QCD SUM RULES: FORM FACTORS AND WAVE FUNCTIONS

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The shape of hadronic distribution amplitudes (DAs) is a critical issue for the perturbative QCD of hard exclusive processes. Recent CLEO data on $\gamma\gamma^* \to \pi^0$ form factor clearly favor a pion DA close to the asymptotic form. We argue that QCD sum rules for the moments of the pion DA $\varphi_\pi(x)$ are unreliable, so that the humpy shape of $\varphi_\pi(x)$ obtained by Chernyak and Zhitnitsky is a result of model assumptions rather than an unambigous consequence of QCD sum rules. This conclusion is also supported by a direct QCD sum rule calculation of the $\gamma\gamma^* \to \pi^0$ form factor which gives a result very close to the CLEO data.

1 Introduction

In this talk, I discuss some general features of QCD sum rule applications to hadronic wave functions and form factors using as examples the pion distribution amplitude $\varphi_\pi(x)$ and transition form factor for the process $\gamma^*\gamma^* \to \pi^0$ in which two virtual photons produce a neutral pion. This process provides an exceptional opportunity to test QCD predictions for exclusive processes. In the lowest order of perturbative QCD, its asymptotic behaviour is due to the subprocess $\gamma^*(q_1) + \gamma^*(q_2) \to \bar{q}(\bar{x}p) + q(xp)$ with x ($\bar{x}$) being the fraction of the pion momentum p carried by the quark produced at the q_1 (q_2) photon vertex. The relevant diagram resembles the handbag diagram of DIS with the pion distribution amplitude (DA) $\varphi_\pi(x)$ instead of parton densities. The asymptotic PQCD prediction is given by[1] ($q_1^2 \equiv -q^2$, $q_2^2 \equiv -Q^2$, $\bar{x} = 1 - x$):

$$F_{\gamma^*\gamma^*\pi^0}^{as}(q^2, Q^2) = \frac{4\pi}{3} \int_0^1 \frac{\varphi_\pi(x)}{xQ^2 + \bar{x}q^2} \, dx \xrightarrow{q^2=0} \frac{4\pi}{3} \int_0^1 \frac{\varphi_\pi(x)}{xQ^2} \, dx \equiv \frac{4\pi f_\pi}{3Q^2} I. \quad (1)$$

Experimentally, the most important situation is when one of the photons is almost real $q^2 \approx 0$ [2,3]. In this case, necessary nonperturbative information is accumulated in the same integral I (see Eq.(1)) which appears in the one-gluon-exchange diagram for the pion electromagnetic form factor[4,5,6].

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The value of I is sensitive to the shape of the pion DA $\varphi_\pi(x)$. Using the asymptotic [4,5] form $\varphi_\pi^{\rm as}(x) = 6f_\pi x\bar{x}$ gives $I = 3$ and $F_{\gamma\gamma^*\pi^0}^{\rm as}(Q^2) = 4\pi f_\pi/Q^2$. If one takes the CZ form [6] $\varphi_\pi^{\rm CZ}(x) = 30f_\pi x\bar{x}(1-2x)^2$, then $I = 5$, and this difference can be used for experimental discrimination between the two forms. One-loop radiative QCD corrections to Eq.(1) are known [7,8,9] and they are under control. Clearly, the asymptotic $1/Q^2$-behaviour cannot be true in the low-Q^2 region, since the $Q^2 = 0$ limit of $F_{\gamma\gamma^*\pi^0}(Q^2)$ is known to be finite and normalized by the $\pi^0 \to \gamma\gamma$ decay rate. From the axial anomaly [10], $F_{\gamma\gamma^*\pi^0}(0) = 1/\pi f_\pi$. Brodsky and Lepage [1] proposed a simple interpolation $\pi f_\pi F_{\gamma\gamma^*\pi^0}^{\rm LO}(Q^2) = 1/(1+Q^2/4\pi^2f_\pi^2)$ between the $Q^2 = 0$ value $1/\pi f_\pi$ and the leading-twist PQCD behavior $4\pi f_\pi/Q^2$ with normalization corresponding to the asymptotic DA. Note that the mass scale $s_o^\pi \equiv 4\pi^2 f_\pi^2 \approx 0.67~GeV^2$ in this monopole formula is close to m_ρ^2. Recent experimental data [3] from CLEO are below the BL-curve and are by almost a factor of 2 lower than the value for the CZ wave function. This result apparently excludes the CZ DA and suggests that the pion DA may be even narrower than $\varphi_\pi^{\rm as}(x)$. Since the CZ model is often perceived as a direct consequence from QCD sum rules, the experimental evidence in favor of a narrow DA may be treated as a failure of the QCD sum rule approach. One should remember, however, that accuracy of QCD sum rules strongly depends on the specific hadronic characteristics to which the sum rule technique is applied. Long ago, in papers [11] written with S. Mikhailov, we argued that CZ sum rules are very unreliable, with the results strongly depending on the assumptions about the size of higher terms in the operator product expansion (OPE).

2 QCD sum rules

QCD sum rules[12] are based on quark-hadron duality, *i.e.*, possibility to describe the same object in terms of either quark or hadronic fields. To calculate f_π, consider the $p_\mu p_\nu$-part of the correlator of two axial currents:

$$\Pi^{\mu\nu}(p) = i \int e^{ipx}\langle 0|T\left(j_{5\mu}^+(x)\,j_{5\nu}^-(0)\right)|0\rangle\,d^4x = p_\mu p_\nu \Pi_2(p^2) - g_{\mu\nu}\Pi_1(p^2). \quad (2)$$

The dispersion relation represents $\Pi_2(p^2)$ as an integral over hadronic spectrum

$$\Pi_2(p^2) = \frac{1}{\pi}\int_0^\infty \frac{\rho^{\rm hadron}(s)}{s-p^2}ds + \text{``subtractions''} \quad (3)$$

with the spectral density $\rho^{\rm hadron}(s)$ determined by projections of the axial current onto hadronic states ($\langle 0|j_{5\mu}(0)|\pi;P\rangle = if_\pi P_\mu$, *etc.*):

$$\rho^{\rm hadron}(s) = \pi f_\pi^2\delta(s-m_\pi^2) + \pi f_{A_1}^2\delta(s-m_{A_1}^2) + \text{``higher states''} \quad (4)$$

2

$(f_\pi^{\mathrm{exp}} \approx 130\, MeV$ in our normalization). On the other hand, when the probing virtuality is negative and large, one can use the OPE

$$\Pi_2(p^2) = \Pi_2^{\mathrm{pert}}(p^2) + \frac{A}{p^4}\langle\alpha_s GG\rangle + \frac{B}{p^6}\alpha_s\langle\bar{q}q\rangle^2 + \ldots \tag{5}$$

where $\Pi_2^{\mathrm{pert}}(p^2) \equiv \Pi_2^{\mathrm{quark}}(p^2)$ is the perturbative version of $\Pi_2(p^2)$ given by a sum of PQCD Feynman diagrams while the condensate terms $\langle GG\rangle$, $\langle\bar{q}q\rangle$, $etc.$, (with calculable coefficients $A, B, etc.$) describe/parameterize the non-trivial structure of the QCD vacuum. The quark amplitude $\Pi_2^{\mathrm{quark}}(p^2)$, can also be written in the dispersion representation (3), with $\rho(s)$ substituted by its perturbative analogue $\rho^{\mathrm{quark}}(s) = \frac{1}{4\pi}\left(1 + \frac{\alpha_s}{\pi} + \ldots\right)$ (quark masses neglected). Hence, the condensate terms describe the difference between the quark and hadron spectra. Treating the condensate values as known, one can try to construct a model for the hadronic spectrum. The simplest model is to approximate all the higher resonances including the A_1 by the quark spectral density starting at some effective threshold s_0:

$$\rho^{\mathrm{hadron}}(s) \approx \pi f_\pi^2 \delta(s - m_\pi^2) + \rho^{\mathrm{quark}}(s)\, \theta(s \geq s_0). \tag{6}$$

Neglecting the pion mass and using the standard values for the condensates $\langle GG\rangle$, $\langle\bar{q}q\rangle^2$, one should adjust s_0 to get an (almost) constant result for the rhs of the SVZ-borelized version of the sum rule

$$f_\pi^2 = \frac{1}{\pi}\int_0^{s_0}\rho^{\mathrm{quark}}(s)e^{-s/M^2}ds \; + \frac{\alpha_s\langle GG\rangle}{12\pi M^2} + \frac{176}{81}\frac{\pi\alpha_s\langle\bar{q}q\rangle^2}{M^4} + \ldots . \tag{7}$$

The magnitude of f_π extracted in this way, is very close to its experimental value $f_\pi^{\mathrm{exp}} \approx 130\, MeV$. Changing the values of the condensates, one would get the best M^2-stability for a different s_0, and the resulting value of f_π would also change. Correlation between the fitted values of f_π and s_0 is manifest in the $M^2 \to \infty$ limit of the sum rule

$$f_\pi^2 = \frac{1}{\pi}\int_0^{s_0}\rho^{\mathrm{quark}}(s)\, ds, \tag{8}$$

giving a local duality relation which states that two densities $\rho^{\mathrm{quark}}(s)$ and $\rho^{\mathrm{hadron}}(s)$ give the same result if one integrates them over the *appropriate* duality interval s_0. The role of the condensates was to determine the size of the duality interval s_0, but after it was fixed, one can write down the relation (8) which does not involve the condensates. In the lowest order, $\rho_0^{\mathrm{quark}}(s) = 1/4\pi$, which gives $s_0 = 4\pi^2 f_\pi^2$. Note, that this is exactly the combination which appeared in the Brodsky-Lepage interpolation formula.

3

3 CZ sum rules and pion DA

Chernyak and A. Zhitnitsky [6] proposed to use QCD sum rules for calculating next moments $\langle \xi^N \rangle$ (where $\xi = 2x - 1$) of the pion DA. They extracted $\langle \xi^2 \rangle$ and $\langle \xi^4 \rangle$ from the relevant SR

$$f_\pi^2 \langle \xi^N \rangle = \frac{3M^2}{4\pi^2} \frac{(1 - e^{-s_0/M^2})}{(N+1)(N+3)} + \frac{\alpha_s \langle GG \rangle}{12\pi M^2} + \frac{16}{81} \frac{\pi \alpha_s \langle \bar{q}q \rangle^2}{M^4}(11 + 4N) \qquad (9)$$

precisely in the same way as the f_π value. Note that the scale determining the magnitude of the hadronic parameters is settled by the ratios of the condensate contributions to the perturbative term. If the condensate contributions in the CZ sum rule (9) would have the same N-behavior as the perturbative term, then the N-dependence of $\langle \xi^N \rangle$ would be determined by the overall factor $3/(N+1)(N+3)$ and the resulting wave function $\varphi_\pi^{\mathrm{as}}(x) = 6f_\pi x(1-x)$ would coincide with the asymptotic form. However, the ratios of the $\langle \bar{q}q \rangle$ and $\langle GG \rangle$-corrections to the perturbative term in Eq. (9) are growing functions of N. In particular, in the $\langle \bar{q}q \rangle$ case, the above mentioned ratio for $N = 2$ is by factor $95/11$ larger than that in the $N = 0$ case. For $N = 4$ the enhancement factor equals $315/11$. As a result, the effective vacuum scales of $(mass)^2$ dimension are by factors $(95/11)^{1/3} \approx 2.1$ and $(315/11)^{1/3} \approx 3.1$ larger than that for the $N = 0$ case. Approximately the same factors ($5^{1/2} \approx 2.2$ and $(35/3)^{1/2} \approx 3.4$) result from the gluon condensate term. Hence, the parameters $s_0^{(N)}$ and the combinations $f_\pi^2 \langle \xi^N \rangle$ straightforwardly extracted from the SR (9) are enhanced compared to $s_0^{N=0} \approx 0.7\, GeV^2$ and $3f_\pi^2/(N+1)(N+3)$, resp., by the factors 2 (for $N = 2$) and 3 (for $N = 4$). These are just the results given in Ref. [6]. To clarify the assumptions implied by such a procedure, we rewrite the CZ sum rule using the standard numerical values for the condensates:

$$\int_0^\infty \rho_N(s) e^{-s/M^2} ds = \frac{M^2}{4\pi^2} \left[\frac{3}{(N+1)(N+3)} \right.$$
$$\left. + 0.1 \left(\frac{0.6}{M^2} \right)^2 + 0.22 \left(1 + \frac{4N}{11} \right) \left(\frac{0.6}{M^2} \right)^3 \right] . \qquad (10)$$

Taking first $N = 0$, we see that for $M^2 = 0.6$ GeV^2 the condensate corrections are by factor 3 smaller than the perturbative term while the exponential e^{-s/M^2} suppresses the A_1 contribution by factor 14 compared to the pion one. Hence, the sum rule looks very reliable since power corrections are small in the region where the s-integral is dominated by the pion. Taking the "first resonance plus effective continuum" model for the spectrum and fitting the sum rule in the $M^2 > 0.6$ GeV^2 region gives $s_0 \approx 0.75$ GeV^2 for the effective threshold, i.e.

4

at the threshold the exponential e^{-s_0/M^2} provides $1/3$ suppression factor for $M^2 = 0.6$ GeV^2, which ensures that the result for f_π^2 is not very sensitive to the model chosen for the higher states.

Now, taking $N = 2$, we observe that for $M^2 = 0.6$ GeV^2 the condensate corrections are by factor 2.4 *larger* than the perturbative term: the $1/M^2$ expansion is apparently useless at such a value of M^2. To bring the size of condensate corrections to less than $1/3$ of the perturbative term, one should take $M^2 > 1.2$ GeV^2. However, for such large M^2 values the exponential e^{-s/M^2} gives practically no suppression at the "old" effective threshold, and results for $\langle \xi^2 \rangle$ would strongly depend on the model for higher states. In particular, the "first resonance plus effective continuum" ansatz gives $s_0^{(2)} \approx 1.5$ GeV^2 and $\langle \xi^2 \rangle \approx 0.4$ which means that with respect to $\langle \xi^2 \rangle$ the pion is dual to much wider interval $0 < s < 1.5$ GeV^2. For $N = 4$ the duality interval obtained in this way is even wider: $s_0^{(4)} \approx 2.2$ GeV^2, i.e., the effective continuum threshold is assumed to be well above the A_1 location.

Of course, one cannot exclude a priori that a different correlator has a different shape of spectral density. Ideally, having the full expression for the right-hand side of the sum rule one could find out $\rho_N(s)$ exactly. Having just few terms of the $1/M^2$-expansion, we can only construct an approximation for the spectrum, the precision of which depends on the relative magnitude of the neglected higher terms. The CZ-procedure is equivalent to assumption that two condensate terms included in their sum rule dominate the expansion not only for $N = 0$ but also for $N = 2$. In fact, it is impossible to check by a direct calculation whether this assumption is true or not, because the number of possible condensates explodes when their dimension increases, and there is no reliable way to determine their values. Still, it is rather easy to establish that coefficients accompanying the condensates $\langle \bar{q} D^2 q\, \bar{q} q \rangle$ with two covariant derivatives D behave like N^3 for large N, i.e. have even larger N-dependent enhancement compared to the perturbative term. In general, the coefficients for $\langle \bar{q} (D^2)^n q\, \bar{q} q \rangle$ condensates behave like N^{n+1} for large N. Hence, one would rather expect that there are *large* higher-condensate corrections to the $\langle \xi^2 \rangle$ sum rule. Only some miraculous cancellation can make them small. No reason for such a cancellation was given.

To summarize: if one takes the CZ sum rule at face value, i.e., assumes that there are no essential corrections to it, the fitting procedure would produce the large CZ value for $\langle \xi^2 \rangle$. However, since the perturbative term decreases with N while the condensate terms rapidly increase with N, the CZ sum rules for $N \leq 2$ is an obvious case when one *must* expect essential corrections.

4 Nonlocal condensates

It is also instructive to write the SR for the pion DA $\varphi_\pi(x)$ itself[11]:

$$f_\pi^2 \varphi_\pi(x) = \frac{M^2}{4\pi^2}(1 - e^{-s_0/M^2})\varphi_\pi^{as}(x) + \frac{\alpha_s\langle GG\rangle}{24\pi M^2}[\delta(x) + \delta(1-x)]$$

$$+ \frac{8}{81}\frac{\pi\alpha_s\langle\bar{q}q\rangle^2}{M^4}\{11[\delta(x) + \delta(1-x)] + 2[\delta'(x) + \delta'(1-x)]\}. \quad (11)$$

The $O(1)$ and $O(N)$ terms in Eq. (9) correspond to the $\delta(x)$ and $\delta'(x)$-terms in Eq.(11) indicating that the vacuum fields are carrying zero fraction of the pion momentum. The operator product expansion (underlying eqs.(9),(11)) is, in fact, a power series expansion over small momenta k of vacuum quarks and gluons. Retaining only the $\langle\bar{q}q\rangle$ and $\langle GG\rangle$-terms (like in eqs.(9),(11)) is just equivalent to the assumption that k is not simply small but exactly equals zero. However, it is much more reasonable to expect that the vacuum quanta have a smooth distribution with a finite width μ. In configuration space, this means that vacuum fluctuations have a finite correlation length of the order of $1/\mu$, so that the two-point condensates like $\langle\bar{q}(0)q(z)\rangle$ die away for $|z|$ large compared to $1/\mu$. In the OPE, $\langle\bar{q}(0)q(z)\rangle$ is expanded in powers of z and the first term $\langle\bar{q}(0)q(0)\rangle$ produces eventually the $\delta(x)$-term, while higher $\langle\bar{q}(0)(D^2)^n q(0)\rangle$ terms give $\delta^n(x)$ contributions resulting in N^n factors in the $\langle\xi^N\rangle$ sum rule. In other words, arranging the $1/M^2$ expansion through the OPE in terms of local operators, one automatically obtains $\langle\xi^N\rangle$ in the form of Taylor expansion in N. Even if the condensate contribution to the $\langle\xi^N\rangle$ sum rule is a rapidly decreasing function of N (which must be the case for any smooth function of ξ), the OPE gives it as a Taylor series in N^n whose terms rapidly increase with N. In such a situation, it is obviously risky to take just the first term of the expansion, e.g., the quark condensate $(11 + 4N)$ factor may well be just the first term of something like $(11 + 4N)\exp[-N\lambda^2/M^2]$ with much smaller value for $N = 2$ than one would expect from $(11 + 4N)$. How much smaller, depends on the value of the scale λ^2. The size of the correlation length of vacuum fluctuations can be estimated using the standard value [13] $\lambda_q^2 \equiv \langle\bar{q}D^2q\rangle/\langle\bar{q}q\rangle = 0.4 \pm 0.1\,GeV^2$ for the average virtuality of the vacuum quarks. One can see that it is not small compared to the relevant hadronic scale $s_0^{N=0} \approx 4\pi^2 f_\pi^2 = 0.7\,GeV^2$, and the constant-field approximation for the vacuum fields is not safe. Using the exponential model $\langle\bar{q}(0)q(z)\rangle = \langle\bar{q}q\rangle\exp[z^2\lambda_q^2/2]$ for the nonlocal condensate gives a QCD sum rule producing the wave functions very close to the asymptotic ones[11]. This study suggests that the humpy form of the CZ wave function is a mere consequence of the approximation that vacuum quarks have zero momentum.

5 QCD sum rule for the $\gamma^* \gamma \to \pi^o$ form factor

Another evidence that the pion DA is close to its asymptotic shape is given by a direct QCD sum rule analysis [14,15] of the $\gamma^* \gamma \to \pi^o$ transition form factor. In this case, one should consider the three-point correlation function

$$\mathcal{F}_{\alpha\mu\nu}(q_1, q_2) = 2\pi i \int \langle 0|T\left\{ j_\alpha^5(Y) J_\mu(X) J_\nu(0) \right\}|0\rangle e^{-iq_1 X} e^{ipY} d^4X \, d^4Y \,, \quad (12)$$

where J_μ is the electromagnetic current. The operator product expansion is simpler when both photon virtualities q^2 are large: $q^2, Q^2 \geq 1 \, GeV^2$. QCD sum rule in this kinematics is given by

$$\pi f_\pi F_{\gamma^*\gamma^*\pi^o}(q^2, Q^2) = 2 \int_0^{s_o} ds \, e^{-s/M^2} \int_0^1 \frac{x\bar{x}(xQ^2 + \bar{x}q^2)^2}{[sx\bar{x} + xQ^2 + \bar{x}q^2]^3} dx$$

$$+ \frac{\pi^2}{9}\langle\frac{\alpha_s}{\pi}GG\rangle \left(\frac{1}{2M^2Q^2} + \frac{1}{2M^2q^2} - \frac{1}{Q^2q^2} \right)$$

$$+ \frac{64}{243}\pi^3\alpha_s\langle\bar{q}q\rangle^2 \left(\frac{1}{M^4}\left[\frac{Q^2}{q^4} + \frac{9}{2q^2} + \frac{9}{2Q^2} + \frac{q^2}{Q^4} \right] + \frac{9}{Q^2q^4} + \frac{9}{Q^4q^2} \right). \quad (13)$$

Keeping only the leading $O(1/Q^2$ and $1/q^2)$-terms one can rewrite it as

$$F_{\gamma^*\gamma^*\pi^o}^{LO}(q^2, Q^2) = \frac{4\pi}{3f_\pi} \int_0^1 \frac{dx}{(xQ^2 + \bar{x}q^2)} \left\{ \frac{3M^2}{2\pi^2}(1 - e^{-s_0/M^2})x\bar{x} \right.$$

$$+ \frac{1}{24M^2}\langle\frac{\alpha_s}{\pi}GG\rangle[\delta(x) + \delta(\bar{x})]$$

$$+ \left. \frac{8}{81M^4}\pi\alpha_s\langle\bar{q}q\rangle^2 \left(11[\delta(x) + \delta(\bar{x})] + 2[\delta'(x) + \delta'(\bar{x})] \right) \right\}. \quad (14)$$

Note, that the expression in curly brackets coincides with the QCD sum rule (11) for the pion DA $f_\pi\varphi_\pi(x)$. Hence, the QCD sum rule approach exactly reproduces the PQCD result (1). One may be tempted to get a QCD sum rule for the integral I by taking $q^2 = 0$ in Eq.(13). The attempt is ruined by power singularities $1/q^2$, $1/q^4$ in the condensate terms. Moreover, the perturbative term in the small-q^2 region has logarithms $\log q^2$ which are a typical example of mass singularities (see, $e.g.$,[16]). All these infrared sensitive terms are produced in a regime when the hard momentum flow bypasses the soft photon vertex, $i.e.$, the EM current $J_\mu(X)$ of the low-virtuality photon is far away from the two other currents $J(0), j^5(Y)$. It is also important to observe that power singularities $1/q^2$, $1/q^4$ are generated precisely by the same $\delta(x)$ and $\delta'(x)$ terms in Eq.(14) which generate the two-hump form for $\varphi_\pi(x)$ in the CZ-approach [6]. As shown in Ref.[11], the humps disappear if one treats the $\delta(x)$

and $\delta'(x)$ terms as the first terms of a formal expansion $\Phi(x) \sim \sum a_n \delta^n(x)$ of smooth functions $\Phi(x)$. Similarly, the $1/q^2$ singularity can be understood as the first term of the large-q^2 expansion of a term like $1/(q^2 + m_\rho^2)$ in powers of $1/q^2$. However, constructing $\Phi(x)$ from two first terms of such expansion is a strongly model-dependent procedure. On the other hand, the small-q^2 behavior of the three-point function is rather severely constrained by known structure of the physical spectrum in the EM-current channel. The procedure developed in Refs.[14,15] allows to subtract all the small-q^2 singularities from the coefficient functions of the original OPE for the 3-point correlation function Eq.(12). They are absorbed in this approach by universal bilocal correlators, which can be also interpreted as moments of the DAs for (almost) real photon

$$\int_0^1 y^n \phi_\gamma^{(i)}(y, q^2) \sim \Pi_n^{(i)}(q^2) = \int e^{iq_1 X} \langle 0|T\{J_\mu(X)\mathcal{O}_n^{(i)}(0)\}|0\rangle d^4X,$$

where $\mathcal{O}_n^{(i)}(0)$ are operators of leading and next-to-leading twist with n covariant derivatives[14,15]. The bilocal contribution to the 3-point function Eq.(12) can be written in a "parton" form as a convolution of the photon DAs and some coefficient functions. The latter originate from a light cone OPE for the product $T\{J(0)j^5(Y)\}$. The amplitude $\mathcal{F}$ is now a sum of its purely short-distance (SD) (regular for $q^2 = 0$) and bilocal (B) parts. Getting the $q^2 \to 0$ limit of $\Pi_n^{(i)}(q_1)$ requires a nonperturbative input obtained from an auxiliary QCD sum rule. After all the modifications outlined above are made, one can write the QCD sum rule for the $\gamma\gamma^* \to \pi^0$ form factor in the $q^2 = 0$ limit:

$$\pi f_\pi F_{\gamma\gamma^*\pi^0}(Q^2) = \int_0^{s_0} \left\{ 1 - 2\frac{Q^2 - 2s}{(s + Q^2)^2} \left(s_\rho - \frac{s_\rho^2}{2m_\rho^2} \right) \right.$$

$$+ 2\frac{Q^4 - 6sQ^2 + 3s^2}{(s + Q^2)^4} \left(\frac{s_\rho^2}{2} - \frac{s_\rho^3}{3m_\rho^2} \right) \right\} e^{-s/M^2} \frac{Q^2 ds}{(s + Q^2)^2}$$

$$+ \frac{\pi^2}{9} \langle \frac{\alpha_s}{\pi} GG \rangle \left\{ \frac{1}{2Q^2 M^2} + \frac{1}{Q^4} - 2\int_0^{s_0} e^{-s/M^2} \frac{ds}{(s + Q^2)^3} \right\} \qquad (15)$$

$$+ \frac{64}{27} \pi^3 \alpha_s \langle \bar{q}q \rangle^2 \lim_{\lambda^2 \to 0} \left\{ \frac{1}{2Q^2 M^4} + \frac{12}{Q^4 m_\rho^2} \left[\log\frac{Q^2}{\lambda^2} - 2 \right. \right.$$

$$+ \int_0^{s_0} e^{-s/M^2} \left(\frac{s^2 + 3sQ^2 + 4Q^4}{(s + Q^2)^3} - \frac{1}{s + \lambda^2} \right) ds \right]$$

$$\left. - \frac{4}{Q^6} \left[\log\frac{Q^2}{\lambda^2} - 3 + \int_0^{s_0} e^{-s/M^2} \left(\frac{s^2 + 3sQ^2 + 6Q^4}{(s + Q^2)^3} - \frac{1}{s + \lambda^2} \right) ds \right] \right\}.$$

8

Here the bilocal contributions are modeled by asymptotic form for the ρ-meson DAs. They are approximately dual to the corresponding perturbative contribution with the ρ-meson duality interval [12] $s_\rho = 1.5\ GeV^2$. The results of

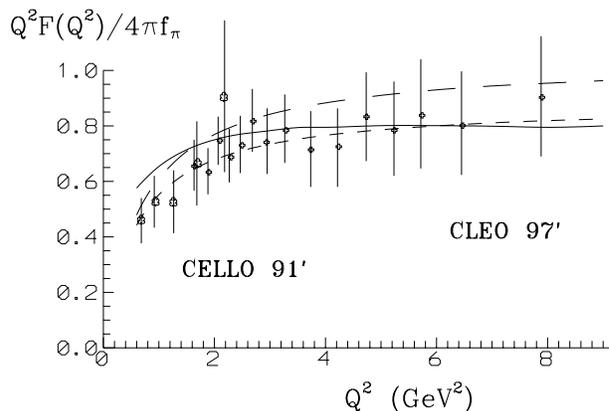Figure 1:

fitting procedure for (15) favor the value $s_0 \approx 0.7\ \mathrm{GeV}^2$ as the effective threshold [14,15]. For this reason, the results of our calculations are well approximated by the local duality prescription [17]

$$\pi f_\pi F_{\gamma\gamma^*\pi^0}^{\mathrm{LD}}(Q^2) = \int_0^{s_0} \rho^{\mathrm{quark}}(s, Q^2)ds = \frac{1}{1 + Q^2/s_0} \qquad (16)$$

which coincides for $s_0 = 4\pi^2 f_\pi^2$ with the BL interpolation formula. In Fig.1, we present our curve (solid line) for $Q^2 F_{\gamma\gamma^*\pi^0}(Q^2)/4\pi f_\pi$ calculated from Eq.(15) for $s_0 = 0.7\ GeV^2$ and $M^2 = 0.8\ GeV^2$. One can observe very good agreement with the new CLEO data [3]. It is also rather close to the BL interpolation/local duality formula (long-dashed line) and the ρ-pole approximation (short-dashed line) $\pi f_\pi F^{VMD}(Q^2) = 1/(1 + Q^2/m_\rho^2)$. It should be noted that the Q^2-dependence of the ρ-pole type emerges due to the fact that the pion duality interval $s_0 \approx 0.67\ GeV^2$ is numerically close to $m_\rho^2 \approx 0.6\ GeV^2$. In the region $Q^2 > Q_*^2 \sim 3\ GeV^2$, our curve for $Q^2 F_{\gamma\gamma^*\pi^0}(Q^2)$ is practically constant, supporting the PQCD expectation (1). The absolute magnitude of our prediction gives $I \approx 2.4$ for the I-integral with an accuracy of about 20%. Comparing the value $I = 2.4$ with $I^{\mathrm{as}} = 3$ and $I^{\mathrm{CZ}} = 5$, we conclude that our result favours a pion DA which is narrower than the asymptotic form. Parametrizing the width of $\varphi_\pi(x)$ by a simple model $\varphi_\pi(x) \sim [x(1-x)]^n$, we obtain that $I = 2.4$ corresponds to $n = 2.5$. The second moment $\langle\xi^2\rangle \equiv \langle(x - \bar{x})^2\rangle$ for such

9

a function is 0.125 (recall that $\langle\xi^2\rangle^{\mathrm{as}} = 0.2$ while $\langle\xi^2\rangle^{\mathrm{CZ}} = 0.43$) which agrees with the lattice calculation [18].

Thus, the old claim [11] that the CZ sum rules [6] for the moments of DAs are unreliable is now supported both by a direct QCD sum rule calculation of the $\gamma^*\gamma\pi^0$ form factor [14,15] producing the result corresponding to a narrow pion DA, and by experimental measurement of this form factor [3] which also favors a pion DA close to the asymptotic form. Since the humpy form of the CZ models for the nucleon DA's [6] has the same origin as in the pion case, there is no doubt that the nucleon DA's are also close to the asymptotic ones. This means that PQCD contributions to nucleon elastic and transition form factors are tiny at available and reachable energies.

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