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THE ASYMPTOTICS OF THE TRANSITION FORM FACTOR $\gamma\gamma^* \to \pi^o$ AND QCD SUM RULES [a]

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In this paper we present the result [1,2] of a direct QCD sum rule calculation of the transition form factor $\gamma\gamma^* \to \pi^0$ in the region of moderately large invariant momentum $Q^2 \geq 1\,GeV^2$ of the virtual photon. In contrast to pQCD, we make no assumptions about the shape of the pion distribution amplitude $\varphi_\pi(x)$. Our results agree with the Brodsky-Lepage proposal that the Q^2-dependence of this form factor is given by an interpolation between its $Q^2 = 0$ value fixed by the axial anomaly and $1/Q^2$ pQCD behaviour for large Q^2, with normalization corresponding to the asymptotic form $\varphi_\pi^{as}(x) = 6f_\pi x(1-x)$ of the pion distribution amplitude. Our prediction for the from factor $F_{\gamma^*\gamma^*\pi^0}(q_1^2 = 0, q_2^2 = -Q^2)$ is in good agreement with new CLEO data.

The transition $\gamma^*\gamma^* \to \pi^0$ of two virtual photons γ^* into a neutral pion provides an exceptional opportunity to test QCD predictions for exclusive processes. In the lowest order of perturbative QCD, its asymptotic behaviour is due to the subprocess $\gamma^*(q_1) + \gamma^*(q_2) \to \bar{q}(\bar{x}p) + q(xp)$ with x $(\bar{x})$ being the fraction of the pion momentum p carried by the quark produced at the q_1 (q_2) photon vertex. The relevant diagram resembles the handbag diagram of DIS, with the main difference that one should use the pion distribution amplitude (DA) $\varphi_\pi(x)$ instead of parton densities. This gives good reasons to expect that pQCD for this process may work at accessible values of spacelike photon virtualities. The asymptotic pQCD prediction is given by [3] ($\bar{x} = 1 - x$):

$$F^{as}_{\gamma^*\gamma^*\pi^0}(q^2, Q^2) = \frac{4\pi}{3}\int_0^1 \frac{\varphi_\pi(x)}{xQ^2 + \bar{x}q^2}\,dx \xrightarrow{q^2=0} \frac{4\pi}{3}\int_0^1 \frac{\varphi_\pi(x)}{xQ^2}\,dx \equiv \frac{4\pi f_\pi}{3Q^2} I. \quad (1)$$

[a] Talk presented by R. Ruskov at the Photon'97 Conference, Egmond aan Zee, The Netherlands, May 10-15, 1997

Experimentally, the most important situation is when one of the photons is almost real $q^2 \approx 0$ [4,5]. In this case, necessary nonperturbative information is accumulated in the same integral I (see eq.(1)) that appears in the one-gluon-exchange diagram for the pion electromagnetic form factor [6,7,8].

The value of I is sensitive to the shape of the pion DA $\varphi_\pi(x)$, mainly to its end-point behaviour. In particular, using the asymptotic form $\varphi_\pi^{as}(x) = 6f_\pi x\bar{x}$ [6,7] gives $F^{as}_{\gamma\gamma^*\pi^0}(Q^2) = 4\pi f_\pi/Q^2$ for the asymptotic behaviour [3]. If one takes the Chernyak-Zhitnitsky form[8] $\varphi_\pi^{CZ}(x) = 30f_\pi x\bar{x}(1-2x)^2$, the integral I increases by a sizable factor of 5/3, and this difference can be used for experimental discrimination between the two forms. One-loop radiative QCD corrections to eq.(1) are known [9,10] and they are under control.

For lower Q^2, power corrections become very important. Indeed, the asymptotic $1/Q^2$-behaviour cannot be true in the low-Q^2 region, since the $Q^2 = 0$ limit of $F_{\gamma\gamma^*\pi^0}(Q^2)$ is known to be finite and normalized by the $\pi^0 \to \gamma\gamma$ decay rate. Theoretically[11], $F_{\gamma\gamma^*\pi^0}(0) = 1/\pi f_\pi$. It is natural to expect that the leading term is close to a simple interpolation $\pi f_\pi F^{LO}_{\gamma\gamma^*\pi^0}(Q^2) = 1/(1+Q^2/4\pi^2 f_\pi^2)$[3] between the $Q^2 = 0$ value and the large-Q^2 asymptotics. This interpolation agrees with experiment[4,5] and implies the asymptotic form[6,7] of the DA for accessible Q^2. It introduces a mass scale $s_o^\pi \equiv 4\pi^2 f_\pi^2 \approx 0.67\, GeV^2$ close to m_ρ^2.

Consider a three-point correlation function[b]

$$\mathcal{F}_{\alpha\mu\nu}(q_1,q_2) = 2\pi i \int \langle 0|T\left\{j^5_\alpha(Y) J_\mu(X)\, J_\nu(0)\right\}|0\rangle e^{-iq_1X}\, e^{ipY} d^4X\, d^4Y\ , \quad (2)$$

where J_μ is the EM current and the axial-vector current has a non-zero projection onto the neutral pion state. The amplitude $\mathcal{F}_{\alpha\mu\nu}(q_1,q_2)$ has a pole for $p^2 = m_\pi^2$ with residue proportional to the form factor of interest. The higher states include A_1 and higher broad pseudovector resonances. Due to asymptotic freedom, their sum for large s rapidly approaches the pQCD spectral density $\rho^{PT}(s,q^2,Q^2)$. Hence, the spectral density of the dispersion relation for the relevant invariant amplitude $\mathcal{F}\left(p^2,q^2,Q^2\right)$ can be written as $\rho\left(s,q^2,Q^2\right) = \pi f_\pi \delta(s-m_\pi^2) F_{\gamma^*\gamma^*\pi^\circ}\left(q^2,Q^2\right) + \theta(s-s_o)\rho^{PT}(s,q^2,Q^2)$, with the parameter s_o being the effective threshold for higher states. To construct a QCD sum rule, we calculate the three-point function $\mathcal{F}(p^2,q^2,Q^2)$ and then its SVZ-transform $\Phi(M^2,q^2,Q^2)$ as a power expansion in $1/M^2$ for large M^2.

The simplest case is when the smaller virtuality q^2 is large: $q^2, Q^2, -p^2 \geq 1\ GeV^2$. Then, to produce a contribution with a power behaviour $(1/p^2)^N$, all three currents should be kept close to each other: all the intervals X^2, Y^2,

[b] Actually, it is a common starting point both for pQCD and QCD SR approaches.

$(X-Y)^2$ should be small. Taking into account the perturbative contribution and the condensate corrections, we obtain a QCD sum rule[1,2]. For $Q^2, q^2 \gg s_0$, keeping only the leading $O(1/Q^2, 1/q^2)$- terms we obtain:

$$F^{LO}_{\gamma^*\gamma^*\pi^\circ}(q^2,Q^2) = \frac{4\pi}{3f_\pi}\int_0^1 \frac{dx}{(xQ^2+\bar{x}q^2)}\left\{\frac{3M^2}{2\pi^2}(1-e^{-s_0/M^2})x\bar{x}\right.$$
$$+\frac{1}{24M^2}\langle\frac{\alpha_s}{\pi}GG\rangle[\delta(x)+\delta(\bar{x})]$$
$$\left.+\frac{8}{81M^4}\pi\alpha_s\langle\bar{q}q\rangle^2\Big(11[\delta(x)+\delta(\bar{x})]+2[\delta'(x)+\delta'(\bar{x})]\Big)\right\}. \qquad (3)$$

Note, that the expression in curly brackets coincides with the QCD sum rule for the pion DA $f_\pi\varphi_\pi(x)$ (see, *e.g.*, ref.[12]). Hence, the QCD sum rules approach is capable to reproduce the pQCD result (1).

An attempt to get a QCD sum rule for the integral I by taking $q^2=0$ in eq.(3) is ruined by power singularities $1/q^2$, $1/q^4$ in the condensate terms. The perturbative term in the small-q^2 region has logarithms $\log q^2$ which are a typical example of mass singularities (see, *e.g.*, [13]). All these infrared sensitive terms are produced in a regime when the hard momentum flow bypasses the soft photon vertex, *i.e.*, the EM current $J_\mu(X)$ of the low-virtuality photon is far away from the two other currents $J(0), j^5(Y)$.

Observe also, that power singularities emerge precisely by the same $\delta(x)$ and $\delta'(x)$ terms in eq.(3) which generate the two-hump form for $\varphi_\pi(x)$ in the CZ-approach [8]. As shown in ref.[12], the $\delta^{(n)}(x)$ terms result from the Taylor expansion of nonlocal condensates like $\langle\bar{q}(0)q(Z)\rangle$.

Our strategy is to subtract all these singularities from the coefficient functions of the original OPE for the 3-point correlation function eq.(2). They are absorbed in this approach by *universal* bilocal correlators (see refs.[1,2]), which can be also interpreted as moments of the DAs for (almost) real photon $\int_0^1 y^n\phi_\gamma^{(i)}(y,q^2) \sim \Pi_n^{(i)}(q^2) = \int e^{iq_1X}\langle 0|T\{J_\mu(X)\mathcal{O}_n^{(i)}(0)\}|0\rangle d^4X$, where $\mathcal{O}_n^{(i)}(0)$ are operators of leading and next-to-leading twist with n covariant derivatives [1,2]. The bilocal contribution to the 3-point function eq.(2) can be written in a "parton" form as a convolution of the photon DAs and some coefficient functions. The last originate from a light cone OPE for the product $T\{J(0)j^5(Y)\}$. The amplitude $\mathcal{F}$ is now a sum of its purely short-distance (SD) (regular for $q^2=0$) and bilocal (B) parts. Getting the $q^2\to 0$ limit of $\Pi_n^{(i)}(q_1)$ requires a nonperturbative input.

After all modifications described above are made, we can write the QCD sum rule for the $\gamma\gamma^* \to \pi^0$ form factor in the $q^2 = 0$ limit:

$$
\begin{aligned}
\pi f_\pi F_{\gamma\gamma^*\pi^0}(Q^2) &= \int_0^{s_0} \left\{ 1 - 2\frac{Q^2 - 2s}{(s+Q^2)^2} \left(s_\rho - \frac{s_\rho^2}{2m_\rho^2} \right) \right. \\
&+ \left. 2\frac{Q^4 - 6sQ^2 + 3s^2}{(s+Q^2)^4} \left(\frac{s_\rho^2}{2} - \frac{s_\rho^3}{3m_\rho^2} \right) \right\} e^{-s/M^2} \frac{Q^2 ds}{(s+Q^2)^2} \\
&+ \frac{\pi^2}{9} \langle \frac{\alpha_s}{\pi} GG \rangle \left\{ \frac{1}{2Q^2M^2} + \frac{1}{Q^4} - 2\int_0^{s_0} e^{-s/M^2} \frac{ds}{(s+Q^2)^3} \right\} \\
&+ \frac{64}{27}\pi^3 \alpha_s \langle \bar{q}q \rangle^2 \lim_{\lambda^2 \to 0} \left\{ \frac{1}{2Q^2M^4} + \frac{12}{Q^4 m_\rho^2} \left[\log\frac{Q^2}{\lambda^2} - 2 \right. \right. \\
&+ \left. \int_0^{s_0} e^{-s/M^2} \left(\frac{s^2 + 3sQ^2 + 4Q^4}{(s+Q^2)^3} - \frac{1}{s+\lambda^2} \right) ds \right] \\
&- \left. \frac{4}{Q^6} \left[\log\frac{Q^2}{\lambda^2} - 3 + \int_0^{s_0} e^{-s/M^2} \left(\frac{s^2 + 3sQ^2 + 6Q^4}{(s+Q^2)^3} - \frac{1}{s+\lambda^2} \right) ds \right] \right\} \quad (4)
\end{aligned}
$$

Here we model the bilocal contributions using the asymptotic form for the DAs of the ρ-meson and making them approximately dual to the corresponding pt-contribution. We use the standard values for the condensates and the ρ-meson

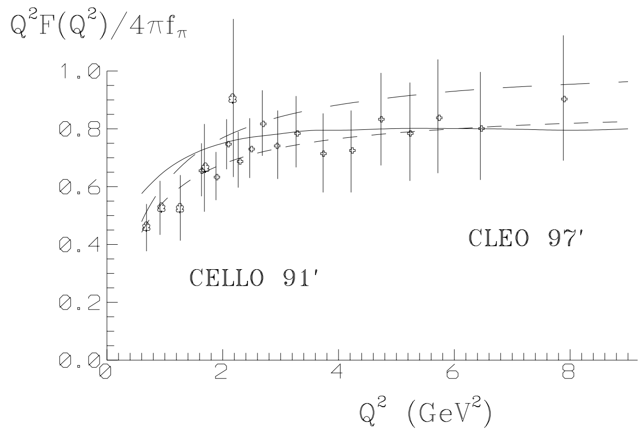

Figure 1:

duality interval $s_\rho = 1.5\ GeV^2$ [14]. Explicit fitting procedure in (4) favours the value $s_0 \approx 0.7\, GeV^2$ for the effective threshold [1,2]. Hence, our calculations support the local duality prescription [15].

In Fig.1, we present our curve (solid line) for $Q^2 F_{\gamma\gamma^*\pi^0}(Q^2)/4\pi f_\pi$ calculated from eq.(4) for $s_0 = 0.7\,GeV^2$. One can observe very good agreement with the new CLEO data [5]. It is rather close to the Brodsky-Lepage interpolation formula [c] (long-dashed line) and the ρ-pole approximation (short-dashed line) $\pi f_\pi F^{VMD}(Q^2) = 1/(1 + Q^2/m_\rho^2)$. It should be noted that the Q^2-dependence of the ρ-pole type emerges due to the fact that the pion duality interval $s_0 \approx 0.7\,GeV^2$ is numerically close to $m_\rho^2 \approx 0.6\,GeV^2$. In the region $Q^2 > Q_*^2 \sim 3\,GeV^2$, our curve for $Q^2 F_{\gamma\gamma^*\pi^0}(Q^2)$ is practically constant, supporting the pQCD expectation (1). The absolute magnitude of our prediction gives $I \approx 2.4$ for the I-integral with an accuracy of about 20%.

Comparing the value $I = 2.4$ with $I^{as} = 3$ and $I^{CZ} = 5$, we conclude that our result favours a pion DA which is narrower than the asymptotic form. Parametrizing the width of $\varphi_\pi(x)$ by a simple model $\varphi_\pi(x) \sim [x(1-x)]^n$, we get that $I = 2.4$ corresponds to $n = 2.5$. The second moment $\langle \xi^2 \rangle \equiv \langle (x - \bar{x})^2 \rangle$ for such a function is 0.125 (recall that $\langle \xi^2 \rangle^{as} = 0.2$ while $\langle \xi^2 \rangle^{CZ} = 0.43$) which agrees with the lattice calculation [16].

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[c] In fact, such interpolation follows from the local duality considerations [15]

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