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Deeply Virtual Compton Scattering: Facing Nonforward Distributions

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Applications of perturbative QCD to deeply virtual Compton scattering process require a generalization of usual parton distributions for the case when long-distance information is accumulated in nonforward matrix elements of quark and gluon operators. We discuss two types of functions parametrizing such matrix elements: double distributions $F(x, y; t)$ and nonforward distribution functions $\mathcal{F}_\zeta(X; t)$ and also their relation to usual parton densities $f_a(x)$.

PROTON SPIN AND DVCS

Recently, X. Ji [1] suggested to use deeply virtual Compton scattering (DVCS) to get information about the total quark and gluon contributions to the spin of the proton. Note, that the angular momentum operator $J^{\mu\nu}$ can be written in terms of the symmetric energy-momentum tensor $T^{\alpha\beta}$:

$$J^{\mu\nu} = \int d^3\vec{x}\, M^{0\mu\nu}(\vec{x}) \ \ ; \ \ M^{\alpha\mu\nu} = T^{\alpha\mu}x^\nu - T^{\alpha\nu}x^\mu \ , \tag{1}$$

which in QCD can be represented as a sum of quark and gluon parts

$$T^{\alpha\beta} = T_q^{\alpha\beta} + T_g^{\alpha\beta} = \frac{1}{4}\bar{\psi}\gamma^{\{\alpha}i\overset{\leftrightarrow}{D}{}^{\beta\}}\psi + \left(\frac{1}{4}g^{\alpha\beta}F^2 - F^{\alpha\mu}F^\beta_{\ \mu}\right) \ . \tag{2}$$

Hence, the quark J_q and gluon J_g contributions to the proton spin can be obtained from the form factors $A_{q,g}(t), B_{q,g}(t)$ of the energy-momentum tensor at zero momentum transfer $t = 0$:

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right] \ \ ; \ \ J_q + J_g = \frac{1}{2} \ . \tag{3}$$

The functions $A_{q,g}(t), B_{q,g}(t)$ are defined by [1]

$$\langle p'|T_{q,g}^{\mu\nu}|p\rangle = \bar{u}(p')\left[A_{q,g}(t)\gamma^{\{\mu}P^{\nu\}} + B_{q,g}(t)P^{\{\mu}\sigma^{\nu\}\alpha}\frac{r_\alpha}{2M}\right.$$
$$\left. +C_{q,g}(t)(r^\mu r^\nu - g^{\mu\nu}t) + D_{q,g}(t)Mg^{\mu\nu}\right]u(p) \tag{4}$$

where $r \equiv p' - p, t \equiv r^2$. To get $J_{q,g}$, one should know both the non-spin-flip quantities $A_{q,g}(0)$ related to total hadron momentum carried by quarks or gluons and spin-flip amplitudes invisible in deep-inelastic cross sections corresponding to exactly forward $r = 0$ virtual Compton amplitude. However, information about $B_{q,g}(t)$ and $A_{q,g}(t)$, is contained in the DVCS amplitude [1]. Anyway, whether the extraction of $B_{q,g}(0)$ is feasible or not, the studies of DVCS, an elastic process exhibiting, in the Bjorken limit, a scaling behavior similar to that of DIS, may be interesting on its own grounds [2,3] (earlier discussions of nonforward Compton-like amplitudes $\gamma^*p \to \gamma^*p'$ with a virtual photon or Z^0 in the final state were given in refs. [4–6]).

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PARTON PICTURE: DOUBLE DISTRIBUTIONS

The kinematics of the process $\gamma^* p \to \gamma p'$ is specified by the initial nucleon momentum p, virtual photon momentum q and momentum transfer $r = p - p'$. We are interested in the situation, when the invariant momentum transfer $t \equiv r^2$ and $p^2 \equiv m_p^2$ are much smaller than the virtuality $q^2 \equiv -Q^2$ of the initial photon and the energy invariant $p \cdot q$, with the Bjorken ratio $Q^2/2(p \cdot q) \equiv x_{Bj}$ fixed. It is helpful to consider first a (formal) limit $p^2 = 0$ and $t = 0$ and take p as a basic light-cone momentum in the Sudakov decomposition. Since the final photon momentum q' is light-like $q'^2 = 0$, it is natural to use q' as another basic light-cone 4-vector. Then $q = q' - x_{Bj} p$. Furthermore, in this limit, the requirement $p'^2 \equiv (p + r)^2 = p^2$ reduces to the condition $p \cdot r = 0$ which can be satisfied only if the two lightlike momenta p and r are proportional to each other: $r = \zeta p$, where ζ coincides with x_{Bj} which satisfies $0 \leq x_{Bj} \leq 1$.

The leading contribution in the large-Q^2, fixed-x_{Bj}, small-t limit is given by DIS-type handbag diagrams in which the long-distance dynamics is described by matrix elements $\langle p - r | \bar\psi_a(0) \gamma_\mu E(0, z; A) \psi_a(z) | p \rangle$ and $\langle p - r | \bar\psi_a(0) \gamma_5 \gamma_\mu E(0, z; A) \psi_a(z) | p \rangle$, where $E(0, z; A)$ is the usual P-exponential of the gluonic A-field along the straight line connecting 0 and z. Though the momenta p and r are proportional to each other $r = \zeta p$, to construct an adequate QCD parton picture, one should make a clear distinction between them. The basic reason is that p and r specify the momentum flow in two different channels. For $r = 0$, the net momentum flows only in the s-channel and the total momentum entering into the composite operator vertex is zero. In this case, the matrix element coincides with the standard distribution function. The partons entering the composite vertex then carry the fractions x_i of the initial proton momentum $(-1 < x_i < 1)$. When x is negative, the parton is interpreted as belonging to the final state and x_i is redefined to secure that the integral always runs over the segment $0 \leq x \leq 1$. In this parton picture, the spectators take the remaining momentum $(1 - x)p$. On the other hand, if the total momentum flowing through the composite vertex is r, the matrix element has the structure of the distribution amplitude in which the momentum r splits into the fractions yr and $(1 - y)r \equiv \bar{y}r$ carried by the quark fields attached to that vertex. In a combined situation, when both p and r are nonzero, the initial quark has the momentum $xp + yr$, while the final one carries the momentum $xp - \bar{y}r$. In more formal terms, this corresponds to the following parameterization of the light-cone matrix elements

$$\langle p - r | \bar\psi_a(0) \hat{z} E(0, z; A) \psi_a(z) | p \rangle |_{z^2=0} = \bar{u}(p - r) \hat{z} u(p) \int_0^1 \int_0^1 \theta(x + y \leq 1)$$

$$\left(e^{-ix(pz) - iy(rz)} F_a(x, y; t) - e^{ix(pz) - i\bar{y}(rz)} F_{\bar{a}}(x, y; t) \right) dy \, dx, \tag{5}$$

$$\langle p - r | \bar\psi_a(0) \gamma_5 \hat{z} E(0, z; A) \psi_a(z) | p \rangle |_{z^2=0} = \bar{u}(p - r) \gamma_5 \hat{z} u(p) \int_0^1 \int_0^1 \theta(x + y \leq 1)$$

$$\left(e^{-ix(pz) - iy(rz)} G_a(x, y; t) + e^{ix(pz) - i\bar{y}(rz)} G_{\bar{a}}(x, y; t) \right) dy \, dx, \tag{6}$$

where $\hat{z} \equiv \gamma_\mu z^\mu$. Though we arrived at the matrix elements (5), (6) in the context of the scaling limit of the DVCS amplitude, they accumulate a process-independent information. The coefficient of proportionality between (pz) and (rz) is just a parameter characterizing "skewedness" of the matrix elements. The fact that, in our case, ζ coincides with the Bjorken variable is specific for the DVCS amplitude. An important feature implied by the representation (5),(6) is the absence of the ζ-dependence in the double distributions $F_a(x, y; t)$ and $G_a(x, y; t)$. This property and spectral constraints $x \geq 0$, $y \geq 0$, $x + y \leq 1$ hold for any Feynman diagram [7]. As a result, both the initial active quark and the spectators carry positive fractions of the light-cone "plus" momentum p: $x + \zeta y$ for the active quark and $(1 - x - y) + (1 - \zeta)y$ for the spectators. However, the fraction of the initial momentum p carried by the "returning" quark is given by $x - \bar{y}\zeta$ and it may take both positive and negative values.

Taking the limit $r = 0$ gives the matrix element defining the parton distribution functions $f_{a, \bar{a}}(x)$, $\Delta f_{a, \bar{a}}(x)$. This observation results in the following reduction formulas for the double distributions $F(x, y; t), G(x, y; t)$:

$$\int_0^{1-x} F_a(x, y; t = 0) \, dy = f_a(x) \quad , \quad \int_0^{1-x} G_a(x, y; t = 0) \, dy = \Delta f_a(x). \tag{7}$$

The parameterization for the matrix elements given above, results in a parton representation for the handbag contributions to the DVCS amplitude:

$$T^{\mu\nu}(p, q, q') = \frac{1}{2(pq')} \sum_a e_a^2 \left[\left(-g^{\mu\nu} + \frac{1}{p \cdot q'} (p^\mu q'^\nu + p^\nu q'^\mu) \right) \left\{ \bar{u}(p') \hat{q}' u(p) T_F^a(\zeta) \right. \right.$$

$$\left. \left. + \frac{1}{2M} \bar{u}(p') (\hat{q}'\hat{r} - \hat{r}\hat{q}') u(p) T_K^a(\zeta) + \{a \to \bar{a}\} \right\} \right. \tag{8}$$

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$$+i\epsilon^{\mu\nu\alpha\beta}\frac{p_\alpha q'_\beta}{(pq')}\left\{\bar{u}(p')\gamma_5\hat{q}'u(p)\,T_G^a(\zeta)+\frac{(q'r)}{2M}\bar{u}(p')\gamma_5 u(p)T_P^a(\zeta)+\{a\to\bar{a}\}\right\}\Bigg]$$

where $T^a(\zeta,t)$ are the functions depending on the scaling variable ζ:

$$T_F^a(\zeta,t)=-\int_0^1 dx\int_0^{1-x}\left(\frac{1}{x-\zeta\bar{y}+i\epsilon}+\frac{1}{x+\zeta y}\right)F_a(x,y;t)\,dy, \tag{9}$$

etc. The terms containing $1/(x-\zeta\bar{y}+i\epsilon)$ generate the imaginary part:

$$\frac{1}{\pi}\,\mathrm{Im}\,T_F^a(\zeta,t)=\int_0^1 F_a(\bar{y}\zeta,y;t)\,dy, \tag{10}$$

with a similar expression for $\mathrm{Im}\,T_A^a(\zeta,t)$. The relation between $\mathrm{Im}\,T(\zeta,t)$ and the double distributions $F_a(x,y;t)$ is not as direct as in the case of forward virtual Compton amplitude, the imaginary part of which is just given by parton densities $f_a(\zeta)$. Note, that the y-integral in eq.(10) is different from that in the reduction formula (7), *i.e.*, though

$$\Phi_a(\zeta,t)\equiv\int_0^1 F_a(\bar{y}\zeta,y;t)\,dy \tag{11}$$

is a function of the Bjorken variable ζ, it does not coincide with $f_a(\zeta)$. To get the real part of the $1/(x-\zeta\bar{y}+i\epsilon)$ terms, one should use the principal value prescription, *i.e.*, $\mathrm{Re}\,T(\zeta,t)$ is related to $F_a(x,y;t)$ through two integrations.

NONFORWARD DISTRIBUTIONS

Since $(rz)=\zeta(pz)$, the variable y appears in eqs.(5),(6) only in the $x+y\zeta\equiv X$ combination, where X can be treated as the *total* fraction of the initial hadron momentum p carried by the active quark. Since $\zeta\le 1$ and $x+y\le 1$, the variable X satisfies a natural constraint $0\le X\le 1$. Integrating the double distribution $F(X-y\zeta,y)$ over y gives the *nonforward parton distribution* [8,7]

$$\mathcal{F}_\zeta(X;t)=\int_0^{\min\{X/\zeta,\bar{X}/\bar{\zeta}\}}F(X-y\zeta,y;t)\,dy \tag{12}$$

where $\bar{\zeta}\equiv 1-\zeta$. The basic distinction between the double distribution $F(x,y;t)$ and the nonforward distribution $\mathcal{F}_\zeta(X;t)$ is that the latter explicitly depends on the skewedness parameter ζ: one deals now with a family of nonforward distributions $\mathcal{F}_\zeta(X,t)$ whose shape changes when ζ is changed.

The fraction $X-\zeta\equiv X'$ of the original hadron momentum p carried by the "returning" parton differs from X by ζ: $X-X'=\zeta$. Since X changes from 0 to 1 and $\zeta\ne 0,1$, the fraction X' can be either positive or negative, *i.e.*, the asymmetric distribution function has two components corresponding to the regions $1\ge X\ge\zeta$ and $0\le X\le\zeta$. In the region $X>\zeta$, the function $\mathcal{F}_\zeta(X)$ can be treated as a generalization of the usual distribution function $f(x)$ for the asymmetric case when the final hadron momentum p' differs by ζp from the initial momentum p. In the region $X<\zeta$ the "returning" parton has a negative fraction $(X-\zeta)$ of the light-cone momentum p. Hence, it is more appropriate to treat it as a parton going out of the hadron and propagating along with the original parton. Writing X as $X=Y\zeta$, we see that both partons carry now positive fractions $Y\zeta p\equiv Yr$ and $\bar{Y}r\equiv(1-Y)r$ of the momentum transfer r. Thus, the nonforward distribution in the region $X=Y\zeta<\zeta$ looks like a distribution amplitude $\Psi_\zeta(Y;t)$ for a $\bar{q}q$-state with the total momentum $r=\zeta p$:

$$\Psi_\zeta(Y;t)=\int_0^Y F((Y-y)\zeta,y;t)\,dy. \tag{13}$$

In terms of $\mathcal{F}_\zeta(X)$, the F-part of the virtual Compton amplitude is

$$T_F^a(\zeta)=-\int_0^1\left[\frac{1}{X-\zeta+i\epsilon}+\frac{1}{X-i\epsilon}\right]\left(\mathcal{F}_\zeta^a(X;t)+\mathcal{F}_\zeta^{\bar{a}}(X;t)\right)\,dX\,. \tag{14}$$

It can be shown that $\mathcal{F}_\zeta^{a,\bar{a}}(X)$ linearly vanish as $X\to 0$ [7]. As a result, the imaginary part is generated by the $1/(X-\zeta+i\epsilon)$ singularity

$$\frac{1}{\pi} \operatorname{Im} T_F^a(\zeta) = \mathcal{F}_\zeta^a(\zeta;t) + \mathcal{F}_\zeta^{\bar{a}}(\zeta;t) \tag{15}$$

Hence, the integral $\Phi(\zeta,t)$ in Eq.(11) is equal to $\mathcal{F}_\zeta(\zeta;t)$, *i.e.*, to the nonforward distribution $\mathcal{F}_\zeta(X;t)$ taken at the point $X = \zeta$. The parameter ζ is present in $\mathcal{F}_\zeta(\zeta;t)$ twice: first as the parameter specifying the skewedness of the matrix element and then as the momentum fraction at which the imaginary part appears. As one may expect, it appears for $X = x_{Bj} = \zeta$, just like in the forward case. Note, however, that the momentum $(X - \zeta)p$ of the "returning" parton vanishes when $X = \zeta$: the imaginary part appears in a highly asymmetric configuration in which the fraction of the original hadron momentum carried by the second parton vanishes. Hence, $\mathcal{F}_\zeta(\zeta)$ in general differs from the function $f(\zeta)$. Experimentally, the imaginary part of the DVCS amplitude can be extracted by measuring the single-spin asymmetry [3].

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