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How to extract the $P_{33}(1232)$ resonance contributions from the amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$ of pion electroproduction on nucleons

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Abstract

Within the dispersion relation approach, solutions of integral equations for the multipoles $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$ are found at $0 \leq Q^2 \leq 3 \ GeV^2$. These solutions should be used as input for the resonance and nonresonance contributions in the analyses of pion electroproduction data in the $P_{33}(1232)$ resonance region. It is shown that the traditional identification of the amplitude $M_{1+}^{3/2}$ (as well of the amplitudes $E_{1+}^{3/2}, S_{1+}^{3/2}$) with the $P_{33}(1232)$ resonance contribution is not right; there is a contribution in these amplitudes which has a nonresonance nature and is produced by rescattering effects in the diagrams corresponding to the nucleon and pion poles. This contribution is reproduced by the dispersion relations. Taking into account nonresonance contributions in the amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}$, the helicity amplitudes $A_p^{1/2}$, $A_p^{3/2}$ and the ratio $E2/M1$ for the $\gamma N \rightarrow P_{33}(1232)$ transition are extracted from experiment at $Q^2 = 0$. They are in good agreement with quark model predictions.

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1   Introduction

It is known that experimental data on form factors of the $\gamma N \to P_{33}(1232)$ transition may play an important role in the investigation of energetic scale of transition to perturbative region of QCD. The conservation of quark helicities in the regime of perturbative QCD leads to the asymptotic relation [1-3]:

$$\frac{G_E}{G_M} \to -1, \quad Q^2 \to \infty \quad (pQCD). \tag{1}$$

In contrast to this at $Q^2 = 0$ quark model predicts the suppression of G_E:

$$\frac{G_E}{G_M} = 0, \quad Q^2 = 0 \quad (quark\ model), \tag{2}$$

which agrees well with experiment. Thus, the transition from nonperturbative region of QCD to perturbative one is characterized by a striking change of the behavior of the ratio $G_E(Q^2)/G_M(Q^2)$, and, therefore, the measurement of this ratio will provide a sensitive test for understanding of mechanisms of transition to the QCD asymptotics.

For the Coulombic form factor predictions of quark model and pQCD coincide with each other:

$$\frac{G_C}{G_M} = 0, \quad Q^2 = 0 \quad (quark\ model), \tag{3}$$

$$\frac{G_C}{G_M} \to 0, \quad Q^2 \to \infty \quad (pQCD), \tag{4}$$

and in this case we have no test for investigation of the energetic scale of the transition to the QCD asymptotics. However, precise measurement of the Q^2-dependence of the Coulombic form factor can be of interest too for the development of realistic models of the nucleon and the $P_{33}(1232)$.

In the nearest future significant progress in the investigation of the $\gamma N \to P_{33}(1232)$ transition form factors is expected due to construction of continuous wave electron accelerators. These form factors will be studied in the reaction of pion electroproduction on nucleons via extraction of the resonance multipole amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$ which carry information on the contribution of the resonance P_{33} to this process (the diagram of Fig. 1d).

It is well known that an extremely fruitful role in the investigation of pion photo-and electroproduction on nucleons belongs to the approach based on dispersion relations. The basis of this approach was founded in the classical works [4,5]. Further, it was developed in numerous works, among which let us mention the papers [6-12]. Analysing the results obtained within dispersion relation approach we came to the conclusion that in order to obtain a proper input for the analysis of expected experimental data in the $P_{33}(1232)$ resonance region it is very useful to use the approach developed in Refs. [6,7]. Let us clarify this statement.

The solutions of integral equations for the multipoles $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$ following from dispersion relations for these multipoles are obtained in Refs. [6,7] in the form which contains two parts. One part is the particular solution of the integral equation generated by the Born term (i.e. by the diagrams of Fig. 1(a-c) corresponding to the nucleon and pion poles). It has definite magnitude fixed by the Born term. Our analysis shows that other contributions to the particular solutions for the multipoles $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$ which can arise from nonresonance multipoles and high energy contributions are negligibly small.

Other part of the solutions corresponds to the homogeneous parts of the integral equations. This part has a certain energy dependence fixed by dispersion relations and an arbitrary weight which should be found from the comparison with experiment.

In the present work (Sec.3) we have repeated the results of Refs. [6,7] for the solutions of dispersion relations for the multipoles $M_{1+}^{3/2}, E_{1+}^{3/2}$ at $Q^2 = 0$. In addition, we have obtained Q^2-evolution of these solutions in the range of Q^2 from 0 to 3 GeV^2. The solutions of dispersion relations for the multipole $S_{1+}^{3/2}$ at $0 \leq Q^2 \leq 3$ GeV^2 are also obtained. The obtained solutions should be considered as an input for the multipoles $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$ in the analysis of future experimental data.

Analysing the solutions of integral equtions for multipole amplitudes obtained within approach of Refs. [6,7] we came also to the conclusion that the traditional identification of the amplitude $M_{1+}^{3/2}$ (as well of the amplitudes $E_{1+}^{3/2}, S_{1+}^{3/2}$) with the contribution of the $P_{33}(1232)$ resonance is not right. The physical interpretation of these solutions

(Sec. 3) shows that the particular solutions of integral equations generated by the Born term should be considered as nonresonance background to the contribution of the P_{33} resonance. These solutions are produced by rescattering effects in the diagrams 1(a-c) corresponding to the Born term. For the first time the presence of such nonresonance contribution in the multipoles $M_{1+}^{3/2}, E_{1+}^{3/2}$ was mentioned in Ref. [13] within dynamical model describing pion photoproduction in the P_{33} region in terms of the diagrams of Fig. 1 taking into account rescattering effects.

In Sec. 4 taking into account nonresonance contributions in the multipoles $M_{1+}^{3/2}, E_{1+}^{3/2}$ the amplitudes $A_{1/2}^p$, $A_{3/2}^p$ for the $\gamma N \rightarrow P_{33}(1232)$ transition are extracted from experiment. The magnitudes of these amplitudes turned out to be smaller than the magnitudes which are traditionally extracted from experiment without taking into account nonresonance background contributions in the amplitude $M_{1+}^{3/2}$. As a result, the traditionally mentioned disagreement between quark model predictions and experiment for the amplitudes $A_{1/2}^p$, $A_{3/2}^p$ turned out to be removed.

2 Dispersion relations for invariant and multipole amplitudes

In this Section we will present very briefly main formulas which are necessary for our calculations. Following the work [11] we choose invariant amplitudes in accordance with the following definition of the hadron current:

$$I^\mu = \bar{u}(p_2)\gamma_5 \left\{ \frac{B_1}{2} \left[\gamma^\mu(\gamma k) - (\gamma k)\gamma^\mu \right] + 2P^\mu B_2 + 2q^\mu B_3 + 2k^\mu B_4 \right.$$
$$\left. -\gamma^\mu B_5 + (\gamma k)P^\mu B_6 + (\gamma k)k^\mu B_7 + (\gamma k)q^\mu B_8 \right\} u(p_1), \tag{5}$$

where k, q, p_1, p_2 are the 4-momenta of virtual photon, pion, initial and final nucleons, respectively, $P = \frac{1}{2}(p_1 + p_2)$, $Q^2 \equiv -k^2$, $B_1, B_2, ...B_8$ are invariant amplitudes which are functions of the invariant variables $s = (k + p_1)^2$, $t = (k - q)^2$, Q^2.

The conservation of the hadron current leads to the relations:

$$4Q^2 B_4 = (s - u)B_2 - 2(t + Q^2 - \mu^2)B_3, \tag{6}$$

4

$$2Q^2 B_7 = -B_5' - (t + Q^2 - \mu^2)B_8, \tag{7}$$

where $B_5' \equiv B_5 - \frac{1}{4}(s-u)B_6$ μ is the pion mass. So, only the six of the eight invariant amplitudes are independent. Let us choose as independent amplitudes following ones: $B_1, B_2, B_3, B_5', B_6, B_8$. For all these amplitudes, except $B_3^{(-)}$, unsubtracted dispersion relations can be written:

$$ReB_i^{(\pm,0)}(s,t,Q^2) = R_i^{(v,s)}\left(\frac{1}{s-m^2} \pm \frac{\eta_i}{u-m^2}\right)$$
$$+\frac{P}{\pi}\int\limits_{s_{thr}}^{\infty} ImB_i^{(\pm,0)}(s',t,Q^2)\left(\frac{1}{s'-s} \pm \frac{\eta_i}{s'-u}\right)ds', \tag{8}$$

where $\pm$ and 0 labels refer to isospin states with a definite symmetry under the interchange $s \leftrightarrow u$, $R_i^{(v,s)}$ are residues in the nucleon poles (they are given in the Appendix), $\eta_1 = \eta_2 = \eta_6 = 1$, $\eta_3 = \eta_5' = \eta_6 = -1$, $s_{thr} = (m+\mu)^2$, m is the nucleon mass. For the amplitude $B_3^{(-)}$ we take the subtraction point at an infinity. In this case using the current conservation condition (6) we have

$$ReB_3^{(-)}(s,t,Q^2) = R_3^{(v)}\left(\frac{1}{s-m^2} + \frac{1}{u-m^2}\right) - \frac{eg}{t-\mu^2}F_\pi(Q^2)$$
$$+\frac{P}{\pi}\int\limits_{s_{thr}}^{\infty} ImB_3^{(-)}(s',t,Q^2)\left(\frac{1}{s'-s} + \frac{1}{s'-u} - \frac{4}{s'-u'}\right)ds'. \tag{9}$$

In order to connect the invariant amplitudes with cross section, helicity and multipole amplitudes, it is convenient to introduce intermediate amplitudes f_i which are related to the invariant amplitudes via:

$$f_1 = \frac{a_1}{8\pi W}\left[(W-m)B_1 - B_5\right], \tag{10}$$

$$f_2 = \frac{a_2}{8\pi W}\left[-(W+m)B_1 - B_5\right], \tag{11}$$

$$f_3 = \frac{a_3}{8\pi W}\left[2B_3 - B_2 + (W+m)\left(\frac{B_6}{2} - B_8\right)\right], \tag{12}$$

$$f_4 = \frac{a_4}{8\pi W}\left[-(2B_3 - B_2) + (W-m)\left(\frac{B_6}{2} - B_8\right)\right], \tag{13}$$

$$f_5 = \frac{a_5}{8\pi W}\left\{\left[Q^2 B_1 + (W-m)B_5 + 2W(E_1 - m)\left(B_2 - \frac{W+m}{2}B_6\right)\right](E_1+m)\right.$$
$$\left. -X\left[(2B_3 - B_2) + (W+m)\left(\frac{B_6}{2} - B_8\right)\right]\right\}, \tag{14}$$

$$f_6 = \frac{a_6}{8\pi W} \left\{ -\left[Q^2 B_1 - (W+m)B_5 + 2W(E_1+m)\left(B_2 + \frac{W-m}{2}B_6\right)\right](E_1 - m) \right.$$
$$\left. + X\left[(2B_3 - B_2) - (W-m)\left(\frac{B_6}{2} - B_8\right)\right] \right\}, \tag{15}$$

$$\tag{16}$$

where

$$a_1 = [(E_1 + m)(E_2 + m)]^{1/2},$$

$$a_2 = [(E_1 - m)(E_2 - m)]^{1/2},$$

$$a_3 = [(E_1 - m)(E_2 - m)]^{1/2}(E_2 + m), \tag{17}$$

$$a_4 = [(E_1 + m)(E_2 + m)]^{1/2}(E_2 - m),$$

$$a_5 = [(E_1 - m)(E_2 + m)]/Q^2,$$

$$a_6 = [(E_1 + m)(E_2 - m)]/Q^2,$$

and

$$X = \frac{k_0}{2}(t - \mu^2 + Q^2) - Q^2 q_0, \tag{18}$$

k_0, q_0, E_1, E_2 are the energies of virtual photon, pion, initial and final nucleons in the c.m.s., $W = s^{1/2}$. The amplitudes f_i are related to cross section, helicity and multipole amplitudes by the relations (A.3-A.5).

Dispersion relations for the multipoles M_{1+}, E_{1+}, S_{1+} can be found from the relations (8,9) using the projection formulas:

$$M_{1+} = \frac{1}{8}\int_{-1}^{1}\left[2f_1 x + f_2(1 - 3x^2) - f_3(1 - x^2)\right]dx,$$

$$E_{1+} = \frac{1}{8}\int_{-1}^{1}\left[2f_1 x + f_2(1 - 3x^2) + (f_3 + 2xf_4)(1 - x^2)\right]dx, \tag{19}$$

$$S_{1+} = \frac{1}{8}\int_{-1}^{1}\left[2f_5 x + (3x^2 - 1)f_6\right]dx.$$

All the subsequent calculations for these multipoles will be made numerically. By this reason we do not specify further these relations, and dispersion relations for M_{1+}, E_{1+}, S_{1+} will be written immediately through the dispersion relations (8,9).

6

3 Solutions of dispersion relations for the multipoles $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$ at $Q^2 \leq 3 \; GeV^2$. The interpretation of these solutions

Let us write the dispersion relations for the multipoles $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$ in the form:

$$M(W, Q^2) = \quad M^B(W, Q^2) + \frac{1}{\pi} \int\limits_{W_{thr}}^{\infty} \frac{ImM(W', Q^2)}{W' - W - i\varepsilon} dW'$$

$$+ \frac{1}{\pi} \int\limits_{W_{thr}}^{\infty} K(W, W', Q^2) ImM(W', Q^2) dW'. \quad (20)$$

Here $M(W, Q^2)$ denotes any of the considered multipoles, $M^B(W, Q^2)$ is the contribution of the Born term into these multipoles, $K(W, W', Q^2)$ is a nonsingular kernel arising from the u-channel contribution into the dispersion integral and the nonsingular part of the s-channel contribution. In the integrand of the relation (20) we did not take into account the coupling of $M(W, Q^2)$ to other multipoles by the following reason. Here we consider only resonance multipoles with large imaginary parts. Contributions of nonresonance multipoles to these multipoles are negligibly small. The couplings of the resonance multipoles with each other are also reasonably small [7]. In the dispersion relation (20) we also did not take into account high energy contributions to the multipoles $M(W, Q^2)$ which, if they exist, should be added to $M^B(W, Q^2)$. Our estimations show that these contributions to the multipoles $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$ are negligibly small and do not affect our results presented below.

In the present work we use the dispersion relations in the $P_{33}(1232)$ resonance region. From the phase shift analyses of πN scattering in this region [14-17] it is known that the resonance amplitude $h_{1+}^{\frac{3}{2}}(W)$ of πN scattering is elastic, and, so, can be written in the form:

$$h_{1+}^{\frac{3}{2}}(W) = sin\delta_{1+}^{\frac{3}{2}}(W)exp(i\delta_{1+}^{\frac{3}{2}}(W)). \quad (21)$$

In this energy region due to the elasticity of the amplitudes (1+) one can use the

Watson theorem [18] for the amplitudes $M(W, Q^2)$ and write them in the form:

$$M(W, Q^2) = exp(i\delta_{1+}^{\frac{3}{2}}(W))|M(W, Q^2)|. \tag{22}$$

From the experimental data [14-17] it is known that for the amplitude $h_{1+}^{\frac{3}{2}}(W)$ the elasticity condition is strictly fulfilled up to $W = 1.5\ GeV$, i.e. up to energies which are much higher in comparison with the energies in the $P_{33}(1232)$ resonance region. By this reason one can assume that the amplitude $h_{1+}^{\frac{3}{2}}(W)$ is elastic at all energies and the condition (22) is valid on the whole physical cut, i.e.

$$ImM(W, Q^2) = h^*(W)M(W, Q^2). \tag{23}$$

Furthemore, as at $W = 1.5\ GeV$, $\delta(W) = 160°$, one can assume that

$$0 < \delta(\infty) \leq \pi. \tag{24}$$

With these conditions the dispersion relation (20) transforms into singular integral equation. At $K(W, W', Q^2) = 0$ this equation has a solution which have the following analytical form (see Ref.[6] and the references therein):

$$M_{K=0}(W, Q^2) = M_{K=0}^{part}(W, Q^2) + c_M M_{K=0}^{hom}(W), \tag{25}$$

where

$$M_{K=0}^{part}(W, Q^2) = M^B(W, Q^2) + \frac{1}{\pi}\frac{1}{D(W)}\int\limits_{W_{thr}}^{\infty}\frac{D(W')h(W')M^B(W', Q^2)}{W' - W - i\varepsilon}dW' \tag{26}$$

is the particular solution of the singular equation, and

$$M_{K=0}^{hom}(W) = \frac{1}{D(W)} = exp\left[\frac{W}{\pi}\int\limits_{W_{thr}}^{\infty}\frac{\delta(W')}{W'(W' - W - i\varepsilon)}dW'\right] \tag{27}$$

is the solution of the homogeneous equation

$$M_{K=0}^{hom}(W) = \frac{1}{\pi}\int\limits_{W_{thr}}^{\infty}\frac{h^*(W')M_{K=0}^{hom}(W')}{W' - W - i\varepsilon}dW', \tag{28}$$

which enters the solution (25) with an arbitrary weight, i.e. multiplied by an arbitrary constant c_M.

Replacing in the Eqs. (25,26) $M^B(W, Q^2)$ by

$$M^B(W, Q^2) + \frac{1}{\pi} \int_{W_{thr}}^{\infty} K(W, W', Q^2) h^*(W') M(W', Q^2) dW', \tag{29}$$

one can transform the dispersion relation (20) into the Fredholm integral equation for the imaginary parts of the multipole amplitudes:

$$\begin{aligned} ImM(W, Q^2) = \quad & ImM_{K=0}^{part}(W, Q^2) + c_M ImM_{K=0}^{hom}(W) + \\ & \frac{1}{\pi} \int_{W_{thr}}^{\infty} f(W, W', Q^2) ImM(W', Q^2) dW' \end{aligned} \tag{30}$$

where

$$ImM_{K=0}^{part}(W, Q^2) = sin\delta(W) \left[M^B(W, Q^2) cos\delta(W) + e^{a(W)} r(W, Q^2) \right], \tag{31}$$

$$r(W, Q^2) = \frac{P}{\pi} \int_{W_{thr}}^{\infty} \frac{e^{-a(W')} sin\delta(W') M^B(W', Q^2)}{W' - W} dW', \tag{32}$$

$$ImM_{K=0}^{hom}(W) = sin\delta(W) e^{a(W)}, \tag{33}$$

$$f(W, W', Q^2) = sin\delta(W) \left[K(W, W', Q^2) cos\delta(W) + e^{a(W)} R(W, W', Q^2) \right], \tag{34}$$

$$R(W, W', Q^2) = \frac{P}{\pi} \int_{W_{thr}}^{\infty} \frac{e^{-a(W'')} sin\delta(W'') K(W'', W', Q^2)}{W'' - W} dW'', \tag{35}$$

$$a(W) = \frac{P}{\pi} \int_{W_{thr}}^{\infty} \frac{W\delta(W')}{W'(W' - W)} dW'. \tag{36}$$

The solution of Eq. (30) contains two parts. One part is determined by the particular solution (26,31) of the singular integral equation (20) with $K(W, W', Q^2) = 0$. It is, in fact, the particular solution of the integral equation (20). Obviously, there are an infinite number of particular solutions of Eq. (20) which differ from each other by the solutions of homogeneous part of this equation. The solution (26,31) and the particular solution of Eqs. (20,30) generated by (26,31) are concrete solutions which are entirely determined

9

by the Born term $M^B(W, Q^2)$ and turns out to be 0 when $M^B(W, Q^2) = 0$. Let us denote this particular solution of Eq. (20) as $M^{part}_{Born}(W, Q^2)$.

Other part of the solution of Eq. (20) is generated by $M^{hom}_{K=0}(W, Q^2)$. It is the solution of the homogeneous part of Eq. (20), i.e. of Eq. (20) with $M^B(W, Q^2) = 0$. This part has an arbitrary weight, which can be found only from some additional conditions, for example, from comparison with experiment.

The solutions $M^{part}_{Born}(W, Q^2)$ and $M^{hom}(W, Q^2)$ can be found from the Fredholm integral equation (30) only by numerical methods. The results of the numerical calculations for the multipoles $M^{3/2}_{1+}, E^{3/2}_{1+}, S^{3/2}_{1+}$ at $Q^2 = 0, 1, 2, 3$ GeV^2 are presented on Figs. 2-5. The particular solutions on Figs. 2-4 are normalized by the form factor corresponding to the dipole formula:

$$G_D(Q^2) = 1/(1 + Q^2/0.71 \text{ GeV}^2)^2. \tag{37}$$

For the multipole $S^{3/2}_{1+}$ the results are presented for the ratio $S^{3/2}_{1+}/|\mathbf{k}|$, as this multipole enter the cross section in the form $S^{3/2}_{1+}/|\mathbf{k}|$ (see Appendix, Eqs. (A.3,A.4)).

At $Q^2 = 0$ the solutions of the integral equation (30) for the multipoles $M^{3/2}_{1+}, E^{3/2}_{1+}$ have been obtained in Refs. [6,7] too. They coincide with our results. For the multipole $M^{3/2}_{1+}$ there is a slight, practically invisible, difference in the particular solutions which is caused by the fact that in Refs. [6,7] high energy contributions to the dispersion relations are taken into account. The coincidence of our particular solutions with those of Refs. [6,7] which, in fact, takes place confirms our statement, that high energy contributions do not affect the particular solutions for the considered multipoles.

The solutions of the homogeneous equation are presented on Fig.5. They are normalized on the same value at the energy $E_L \equiv (W^2 - m^2)/2m = 0.34$ GeV corresponding to the center of the $P_{33}(1232)$ resonance.

Let us discuss now the interpretation of the obtained solutions. Suppose, one describes the amplitudes $M^{3/2}_{1+}, E^{3/2}_{1+}, S^{3/2}_{1+}$ within some dynamical model in terms of contributions of the diagrams of Fig.1 taking into account rescattering effects. The examples of such models can be found in Refs.[13,19]. In such approach the rescattering effects produce imaginary part in the Born term which is by itself real. The imaginary part produced

10

by the final state interaction in the diagrams of Fig.1(a-c) should be considered as non-resonance background to the P_{33} resonance contribution in the multipole amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$. For the first time this was mentioned in Ref. [13]. The attempts to calculate this nonresonance contribution within dynamical models are connected with uncertainties coming from the cutoff procedure and the method of taking into account off-shell effects. In dispersion relation approach due to the elasticity of the $h_{1+}^{3/2}$ amplitude of πN scattering up to quite large energies and, as a result, due to the validity of the Watson theorem up to energies which are much larger in comparison with the energies in the P_{33} resonance region, there is the possibility to find the nonresonance contribution in the model independent way. As is seen from Eq. (26) the contribution produced by the final state interaction in the Born term caused by the resonance πN scattering is reproduced by dispersion relations in the form of particular solution of the integral equation (20) generated by the Born term. The whole particular solution satisfies the requirements of unitarity and crossing symmetry. In Refs. [6,7] it is shown that the factor $D(W)$ in Eq. (26) is responsible for modification of amplitudes at small distances. By the shape and magnitude the nonresonance contributions into the multipoles $M_{1+}^{3/2}$ and $E_{1+}^{3/2}$ at $Q^2 = 0$ obtained in this work in the form of particular solutions of Eq. (20) are very close to those obtained within dynamical model of Ref. [19].

The contribution of the diagram of Fig.1d together with rescattering effects should be identified with the solution of the homogeneous part of Eq. (20). The rescattering effects modify the vertices in this diagram. As a result, in the center of the $P_{33}(1232)$ resonance the vertices $\gamma^* N P_{33}(1232)$ and $\pi N P_{33}(1232)$ should be considered as dressed vertices. The dressed vertex $\pi N P_{33}$ can be found from experimental data on the width of the $P_{33} \to \pi N$ decay. Let us note that as is seen from Fig. 5 the shapes of the homogeneous solutions, i.e. the shapes of the P_{33} contribution, are slightly different for different multipoles and for different values of Q^2. The presence of such difference is natural due to rescattering effects which can be different for different multipoles and at different Q^2. For the comparison with predictions of quark model, QCD and other models the magnitudes of the $\gamma^* N \to P_{33}(1232)$ form factors extracted from multipoles

at $W = W_r = 1.232\ GeV$ should be used.

For the analysis of pion electroproduction data a phenomenological approach proposed by Walker [20] is widely used. In this approach multipole amplitudes, including the amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$, are parameterized in terms of resonances taken in the Breit-Wigner form and smooth background contribution. The Breit-Wigner formula for the multipoles $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}|\mathbf{k}_r|/|\mathbf{k}|$ used by Walker has the form:

$$M_{B-W}(W) = \frac{W_r \mathbf{\Gamma}}{W_r^2 - W^2 - iW_r\mathbf{\Gamma}} \left(\frac{\mathbf{q_r}}{\mathbf{q}}\right)^2 \frac{|\mathbf{k}|}{|\bar{\mathbf{k}}_r|}, \tag{38}$$

where

$$\mathbf{\Gamma} = \Gamma\left(\frac{|\mathbf{q}|}{|\mathbf{q_r}|}\right)^3 \frac{\mathbf{q}_r^2 + X^2}{\mathbf{q}^2 + X^2}, \tag{39}$$

$\Gamma = 0.114\ GeV$, $X = 0.167\ GeV$, $\mathbf{q_r}$ and $\bar{\mathbf{k}}_r$ are the momenta of pion and real photon in c.m.s. in the center of the $P_{33}(1232)$ resonance.

Dispersion relations allow to check the Walker approach for the multipoles $M_{1+}^{3/2}, E_{1+}^{3/2}$, $S_{1+}^{3/2}$ in the $P_{33}(1232)$ resonance region. As is seen from the obtained results the nonresonance background in these multipoles has nontrivial behavior and can not be described by a smooth function. The shape of the $P_{33}(1232)$ resonance contribution is fixed in dispersion relation approach by the solution of the homogeneous part of Eq. (20). As is seen from Fig. 5 it also differs from that of the Breit-Wigner formula. So, the input for the multipoles $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$ obtained in this work within dispersion relation approach does not coincide with that in the phenomenological approach proposed by Walker.

4 Comparison with experiment at $Q^2 = 0$. Amplitudes $A_{1/2}^p$, $A_{3/2}^p$ and $E2/M1$ ratio for the $\gamma N \rightarrow P_{33}(1232)$ transition extracted from experiment.

In this Section we will present our results on the description of experimental data for the multipole amplitudes $M_{1+}^{3/2}$ and $E_{1+}^{3/2}$ at $Q^2 = 0$ which are evaluated with high accuracy from existing experimental data in the multipole analysis of Ref. [21]. In the considered

approach these data should be described as a sum of the particular and homogeneous solutions of the integral equations for the multipoles $M_{1+}^{3/2}$ and $E_{1+}^{3/2}$ obtained in the previous Section. The particular solutions have definite magnitude fixed by the Born term. The weights of the homogeneous solutions should be found from the requirement of the best description of the experimental data. For this aim we have used fitting procedure. The obtained results together with the experimental data are presented on Figs. 6,7. In order to demonstrate the role of the nonresonance background contributions on these figures separately the particular solutions generated by the Born term are given. The homogeneous solutions taking with the weights obtained in the result of fitting the experimental data are also presented. The obtained homogeneous solutions give the following values of multipoles $M_{1+}^{3/2}$ and $E_{1+}^{3/2}$ corresponding to the contribution of the $P_{33}(1232)$ resonance:

$$M_{1+}^{3/2}(\gamma N \to P_{33}(1232)) = 4.22 \pm 0.12 \ \mu b^{1/2}, \tag{40}$$

$$E_{1+}^{3/2}(\gamma N \to P_{33}(1232)) = -0.055 \pm 0.011 \ \mu b^{1/2}. \tag{41}$$

In Table 1 we present the ratio E_{1+}/M_{1+} for the transition $\gamma N \to P_{33}(1232)$ which follows from (40,41) and the helicity amplitudes for this transition obtained by the following formulas:

$$A_{1+}^{3/2} = -\left[\frac{3}{8\pi}\frac{|\mathbf{k}|}{|\mathbf{q}|}\frac{m}{M}\frac{\Gamma_\pi}{\Gamma^2}\right]^{1/2} A_{1/2}^p, \tag{42}$$

$$B_{1+}^{3/2} = \left[\frac{1}{2\pi}\frac{|\mathbf{k}|}{|\mathbf{q}|}\frac{m}{M}\frac{\Gamma_\pi}{\Gamma^2}\right]^{1/2} A_{3/2}^p, \tag{43}$$

where $A_{1+} = (M_{1+} + 3E_{1+})/2$, $B_{1+} = E_{1+} - M_{1+}$ and $M, \Gamma, \ \Gamma_\pi$ are the $P_{33}(1232)$ mass, total and partial widths. In Table 1 we present also the ranges of the amplitudes $A_{1/2}^p$, $A_{3/2}^p$ from Ref. [17] which are extracted from existing experimental data without taking into account nonresonance contribution in the multipole $M_{1+}^{3/2}$. The magnitudes of these amplitudes desagree with quark model predictions. As it is seen from our results this desagreement is removed due to taking into account the nonresonance background produced by rescattering effects in the diagrams corresponding to the Born term.

5 Conclusion

The set of the following features of the multipole amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$ in the $P_{33}(1232)$ resonance region:

a. the elasticity of the corresponding amplitude $h_{1+}^{3/2}$ of πN scattering up to energies which are much higher than the energies in the P_{33} resonance region and, as a result, the validity of the Watson theorem in this energy region with the known phase $\delta_{1+}^{3/2}(W)$;

b. the smallness of high energy contributions and other multipole contributions to the dispersion relations for the multipoles $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$,

allows to transform the dispersion relations for these multipoles into the integral equations of the Fredholm type, where the terms which are responsible for the inhomogeneouty of these equations are determined only by the Born terms. This allowed us to make strict conclusions on the nonresonance contributions into the multipoles $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$ and on the shape of the resonance contributions which are equivalent to the solutions of the homogeneous parts of the integral equations. The weights of resonance contributions are not fixed by dispersion relations. These are the only unknown parameters in the multipoles $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$ which should be found from experimental data.

Appendix

The residues in Eq. (8) are equal to:

$$R_1^{(v,s)} = \frac{ge}{2}(F_1^{(v,s)} + 2mF_2^{(v,s)}),$$

14

$$R_2^{(v,s)} = -\frac{ge}{2}F_1^{(v,s)}(Q^2),$$
$$R_3^{(v,s)} = -\frac{ge}{4}F_1^{(v,s)}(Q^2),$$
$$R_5'^{(v,s)} = \frac{ge}{4}(\mu - Q^2 - t)F_2^{(v,s)}(Q^2),$$ \hfill (A.1)
$$R_6^{(v,s)} = geF_2^{(v,s)}(Q^2),$$
$$R_8^{(v,s)} = \frac{ge}{2}F_2^{(v,s)}(Q^2),$$

where in accordance with existing experimental data we have:

$$e^2/4\pi = 1/137, \quad g^2/4\pi = 14.5,$$
$$F_1^{(v,s)} = \left(1 + \frac{g^{(v,s)}\tau}{1+\tau}\right)G_D(Q^2),$$
$$F_2^{(v,s)} = \frac{g^{(v,s)}}{2m}\frac{G_D(Q^2)}{1+\tau},$$ \hfill (A.2)
$$F_\pi(Q^2) = 1/(1 + Q^2/0.59 \text{ GeV}^2),$$
$$\tau = Q^2/4m^2, \quad g^{(v)} = 3.7, \quad g^{(s)} = -0.12.$$

Amplitudes f_i introduced in Sec. 2 are related to the multipole and helicity amplitudes and to the cross section in the following way:

$$f_1 = \sum \left\{(lM_{l+} + E_{l+})P'_{l+1}(x) + [(l+1)M_{l-} + E_{l-}]P'_{l-1}(x)\right\},$$
$$f_2 = \sum [(l+1)M_{l+} + lM_{l-}]P'_l(x),$$
$$f_3 = \sum \left[(E_{l+} - M_{l+})P''_{l+1}(x) + (E_{l-} + M_{l-})P''_{l-1}(x)\right],$$ \hfill (A.3)
$$f_4 = \sum (M_{l+} - E_{l+} - M_{l-} - E_{l-})P''_l(x),$$
$$f_5 = \sum \left[(l+1)S_{l+}P'_{l+1}(x) - lS_{l-}P'_{l-1}(x)\right],$$
$$f_6 = \sum [lS_{l-} - (l+1)S_{l+}]P'_l(x),$$

$$H_1 \equiv f_{+,+-} = f_{-,-+} = -\cos\frac{\theta}{2}\sin\theta(f_3 + f_4)/\sqrt{2},$$
$$H_2 \equiv f_{-,++} = -f_{+,--} = -\sqrt{2}\cos\frac{\theta}{2}\left[f_1 - f_2 - \sin^2\frac{\theta}{2}(f_3 - f_4)\right],$$
$$H_3 \equiv f_{+,-+} = f_{-,+-} = \sin\frac{\theta}{2}\sin\theta(f_3 - f_4)/\sqrt{2},$$ \hfill (A.4)

$$H_4 \equiv f_{+,++} = f_{-,--} = \sqrt{2}\sin\frac{\theta}{2}\left[f_1 + f_2 + \cos^2\frac{\theta}{2}(f_3 + f_4)\right],$$

$$H_5 \equiv f_{-,-0} = -f_{+,+0} = -\frac{Q}{|\mathbf{k}|}\cos\frac{\theta}{2}(f_5 + f_6),$$

$$H_6 \equiv f_{+,-0} = f_{-,+0} = \frac{Q}{|\mathbf{k}|}\sin\frac{\theta}{2}(f_5 - f_6),$$

$$\frac{\bar{k}}{|\mathbf{q}|}\frac{d\sigma}{d\Omega_\pi} = \frac{1}{2}\left(|H_1|^2 + |H_2|^2 + |H_3|^2 + |H_4|^2\right) +$$
$$+\varepsilon\left(|H_5|^2 + |H_6|^2\right) - \varepsilon\cos 2\varphi Re\left(H_4 H_1^* - H_3 H_2^*\right) - \qquad\qquad (A.5)$$
$$-\cos\varphi\left[\varepsilon(1+\varepsilon)\right]^{1/2} Re\left[H_5^*(H_1 - H_4) + H_6^*(H_2 + H_3)\right],$$

where $x = \cos\theta$, θ and φ are the polar and azimuthal angles of the pion in c.m.s., $\mathbf{k}$ and $\mathbf{q}$ are the momenta of the photon and the pion in this system, ε is the polarization factor of the virtual photon, $f_{\mu_2,\mu_1\lambda}$ are the helicity amplitudes, λ, μ_1, μ_2 are the helicities of photon, initial and final nucleons, respectively, $\bar{k} = (W^2 - m^2)/2W$.

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Figure Captions

Fig. 1 Diagrams corresponding to the contributions of the Born term (nucleon and pion poles) and the $P_{33}(1232)$ resonance to the production of pions on nucleons by virtual photons.

Fig. 2 The multipole amplitude $M_{1+}^{3/2}$. The imaginary parts of the partial solutions of the dispersion relations (20) for this amplitude at different Q^2 generated by the Born term. The solutions are divided by the dipole form factor (37). These solutions should be considered as background contributions (to the $P_{33}(1232)$ resonance) produced by rescattering effects in the diagrams corresponding to the Born term. $E_L \equiv \frac{W^2 - m^2}{2m}$.

Fig. 3 The multipole amplitude $E_{1+}^{3/2}$. The legend is as for Fig. 2.

Fig. 4 The multipole amplitude $S_{1+}^{3/2}/|\mathbf{k}|$. The legend is as for Fig. 2.

Fig. 5 The imaginary parts of the solutions of the dispersion relations (20) with $M^B(W, Q^2)=0$ for the multipoles $M_{1+}^{3/2}$, $E_{1+}^{3/2}$, $S_{1+}^{3/2}$ at different Q^2. These solutions represent the shape of the $P_{33}(1232)$ resonance contribution into multipoles. The curves 1-4 correspond to the following multipoles: (1) $S_{1+}^{3/2}|\mathbf{k}_r|/|\mathbf{k}|$ at $Q^2 = 0$; (2) $E_{1+}^{3/2}$ at $Q^2 = 0$; (3) $M_{1+}^{3/2}$ at $Q^2 = 0$ and $E_{1+}^{3/2}$, $S_{1+}^{3/2}|\mathbf{k}_r|/|\mathbf{k}|$ at $Q^2 = 1 - 3\ GeV^2$; (4) $M_{1+}^{3/2}$ at $Q^2 = 1 - 3\ GeV^2$. For comparison the shapes of these multipoles corresponding to the Breit-Wigner formula (38) are presented: dotted line - $Q^2 = 0$, dashed line - $Q^2 = 1 - 3\ GeV^2$.

Fig. 6 The multipole amplitude $M_{1+}^{3/2}$ at $Q^2 = 0$. Our results for the imaginary part of this amplitude (solid line) in comparison with experimental data [21]. Separately, the nonresonance background contribution given by the particular solution of eq. (20) (dashed line) and the resonance contribution given by the solution of the homogeneous part of this equation (dotted line) are presented.

Fig. 7 The multipole amplitude $E_{1+}^{3/2}$ at $Q^2 = 0$. The legend is as for Fig. 6.
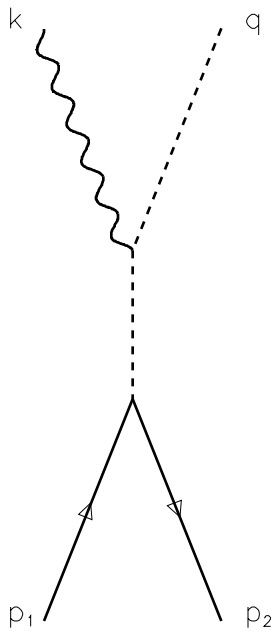
Table Captions

Table 1 Helicity amplitudes and $E2/M1$ ratio for the $\gamma N \to P_{33}(1232)$ transition. Our results are extracted from the phase shift analysis data of Ref. [21] taking into account the nonresonance contributions into multipoles given by the particular solutions of the dispersion relations generated by the Born term.
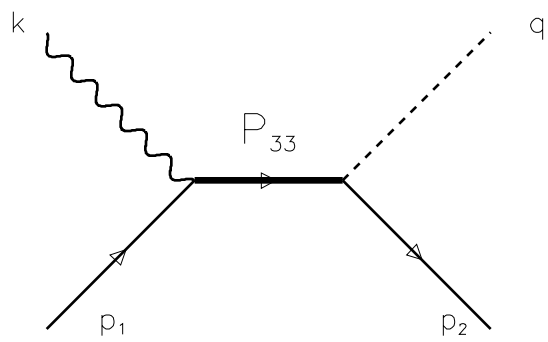
k

q

p₁

p₂

a)

k

q

p₁

p₂

b)

k

q

p₁

p₂

c)

k

q

P₃₃

p₁

p₂
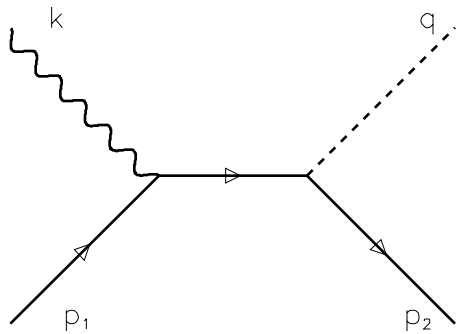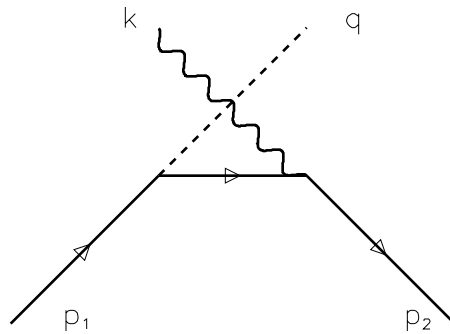
d)

Fig.1

Table 1: Helicity amplitudes and E2/M1 ratio for the γN→P$_{33}$(1232) transition

	$A^p_{1/2}(10^{-3}GeV^{-1/2})$	$A^p_{3/2}(10^{-3}GeV^{-1/2})$	$E2/M1$
Our results	-109 ± 3	-198 ± 6	-0.013 ± 0.003
Exp. data from [17]	-140 - (-160)	-250 - (-270)	-
Nonrelativistic quark model	-101	-175	0
Relativistic quark model [2]	-111	-207	-0.021

Fig.2

Fig.3

Fig.4

Fig.5

Fig.6

Fig.7