f_a(x)$. In particular, in the limit $t \to 0, \zeta \to 0$, the limiting curves for $\mathcal{G}_\zeta(X,t)$ reproduce $\Delta f_a(X)$:

$$\mathcal{G}^a_{\zeta=0}\,(X,t=0) = \Delta f_a(X) \;\;\; ; \;\;\; \mathcal{G}^{\bar a}_{\zeta=0}\,(X,t=0) = \Delta f_{\bar a}(X). \qquad (2)$$

On the other hand, in the region $X < \zeta$, both quarks should be treated as going out of the nucleon matrix element, with momenta Xp and $(\zeta - X)p$, respectively. Now, one can define $Y = X/\zeta$ and treat the function $\mathcal{G}^a_\zeta(X)$ as the distribution amplitude $\Psi^a_\zeta(Y)$. In particular, the $\mathcal{G}$-part in this region can be written as

[‡]The off-forward parton distributions introduced by X. Ji [1,3] (see also [8]) and non-diagonal distributions of Collins, Frankfurt and Strikman [7] can be related to nonforward distributions (see [4]) but do not coincide with them.

$$\zeta\,\bar{u}(p')\hat{z}u(p)\int_0^1\left[e^{-iY(rz)}\mathcal{G}^a_\zeta(\zeta Y)+e^{-i(1-Y)(rz)}\mathcal{G}^{\bar{a}}_\zeta(\zeta Y)\right]dY=\zeta\,\bar{u}(p')\hat{z}u(p)\int_0^1 e^{-iY(rz)}\Psi^a_\zeta(Y)\,dY\,, \tag{3}$$

where the distribution amplitude $\Psi^a_\zeta(Y)$ is defined by $\Psi^a_\zeta(Y)=\mathcal{G}^a_\zeta(Y\zeta)+\mathcal{G}^{\bar{a}}_\zeta(\bar{Y}\zeta)$. The function $\Psi^a_\zeta(Y)$ gives the probability amplitude that the initial nucleon with momentum p is composed of the final nucleon with momentum $p'\equiv p-r$ and a $\bar{q}q$ pair in which the pair momentum r is shared in fractions Y and $1-Y\equiv\bar{Y}$.

For gluons, the nonforward distribution $\mathcal{G}^g_\zeta(X;t)$ is defined through the matrix element

$$\begin{aligned}&\langle p'\,|\,z_\mu z_\nu G^a_{\mu\alpha}(0)E^{ab}(0,z;A)\tilde{G}^b_{\alpha\nu}(z)\,|\,p\rangle|_{z^2=0}\\&=\bar{u}(p')\hat{z}\gamma_5 u(p)\,(z\cdot p)\int_0^1\frac{i}{2}\left[e^{-iX(pz)}-e^{i(X-\zeta)(pz)}\right]\mathcal{G}^g_\zeta(X;t)\,dX\\&+\bar{u}(p')\frac{(rz)}{M}\gamma_5 u(p)(z\cdot p)\int_0^1\frac{i}{2}\left[e^{-iX(pz)}-e^{i(X-\zeta)(pz)}\right]\mathcal{P}^g_\zeta(X;t)\,dX\,.\end{aligned} \tag{4}$$

As usual, $\tilde{G}_{\alpha\nu}=\frac{1}{2}\epsilon_{\alpha\nu\beta\mu}G^{\beta\mu}$. Since there are no "antigluons", the exponentials $e^{-iX(pz)}$ and $e^{i(X-\zeta)(pz)}$ are accompanied here by the same function $\mathcal{G}^g_\zeta(X;t)$. Again, the contribution from the region $0<X<\zeta$ can be written as

$$i\bar{u}(p')\hat{z}\gamma_5 u(p)\,(z\cdot r)\int_0^1 e^{-iY(rz)}\,\Psi^g_\zeta(Y;t)\,dY+\text{“}\mathcal{P}\text{” term,} \tag{5}$$

with the $Y\leftrightarrow\bar{Y}$ antisymmetric generalized distribution amplitude $\Psi^g_\zeta(Y;t)$ given by

$$\Psi^g_\zeta(Y;t)=\frac{1}{2}\left(\mathcal{G}^g_\zeta(Y\zeta;t)-\mathcal{G}^g_\zeta(\bar{Y}\zeta;t)\right)\,. \tag{6}$$

In the formal $t=0$ limit, the nonforward distributions $\mathcal{G}^g_\zeta(X;t)$, $\mathcal{P}^g_\zeta(X;t)$ convert into the asymmetric distribution functions $\mathcal{G}^g_\zeta(X)$, $\mathcal{P}^g_\zeta(X)$. Finally, in the $\zeta=0$ limit, $\mathcal{G}^g_\zeta(X)$ reduces to the usual polarized gluon density

$$\mathcal{G}^g_{\zeta=0}(X)=X\Delta g(X). \tag{7}$$

Under pQCD evolution, the gluonic operator

$$\mathcal{O}_g(uz,vz)=z_\mu z_\nu G^a_{\mu\alpha}(uz)E^{ab}(uz,vz;A)\tilde{G}^b_{\alpha\nu}(vz) \tag{8}$$

mixes with the flavor-singlet quark operator

$$\mathcal{O}_Q(uz,vz)=\sum_{a=1}^{N_f}\mathcal{O}^{(+)}_a(uz,vz) \tag{9}$$

where

$$\mathcal{O}^{(+)}_a(uz,vz)=\frac{1}{2}\left[\bar{\psi}_a(uz)\hat{z}\gamma_5 E(uz,vz;A)\psi_a(vz)+\bar{\psi}_a(vz)\hat{z}\gamma_5 E(vz,uz;A)\psi_a(uz)\right]. \tag{10}$$

The nonforward distribution function $\mathcal{G}^Q_\zeta(X;t)$ for the flavor-singlet quark combination (9)

$$\langle\, p',s'|\,\mathcal{O}_Q(uz,vz)\,|\,p,s\rangle|_{z^2=0}=\bar{u}(p',s')\hat{z}\gamma_5 u(p,s)\int_0^1\frac{1}{2}\left[e^{-ivX(pz)+iuX'(pz)}+e^{ivX'(pz)-iuX(pz)}\right]\mathcal{G}^Q_\zeta(X;t)\,dX\;+\text{“}\mathcal{P}\text{” term,} \tag{11}$$

(where $X'\equiv X-\zeta$) can be expressed as the sum of "$a+\bar{a}$" distributions:

$$\mathcal{G}^Q_\zeta(X;t)=\sum_{a=1}^{N_f}(\mathcal{G}^a_\zeta(X;t)+\mathcal{G}^{\bar{a}}_\zeta(X;t))\,. \tag{12}$$

Writing the contribution from the $0 < X < \zeta$ region as

$$\zeta \bar{u}(p')\hat{z}\gamma_5 u(p)\,(z\cdot r)\int_0^1 e^{-iY(rz)}\Psi_\zeta^Q(Y;t)\,dY + \text{“}\mathcal{P}\text{” term}, \tag{13}$$

we introduce the flavor-singlet quark distribution amplitude $\Psi_\zeta^Q(Y;t)$ which has the symmetry property $\Psi_\zeta^Q(Y;t) = \Psi_\zeta^Q(\bar{Y};t)$ with respect to the $Y \leftrightarrow \bar{Y}$ transformation.

III. EVOLUTION EQUATIONS FOR LIGHT-RAY OPERATORS

Near the light cone $z^2 \sim 0$, the bilocal operators $\mathcal{O}(uz, vz)$ develop logarithmic singularities $\ln z^2$. Calculationally, these singularities manifest themselves as ultraviolet divergences for operators taken on the light cone. The divergences are removed by a subtraction prescription characterized by some scale μ: $\mathcal{G}_\zeta(X;t) \to \mathcal{G}_\zeta(X;t;\mu)$. At one loop, the set of evolution equations for the flavor-singlet light-ray operators has the following form (cf. [9,10]):

$$\mu\frac{d}{d\mu}\,\mathcal{O}_a(0,z) = \int_0^1\int_0^1 \sum_b A_{ab}(u,v)\mathcal{O}_b(uz,\bar{v}z)\,\theta(u+v\le 1)\,du\,dv\,, \tag{14}$$

where $a,b = g,Q$ and $\bar{v} \equiv 1-v$, $\bar{u} \equiv 1-u$. For flavor-nonsinglet distributions, there is no mixing, and their evolution is generated by the QQ-kernel alone. To calculate the kernels, we incorporated the approach [10] based on the background-field method. Below we present our results in the form similar to that used in refs. [6,4]:

$$A_{QQ}(u,v) = \frac{\alpha_s}{\pi}\,C_F\left(1+\frac{3}{2}\,\delta(u)\delta(v) + \left\{\delta(u)\left[\frac{\bar{v}}{v} - \delta(v)\int_0^1\frac{d\tilde{v}}{\tilde{v}}\right] + \{u\leftrightarrow v\}\right\}\right)\,, \tag{15}$$

$$A_{gQ}(u,v) = \frac{\alpha_s}{\pi}\,C_F\Big(\delta(u)\delta(v) - 2\Big)\,, \tag{16}$$

$$A_{Qg}(u,v) = \frac{\alpha_s}{\pi}\,N_f\,(1-u-v)\,, \tag{17}$$

$$A_{gg}(u,v) = \frac{\alpha_s}{\pi}\,N_c\left(4(1-u-v) + \frac{\beta_0}{2N_c}\,\delta(u)\delta(v) + \left\{\delta(u)\left[\frac{\bar{v}^2}{v} - \delta(v)\int_0^1\frac{d\tilde{v}}{\tilde{v}}\right] + \{u\leftrightarrow v\}\right\}\right)\,. \tag{18}$$

Independently, these kernels were calculated by Blumlein, Geyer and Robaschik [11,12]. Their results agree with ours.

IV. EVOLUTION EQUATIONS FOR NONFORWARD DISTRIBUTIONS

Inserting the light-ray evolution equations (14) between chosen hadronic states and parametrizing matrix elements by appropriate distributions, one can get the “old” DGLAP [13–15] and BL-type [16–18] evolution kernels as well as calculate the new kernels $\Delta W_\zeta^{ab}(X,Z)$ governing the evolution of nonforward parton distributions:

$$\mu\frac{d}{d\mu}\,\mathcal{G}_\zeta^a(X;t;\mu) = \int_0^1 \sum_b\,\Delta W_\zeta^{ab}(X,Z)\,\mathcal{G}_\zeta^b(Z;t;\mu)\,dZ\,. \tag{19}$$

Extracting $\Delta W_\zeta^{ab}(X,Z)$ from the light-ray kernels $A_{ab}(u,v)$, one should take into account the extra (pz) factor in the rhs of the gluon distribution definition, which under the Fourier transformation with respect to (pz) results in the differentiation $\partial/\partial X$. Thus, it is convenient to introduce first the auxiliary kernels $\Delta M_\zeta^{ab}(X,Z)$ directly related to the light-ray kernels $A(u,v)$ by

$$\Delta M_\zeta^{ab}(X,Z) = \int_0^1\int_0^1 A_{ab}(u,v)\,\delta(X-\bar{u}Z+v(Z-\zeta))\,\theta(u+v\le 1)\,du\,dv\;. \tag{20}$$

The ΔW-kernels are obtained from the ΔM-kernels using

$$\Delta W_\zeta^{gg}(X,Z) = \Delta M_\zeta^{gg}(X,Z) \quad , \quad \Delta W_\zeta^{QQ}(X,Z) = \Delta M_\zeta^{QQ}(X,Z), \tag{21}$$

$$\frac{\partial}{\partial X}\Delta W_\zeta^{gQ}(X,Z) = -\Delta M_\zeta^{gQ}(\widetilde{X},Z)\, d\widetilde{X} \quad , \quad \Delta W_\zeta^{Qg}(X,Z) = -\frac{\partial}{\partial X}\,\Delta M_\zeta^{Qg}(X,Z)\,. \tag{22}$$

Hence, to get $\Delta W_\zeta^{gQ}(X,Z)$ we should integrate $\Delta M_\zeta^{gQ}(X,Z)$ with respect to X. We fix the integration constant by the requirement that $\Delta W_\zeta^{gQ}(X,Z)$ vanishes for $X > 1$. Then

$$\Delta W_\zeta^{gQ}(X,Z) = \int_X^1 \Delta M_\zeta^{gQ}(\widetilde{X},Z)\, d\widetilde{X}\,. \tag{23}$$

Integrating the delta-function in eq.(20) produces four different types of the θ-functions, each of which corresponds to a specific component of the kernel governing the evolution of the nonforward distributions.

V. BL-TYPE EVOLUTION KERNELS

When $\zeta = 1$, $\mathcal{G}_\zeta(X)$ reduces to a distribution amplitude whose evolution is governed by the BL-type kernels:

$$\Delta W_{\zeta=1}^{ab}(X,Z) = V^{ab}(X,Z). \tag{24}$$

Taking $\zeta = 1$ in Eq.(20) we obtain

$$\Delta M_{\zeta=1}^{ab}(X,Z) \equiv U^{ab}(X,Z) = \int_0^1 \int_0^1 A_{ab}(u,v)\, \delta(X - \bar{u}Z - v(1-Z))\, \theta(u+v \le 1)\, du\, dv\,. \tag{25}$$

In fact, the BL-type kernels appear as a part of the nonforward kernel $W_\zeta^{ab}(X,Z)$ even in the general $\zeta \neq 1, 0$ case. As explained earlier, if X is in the region $X \le \zeta$, then the function $\mathcal{G}_\zeta(X)$ can be treated as a distribution amplitude $\Psi_\zeta(Y)$ with $Y = X/\zeta$. For this reason, when both X and Z are smaller than ζ, the kernels $W_\zeta^{ab}(X,Z)$ simply reduce to the BL-type evolution kernels $V^{ab}(X/\zeta, Z/\zeta)$. Indeed, the relation (20) can be written as

$$\Delta M_\zeta^{ab}(X,Z) = \frac{1}{\zeta}\int_0^1 \int_0^1 A_{ab}(u,v)\, \delta\left(X/\zeta - \bar{u}Z/\zeta - v(1-Z/\zeta)\right)\, \theta(u+v \le 1)\, du\, dv\,. \tag{26}$$

Comparing this expression with the representation for the $U^{ab}(X,Z)$ kernels, we conclude that in the region where $X/\zeta \le 1$ and $Z/\zeta \le 1$, the kernels $\Delta M_\zeta^{ab}(X,Z)$ are given by

$$\Delta M_\zeta^{ab}(X,Z)|_{0\le\{X,Z\}\le\zeta} = \frac{1}{\zeta}\, U^{ab}\left(X/\zeta, Z/\zeta\right)\,. \tag{27}$$

Now, using the expressions connecting the ΔW- and ΔM-kernels, we obtain the following relations between the nonforward evolution kernels $\Delta W_\zeta^{ab}(X,Z)$ in the region $0 \le \{X,Z\} \le \zeta$ and the BL-type kernels $V^{ab}(X,Z)$:

$$\Delta W_\zeta^{QQ}(X,Z) = \frac{1}{\zeta}\, V^{QQ}\left(X/\zeta, Z/\zeta\right) \;;\; \Delta W_\zeta^{gQ}(X,Z) = V^{gQ}\left(X/\zeta, Z/\zeta\right)\;;$$
$$\Delta W_\zeta^{Qg}(X,Z) = \frac{1}{\zeta^2}\, V^{Qg}\left(X/\zeta, Z/\zeta\right) \;;\; \Delta W_\zeta^{gg}(X,Z) = \frac{1}{\zeta}\, V^{gg}\left(X/\zeta, Z/\zeta\right)\,. \tag{28}$$

The kernels $V^{ab}(X,Z)$, in their turn, are derived from the auxiliary kernels $U^{ab}(X,Z)$. Due to the symmetry property $A_{ab}(u,v) = A_{ab}(v,u)$ the kernels $U^{ab}(X,Z)$ satisfy $U^{ab}(\bar{X},\bar{Z}) = U^{ab}(X,Z)$. Hence, it is sufficient to know the U-kernels in the $X \le Z$ region only:

$$U^{ab}(X,Z) = \theta(X \le Z)\, U_0^{ab}(X,Z) + \theta(Z \le X) U_0^{ab}(\bar{X},\bar{Z})\,,$$

with the basic function $U_0^{ab}(X,Z) \equiv \theta(X \le Z)\, U^{ab}(X,Z)$ given by

$$U_0^{ab}(X,Z) = \frac{1}{Z}\int_0^X A_{ab}\left(\bar v - (X-v)/Z, v\right) dv \,. \tag{29}$$

Using Eqs.(15)-(18), the $A \to U_0$ conversion formulas

$$\delta(u)\,\delta(v) \to \delta(Z-X) \quad , \quad 1 \to \frac{X}{Z} \quad , \quad \delta(u)\,\frac{\bar v}{v} \to 0 \quad , \quad \delta(u)\,\left(\frac{\bar v}{v}\right)^2 \to 0,$$
$$\delta(v)\,\frac{\bar u}{u} \to \left(\frac{X}{Z}\right)\frac{1}{Z-X} \quad , \quad \delta(v)\,\frac{\bar u^2}{u} \to \left(\frac{X}{Z}\right)^2 \frac{1}{Z-X} \quad , \quad u+v \to \frac{X}{Z}\left(1-\frac{X}{2Z}\right) \tag{30}$$

and Eqs.(20)-(24), (28) we get the BL-type kernels

$$V^{QQ}(X,Z) = \frac{\alpha_s}{\pi}\,C_F\left\{\left[\frac{X}{Z}\left(1+\frac{1}{Z-X}\right)\theta\,(X<Z)\right]_+ + \{X \to \bar X, Z \to \bar Z\}\right\} \tag{31}$$

$$V^{Qg}(X,Z) = \frac{\alpha_s}{\pi}\,N_f\left\{-\frac{X}{Z^2}\,\theta\,(X<Z) + \frac{\bar X}{\bar Z^2}\,\theta\,(X>Z)\right\}, \tag{32}$$

$$V^{gQ}(X,Z) = \frac{\alpha_s}{\pi}\,C_F\left\{\frac{X^2}{Z}\,\theta\,(X<Z) - \frac{\bar X^2}{\bar Z}\,\theta\,(X>Z)\right\}, \tag{33}$$

$$V^{gg}(X,Z) = \frac{\alpha_s}{\pi}\,N_c\left\{\frac{2X^2-X-Z}{Z^2}\,\theta\,(X<Z) + \left[\frac{\theta\,(X<Z)}{Z-X}\right]_+ + \{X \to \bar X, Z \to \bar Z\}\right.$$
$$\left. + \frac{\beta_0}{2N_c}\,\delta(X-Z)\right\}, \tag{34}$$

calculated originally in [17,18] for flavor-singlet pseudoscalar meson distribution amplitudes. With respect to integration over $0 \le X \le 1$, the "plus"-prescription for a function $V(X,Z)$ is defined by

$$[V(X,Z)]_+ = V(X,Z) - \delta\,(X-Z)\int_0^1 V(Y,Z)\,dY \,. \tag{35}$$

The BL-type kernels also govern the evolution in the region corresponding to transitions from a fraction Z which is larger than ζ to a fraction X which is smaller than ζ. Indeed, using the δ-function to calculate the integral over u, we get

$$\Delta M_\zeta^{ab}(X,Z)|_{X\le\zeta\le Z} = \frac{1}{Z}\int_0^{X/\zeta} A_{ab}\Big(\left[1-X/Z-v(1-\zeta/Z)\right], v\Big)\,dv\,, \tag{36}$$

which has the same analytic form (29) as the expression for $M_\zeta^{ab}(X,Z)$ in the region $X \le Z \le \zeta$. For QQ, gg and Qg kernels, this automatically means that $\Delta W_\zeta^{ab}(X,Z)|_{X\le\zeta\le Z}$ is given by the same analytic expression as $\Delta W_\zeta^{ab}(X,Z)$ for $X < Z < \zeta$. Because of integration in Eq.(23), to get $\Delta W_\zeta^{gQ}(X,Z)$ one should also know $\Delta M_\zeta^{gQ}(X,Z)$ in the region $\zeta \le X \le Z$. However, our explicit calculation confirms that $\Delta W_\zeta^{gQ}(X,Z)$ in the transition region $X \le \zeta \le Z$ is given by the same expression as $\Delta W_\zeta^{gQ}(X,Z)$ for $X < Z \le \zeta$.

In application to parton distributions related to nonforward matrix elements, X. Ji was the first [3] who calculated analogous kernels $P'(x,\xi)$ which govern the evolution of his off-forward parton distributions $\tilde H(x,t;\mu)$ in the $-\xi/2 < x < \xi/2$ region (in our variables this region corresponds to $0 < X < \zeta$). He used a direct momentum-representation approach in the light-cone gauge. After proper redefinitions (discussed in [4]), we reproduced his expressions for the first three kernels. For the gluon-gluon kernel, our result formally differs from that obtained by X. Ji [3]. However, due to the symmetry properties of the gluon distribution in the X. Ji approach, the relevant integral vanishes and the difference does not contribute to the evolution. Blumlein *et al.* [11] derive the "extended"

BL-kernels [8] from the light-ray evolution equations. For $X \neq Z$, we agree with their results except for the gQ-kernel and up to obvious misprints in the QQ and gg-kernels [§].

VI. GENERALIZED DGLAP KERNELS

When $X > \zeta$, we can treat the asymmetric distribution function $\mathcal{G}_\zeta^a(X)$ as a generalization of the usual distribution function $\Delta f_a(X)$ for a skewed kinematics. Hence, evolution in the region $\zeta < X \leq 1$, $\zeta < Z \leq 1$ is close to that generated by the DGLAP equation. In particular, it has the basic property that the evolved fraction X cannot be larger than the original fraction Z. The relevant kernels are given by

$$\Delta M_\zeta^{ab}(X,Z)|_{\zeta \leq X \leq Z \leq 1} = \frac{Z-X}{ZZ'} \int_0^1 A_{ab}\left(\bar{w}\,(1-X/Z)\,,\; w\,(1-X'/Z')\right) dw\,, \tag{37}$$

where $X' \equiv X - \zeta$ and $Z' \equiv Z - \zeta$ are the "returning" partners of the original fractions X, Z. Note, that since $Z - X = Z' - X'$, the kernels $\Delta M_\zeta^{ab}(X,Z)$ are given by functions symmetric with respect to the interchange of X, Z with X', Z'. Using the table for transition from the A_{ab}-kernels to the ΔM^{ab}-kernels in the region $\zeta \leq X \leq Z \leq 1$

$$\delta(u)\,\delta(v) \to \delta(Z-X) \;\; ; \;\; 1 \to \frac{Z-X}{ZZ'} \;\; ; \;\; (u+v) \to \frac{Z-X}{2ZZ'}\left[2 - \frac{X}{Z} - \frac{X'}{Z'}\right] \;\; ;$$
$$\left(\delta(u)\,\frac{\bar{v}}{v} + \delta(v)\,\frac{\bar{u}}{u}\right) \to \frac{1}{Z-X}\left[\frac{X}{Z} + \frac{X'}{Z'}\right] ; \left(\delta(u)\,\frac{\bar{v}^2}{v} + \delta(v)\,\frac{\bar{u}^2}{u}\right) \to \frac{1}{Z-X}\left[\left(\frac{X}{Z}\right)^2 + \left(\frac{X'}{Z'}\right)^2\right], \tag{38}$$

and Eqs.(21), (22), we obtain the kernels $\Delta P_\zeta^{ab}(X,Z) \equiv \Delta W_\zeta^{ab}(X,Z)|_{\zeta \leq X \leq Z \leq 1}$:

$$\Delta P_\zeta^{QQ}(X,Z) = \frac{\alpha_s}{\pi} C_F \left\{ \frac{1}{Z-X}\left[1 + \frac{XX'}{ZZ'}\right] \theta\,(X<Z) \right.$$
$$\left. + \delta(X-Z)\left[\frac{3}{2} - \int_0^1 \frac{du}{u} - \int_0^1 \frac{dv}{v}\right]\right\} \to \frac{1}{Z}\Delta P_{QQ}(X/Z)\,, \tag{39}$$

$$\Delta P_\zeta^{Qg}(X,Z) = \frac{\alpha_s}{\pi} N_f \frac{1}{ZZ'}\left\{\frac{X}{Z} + \frac{X'}{Z'} - 1\right\} \to \frac{1}{Z^2}\Delta P_{Qg}(X/Z)\,, \tag{40}$$

$$\Delta P_\zeta^{gQ}(X,Z) = \frac{\alpha_s}{\pi} C_F \left\{\frac{X}{Z} + \frac{X'}{Z'} - \frac{XX'}{ZZ'}\right\} \to \frac{X}{Z}\Delta P_{gQ}(X/Z)\,, \tag{41}$$

$$\Delta P_\zeta^{gg}(X,Z) = \frac{\alpha_s}{\pi} N_c \left\{\left(2\left[\frac{X}{Z} + \frac{X'}{Z'}\right]\frac{Z-X}{ZZ'} + \frac{1}{Z-X}\left[\left(\frac{X}{Z}\right)^2 + \left(\frac{X'}{Z'}\right)^2\right]\right)\theta(X<Z)\right.$$
$$\left. + \;\delta(X-Z)\left[\frac{\beta_0}{2N_c} - \int_0^1 \frac{du}{u} - \int_0^1 \frac{dv}{v}\right]\right\} \to \frac{X}{Z^2}\Delta P_{gg}(X/Z)\,. \tag{42}$$

The formally divergent integrals over u and v provide here the usual "plus"-type regularization of the $1/(Z-X)$ singularities. The prescription following from Eqs.(37),(38) is that combining the $1/(Z-X)$ and $\delta(Z-X)$ terms into $[\mathcal{G}_\zeta(Z) - \mathcal{G}_\zeta(X)]/(Z-X)$ in the convolution of $\Delta P_\zeta(X,Z)$ with $\mathcal{G}_\zeta(Z)$ one should change $u \to 1 - X/Z$ and $v \to 1 - X'/Z'$.

As expected, the $\Delta P_\zeta^{ab}(X,Z)$ kernels have a symmetric form. The arrows indicate how the nonforward kernels $\Delta P_\zeta^{ab}(X,Z)$ are related to the DGLAP kernels in the $\zeta = 0$ limit when $Z = Z'$ and $X = X'$. Deriving these relations, one should take into account that the gluonic asymmetric distribution function $\mathcal{G}_\zeta^g(X)$ reduces in the $\zeta \to 0$ limit to $X\Delta g(X)$ rather than to $\Delta g(X)$.

[§]We are grateful to J. Blumlein who informed us that the authors of ref. [11] agree with our results.

After the appropriate redefinitions, we managed to reproduce from our results all four kernels $\Delta P_{ab}(x,\xi)$ (relevant to the $x > \xi/2$ region) calculated by X. Ji [3].

Note, that in the region $Z > \zeta$ the evolved fraction X is always smaller than Z. Furthermore, if $Z \leq \zeta$ then also $X \leq \zeta$, *i.e.*, distributions in the $X > \zeta$ regions are not affected by the distributions in the $X < \zeta$ regions. Hence, information about the initial distribution in the $Z > \zeta$ region is sufficient for calculating its evolution in this region. This situation may be contrasted with the evolution of distributions in the $Z < \zeta$ regions: in that case one should know the nonforward parton distributions in the whole domain $0 < Z < 1$.

VII. CONCLUSIONS

In this letter, we discussed the calculation of the evolution kernels $\Delta W_\zeta(X,Z)$ for nonforward parton distributions $\mathcal{G}_\zeta(X,t)$ sensitive to parton helicities. We presented the evolution kernels for the relevant light-ray operators and demonstrated how one can obtain from them the components of the nonforward kernels $\Delta W_\zeta(X,Z)$. Our results have a transparent relation with DGLAP and BL-type kernels and a compact form convenient for further practical applications such as numerical studies of the evolution of nonforward distributions.

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