Consistent Analysis of $O(\alpha_s)$ Corrections to Pion Elastic Form Factor

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We examine the role of $O(\alpha_s)$ perturbative corrections to the pion elastic form factor $F_\pi(Q^2)$. We express the quark current three-point function in terms of light cone variables and use Borel transformation to simultaneously model the Feynman mechanism contribution determined by the soft part of the pion light cone wave function and the hard term involving one gluon exchange. We find that for $Q^2 \sim 4 \text{ GeV}^2$ the total $O(\alpha_s)$ contribution may reach 30% of the soft contribution, even though its hard, factorization scale dependent part remains rather small.

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I. INTRODUCTION

The interplay between contributions due to Feynman [1] and hard-scattering scenarios [2], the two basic mechanisms determining the large-Q^2 behavior of hadronic form factors, is a crucial problem [3,4] for studies of exclusive processes in quantum chromodynamics. To make a meaningful comparison of their magnitude, one should be able to calculate both contributions within the same self-consistent framework. To implement such a program, and especially in order to be able to take into account effects due to the nonperturbative part of the hadronic wave functions it is natural to rely on the light cone quantization [5] which enables a Fock space representation for the hadronic current matrix elements. An important issue which has to be addressed when studying exclusive amplitudes in the region where both hard and soft QCD processes are important, is how to implement factorization, $i.e.$, separate perturbative contributions from those intrinsic to the bound-state wave function [5,3,4]. In the light cone quantization, the pion form factor can (in principle) be determined from

$$F_\pi(Q^2) = \hat{\Psi} \otimes \hat{\Psi} = \sum_n \int [dx_i d\mathbf{k}_{i\perp}]_n \hat{\Psi}_n(x_i, \mathbf{k}_{i\perp}) \hat{\Psi}_n(x_i, \mathbf{k}_{i\perp} + \delta_i \mathbf{q}_\perp), \tag{1}$$

where the summation extends over all quark/gluon Fock sectors which have a nonvanishing overlap with the pion, $\hat{\Psi}_n$ are the corresponding wave functions, $[dx_i d\mathbf{k}_{i\perp}]_n$ is the relativistic measure within the n-particle sector and $\delta_i = (1 - x_i)$ or $-x_i$ depending on whether i refers to the struck quark or a spectator, respectively. Instead of having to deal with an infinite set of wave functions, $\hat{\Psi}_n$ which describe both the low and high momentum partons one can use the factorization procedure which allows to express high momentum tails of these wave functions in terms of suitably defined soft wave functions Ψ_n with nonvanishing support only at low momenta. In principle, this may be achieved by constructing an effective Hamiltonian which has nonvanishing matrix elements within the low momentum subspace. This effective Hamiltonian incorporates couplings between low and high momentum degrees of freedom present in the bare Hamiltonian, through a set of effective potentials connecting the low momentum states. The eigenstates of the effective Hamiltonian determine soft hadronic wave functions, Ψ_n. Because of asymptotic freedom, the removal of couplings to high momentum Fock states and construction of effective operators can be performed in a power series expansion in α_s. A similar procedure when applied to the current $\bar{q}\gamma^+ q$ yields an effective current operator. The contribution from the lowest order one-body current, as given by Eq. (1) is modified in higher orders by contributions from two-body and more complicated operator matrix elements resulting in

$$F_\pi(Q^2) = \Psi \otimes T \otimes \Psi \tag{2}$$
$$\equiv \sum_{nm} \int [dx_i d\mathbf{k}_{i\perp}]_n [dy_i d\mathbf{l}_{i\perp}]_m \Psi_n(x_i, \mathbf{k}_{i\perp}; \mu) T_{nm}(x_i, \mathbf{k}_{i\perp}; y_i, \mathbf{l}_{i\perp}; \mathbf{q}_\perp; \mu) \Psi_m(y_i, \mathbf{l}_{i\perp}; \mu),$$

where summation over n, m now extends over the low momentum states only, and T_{nm} are the partonic matrix elements of the effective current operator. The dependence on the scale separating low and high momenta is indicated by μ. The matrix elements, T_{mn} mix low momentum partons described by the wave functions, Ψ with intermediate high momentum states. The free energy of these intermediate states is bounded from below by μ and in order for the perturbative approach to make sense μ has to be chosen to be much larger than Λ_{QCD} so that $\alpha_s = \alpha(\mu)$ is small. At large momentum transfer we may ignore the dependence of T_{mn} on small transverse momenta $\mathbf{k}_\perp$ and $\mathbf{l}_\perp$ present in the soft wave functions. After that, $\Psi_n(x_i, \mathbf{k}_{i\perp}; \mu)$ and $\Psi_m(y_i, \mathbf{l}_{i\perp}; \mu)$ can be integrated over $\mathbf{k}_\perp$ and $\mathbf{l}_\perp$, respectively to produce the distribution amplitudes ($e.g.$, $\phi(x, \mu^2)$ for the pion) depending only on longitudinal momenta. To avoid large logarithms $O(\alpha_s \log(Q^2/\mu^2))$ emerging from the effective current matrix elements, we can set $\mu \sim |\mathbf{q}_\perp| = \sqrt{Q^2}$, absorbing the logarithms into the evolution of the distribution amplitudes, $\phi(x, Q^2)$. In particular, the latter enter the well-known formula for the pion form factor [6,5,7]:

$$F(Q^2) = \frac{16\pi C_F \alpha_s(Q^2)}{Q^2} \left[\int \frac{\phi(x, Q^2)}{x} dx \right]^2. \tag{3}$$

In the light cone gauge $A^+ = 0$, the distribution amplitudes are determined by matrix elements of composite operators in which the covariant derivatives D^+ coincide with the ordinary ∂^+ ones. In other words, to the leading

order in α_s and $1/Q^2$, only the minimal, valence Fock sector contributes. Conversely, any attempt to improve the leading order formula, for example by keeping the transverse momentum dependence of T_{nm}, requires to take into account the presence of nonvalence degrees of freedom.

Another important observation, supported by QCD sum rules [8,9] and by various constituent quark model studies [10–12], is that the soft contribution to the form factor may be quite substantial. To make a self-consistent comparison of the relative size of the purely soft term and those containing the hard gluon exchange, one should be able to calculate them using the same formalism, which is not necessarily based on the $1/Q^2$ expansion but which rather enables to compute the perturbative corrections to the soft form factor. In other words, we propose to take advantage of the formula given in Eq. (2), retaining there the soft wave functions, rather than expressing the form factor in terms of the distribution amplitudes as given in Eq. (3) for the leading contribution.

In practice, to develop a 3-dimensional renormalization program for a Hamiltonian which also produces light-cone bound states is rather difficult. One of the reasons is that cut-offs violate Lorentz and gauge symmetry, and thus it is highly nontrivial to achieve the necessary cancellation of IR divergences.

It is also unclear how one could systematically incorporate the pQCD radiative corrections into a bound-state approach with phenomenological low-energy interaction kernels.

A simple approach which enables the connection between the three-dimensional soft wave functions and the high energy scattering kernels of perturbative QCD was proposed in Ref. [13] where it was also applied to describe the soft and hard contributions to the heavy-to-light meson transition form factor. The method relies on Green's functions just like the QCD sum rule approach [14]. Let us recall that the QCD sum rule method uses the quark-hadron duality relation

$$\int_0^\infty \rho^{\text{hadron}}(s) e^{-s/M^2}\, ds = \int_0^\infty \rho^{\text{quark}}(s) e^{-s/M^2}\, ds + \sum_N \frac{\langle \mathcal{O}_N \rangle}{(M^2)^N}, \tag{4}$$

between the hadronic spectral density $\rho^{\text{hadron}}(s)$ on one side and the perturbative spectral density $\rho^{\text{quark}}(s)$ and the condensates $\langle \mathcal{O}_N \rangle$ on the other. The exponential weight e^{-s/M^2} in this relation results from the Borel transformation which both emphasizes the lowest resonance contribution into the dispersion integral and improves the convergence properties of the condensate series generated by the operator product expansion. For an appropriate choice of the Borel parameter M, the left-hand side of the duality relation is rather closely approximated by the lowest resonance contribution while the right-hand side is essentially given by the perturbative term alone. In such a situation, one can approximate the lowest resonance contribution by the Borel transform of the relevant Green's function. Such an approximation may be rather accurate when the Borel parameter is taken within a specific range for which the higher state contribution into the left-hand side is close in magnitude to the condensate contribution to the right-hand side. As we will see below, such an ansatz is analogous to the exponential (oscillator-type) model for the soft pion wave function. Calculating the $\alpha_s(M^2)$ expansion for the Green function (*i.e.*, higher-order terms in $\rho^{\text{quark}}(s)$), we can construct a unified self-consistent model which includes both the lowest Fock state contribution and the higher Fock terms generated by the radiative corrections. The model, e.g., automatically preserves such an important property as gauge invariance. The latter is crucial for cancellation of infrared divergences.

In application to form factors, as described in Ref. [13], the Borel transformation is used as a method which, on a loop-by-loop basis, enables to explicitly identify the wave-function-like contributions to the form factor which have the structure of the Fock-space decomposition Eq. (2). An important advantage of this method over a naive summation of perturbative corrections to model wave functions, is that no *ad hoc* assumptions about how to connect soft and hard regimes have to be made. The formalism constrains how the IR divergences are distributed among the wave functions and scattering kernels to guarantee the IR finiteness and factorization scale independence of a matrix element. As we will see, this requires that both the higher Fock space contributions, Ψ_n, with, e.g., $n = \bar{q}qg$, and the radiative corrections to the current matrix elements, T_{nm} should be present.

The paper is organized as follows. In the next section we discuss the model and details of the form factor calculation. The Sudakov suppression of the electromagnetic vertex is discussed in Sec.III. Conclusions are summarized in Sec.IV.

II. MODEL SPECIFICATION

As an illustration of the method, consider a two-point amplitude defined by

$$(p^+)^2 \Pi(p^2) = -i \int d^4 x e^{ipx} \langle 0|T\{\bar{q}(x)\gamma^+\gamma_5 q(x), \bar{q}(0)\gamma^+\gamma_5 q(0)\}|0\rangle. \tag{5}$$

After the Borel transformation,

$$\Pi(p^2) \rightarrow \Pi(\beta) = \frac{1}{2\pi i} \oint dp^2 e^{-p^2/2\beta^2} \Pi(p^2) = \int ds e^{-s/2\beta^2} \rho(s), \tag{6}$$

where $\rho(s) \equiv (\Pi(s - i\epsilon) - \Pi(s + i\epsilon))/2\pi i$, the bare quark loop contribution to Π yields,

$$\Pi^{\text{quark}}(\beta) = \int ds e^{-s/2\beta^2} \rho^{\text{quark}}(s) = 4 \times 2 \times 3 \int [dx_i d\mathbf{k}_{i\perp}]_2 \exp\left[-\frac{1}{2\beta^2}\sum_{i=1}^{2} \frac{\mathbf{k}_{i\perp}^2 + m_i^2}{x_i}\right], \tag{7}$$

with the numerical factors in front of the integral coming form the trace over Dirac indices, the quark current matrix element $\bar{u}(\lambda)\gamma^+\gamma_5 v(\lambda') = 2\delta_{\lambda,-\lambda'}$, and sum over three colors respectively; the measure is given by

$$[dx_i d\mathbf{k}_{i\perp}]_2 = 16\pi^3 \delta\left(1 - \sum_i x_i\right) \delta\left(\sum_i \mathbf{k}_{i\perp}\right) \prod_i \frac{dx_i d^2\mathbf{k}_{i\perp}}{16\pi^3}. \tag{8}$$

The quark spectral function, ρ^{quark} is given by

$$\rho^{\text{quark}}(s) = 2\sqrt{6} \int [dx d^2\mathbf{k}_\perp]_2 \, 2\sqrt{6} \, \delta\left(s - \sum_i \frac{\mathbf{k}_{i\perp}^2 + m_i^2}{x_i}\right). \tag{9}$$

The contribution of the lowest hadronic state to $\Pi(\beta)$ is given by

$$\Pi^{\text{hadron}}(\beta) = f_h^2 \exp\left[-\frac{M_h^2}{2\beta^2}\right], \tag{10}$$

where in our case $M_h = M_\pi$ is the mass of the $(J^P = 0^-)$ ground state, the pion, and $f_h = f_\pi$ is the pion decay constant

$$\langle 0|\bar{q}(0)\gamma^+\gamma_5 q(0|P\rangle = if_\pi P^+. \tag{11}$$

Therefore, if we assign

$$\Psi_2(x_i, k_{i\perp}) = \frac{2\sqrt{6}}{f_\pi} \exp\left(\frac{1}{2\beta^2}\left[M_\pi^2 - \sum_{i=1}^{2} \frac{\mathbf{k}_{i\perp}^2 + m_i^2}{x_i}\right]\right), \tag{12}$$

then the decay constant f_π may be expressed in terms of this valence wave function:

$$f_\pi = 2\sqrt{6} \int [dx_i d\mathbf{k}_{i\perp}]_2 \Psi_2(x_i, k_{i\perp}). \tag{13}$$

This expression agrees with the one obtained using the light cone quantization directly. In the following we will drop the subscript on the wave function and on the light cone measure when referring to the valence $\bar{q}q$ component of the ground state wave function. In the valence sector, the individual quark momenta x_i, $\mathbf{k}_{i\perp}$ may be replaced by the Jacobi variables defined by $x = x_1$, $x_2 = 1 - x_1$ and $\mathbf{k}_\perp = \mathbf{k}_{1\perp} = -\mathbf{k}_{2\perp}$ such that

$$[dx_i d\mathbf{k}_{i\perp}]_2 = [dx d\mathbf{k}_\perp] = \frac{dx d^2\mathbf{k}_\perp}{16\pi^3}. \tag{14}$$

In addition to the Borel transformation we will use also another smearing procedure corresponding to the "local duality" prescription. Its origin can be briefly explained in the following way. In QCD sum rule calculations one typically uses the following ansatz for the hadronic spectral function, $\rho^{\text{hadron}}(s)$

$$\rho^{\text{hadron}}(s) \approx f_h^2 \delta(s - M_h^2) + \rho^{\text{quark}}(s)\theta(s - s_0), \tag{15}$$

with the effective continuum threshold s_0 fitted from the relevant QCD sum rule Eq. (4) by requiring the most stable [M^2-independent] result for physical quantities. In many cases, the stability persists in the whole large-M^2 region. Fitting s_0 and then taking the limit $M \to \infty$ for the Borel parameter in Eq. (4) gives [15]

$$f_h^2 = \int_0^{s_0} \rho^{\text{quark}}(s)\, ds. \tag{16}$$

The local duality states that the two densities, ρ^{hadron} and ρ^{quark} give the same result provided the duality interval s_0 is properly chosen. Comparing Eqs. (9) and (13) results in

$$f_\pi = 2\sqrt{6} \int [dx_i d^2 \mathbf{k}_{i\perp}]_2 \Psi^{\text{LD}}(x_i, \mathbf{k}_{i\perp}), \tag{17}$$

where the "local duality" pion wave function Ψ^{LD} is defined by

$$\Psi^{\text{LD}}(x_i, \mathbf{k}_{i\perp}) = \frac{2\sqrt{6}}{f_\pi} \theta \left(s_0 - \sum_i^2 \frac{\mathbf{k}_{i\perp}^2 + m_i^2}{x_i} \right). \tag{18}$$

The parameter $s_0 = 4\pi^2 f_\pi^2 \sim (830\text{MeV})^2$ serves here as a transverse momentum cutoff and plays a role similar to the wave function scale β in the approach based directly on the Borel transformation. In the following, we will continue using the Borel transformation when deriving analytical expressions. But we will also give numerical results obtained for the local duality wave function.

A. Soft contribution

To obtain a model expression for the soft contribution to $F_\pi(Q^2)$ in a form of an integral over the valence light cone wave function, consider the three-point function

$$(p^+)^3 T(p^2, p'^2) = -\int d^4x d^4y e^{ipx - ip'y} \langle 0|T\bar{q}(x)\gamma^+\gamma_5 q(x), \bar{q}(0)\gamma^+ q(0), \bar{q}(y)\gamma^+\gamma_5 q(y)|0\rangle \tag{19}$$

in the kinematical region defined by $p'^+ - p^+ = q^+ = 0$ and $(p' - p)^2 = q^2 = -\mathbf{q}_\perp^2 = -Q^2$. The quark loop contribution to the double Borel transform ation,

$$T(p^2, p'^2) \to T(\beta) = \int ds ds' e^{-s/2\beta^2} e^{-s'/2\beta^2} \rho(s,'s), \tag{20}$$

is

$$\rho^{\text{quark}}(s, s') = SpT(s, s') = 2(2\sqrt{6})^2 \delta\left(s - \sum_i^2 \frac{\mathbf{k}_{i\perp}^2 + m_i^2}{x_i} \right) \delta\left(s' - \sum_i^2 \frac{(k_{i\perp} + \delta_i \mathbf{q}_\perp)^2 + m_i^2}{x_i} \right), \tag{21}$$

and therefore the $q\bar{q}$ contribution yields

$$T^{\text{quark}}(\beta) = 2f_\pi^2 \exp\left(-\frac{2M_\pi^2}{2\beta^2} \right) \int [dx d\mathbf{k}_\perp] \Psi(x, \mathbf{k}_\perp)\Psi(x, \mathbf{k}_\perp - x\mathbf{q}_\perp). \tag{22}$$

The ground state hadronic contribution to T is given by

$$T^{\text{hadron}}(\beta) = 2f_\pi^2 F_\pi(Q^2) \exp\left(-\frac{2M_\pi^2}{2\beta^2}\right). \tag{23}$$

Identifying Eq. (23) and Eq. (22) yields the standard expression for the valence quark wave function contribution to $F_\pi(Q^2)$,

$$F_\pi(Q^2) = \int [dx\,d\mathbf{k}_\perp] \Psi(x, \mathbf{k}_\perp) \Psi(x, \mathbf{k}_\perp - x\mathbf{q}_\perp). \tag{24}$$

As illustrated on the two examples above, for a single quark loop we have generated expressions for hadronic matrix elements in terms of model light cone wave functions. As will be shown below, this remains to be the case beyond the single quark loop. When the higher loops are considered, the nonvalence wave functions emerge.

In the following we shall work in the chiral limit and set $M_\pi = 0$. With the local duality wave function given by Eq. (18), the normalization condition for the form factor, $F_\pi(0) = 1$ is satisfied automatically. Taking the wave function (12) based on the Borel transformation, produces only a half of the necessary form factor normalization at $Q^2 = 0$. In the standard light-cone approach, the remainder comes from contributions due to the higher Fock components like $\bar{q}G\ldots Gq$ corresponding to the "intrinsic" gluon content of the hadron. The intrinsic gluons should be contrasted with those emerging from explicit radiative corrections to the two-body wave function. Since the local duality wave function provides 100% of the $Q^2 = 0$ normalization, one can interpret it as an *effective* two-body wave function absorbing in itself the soft part of the higher-Fock states. In the spirit of effective wave functions, we will also modify the Gaussian wave function and instead of Eq. (12) we will use

$$\Psi_2(x_i, k_{i\perp}) = \frac{2\sqrt{6}}{f_\pi} N \exp\left(\frac{1}{2\beta^2}\left[M_\pi^2 - \sum_{i=1}^{2} \frac{\mathbf{k}_{i\perp}^2 + m_i^2}{x_i}\right]\right), \tag{25}$$

with the extra normalization constant N and the parameter β fixed by Eq. (13) and $F_\pi(0) = 1$ which lead to $N = 2$, $\beta = \pi f_\pi \sim 400\,\text{MeV}$.

B. One-gluon exchange and hard contribution

The one-gluon-exchange diagrams are shown in Fig. 1b-c. In addition, to order α_s there are three self-energy type diagrams from dressing of the propagators of the triangle diagram shown in Fig. 1a. The hard-gluon-exchange contribution to F_π will be defined as an IR-regular part of two diagrams (Figs. 1c,d) corresponding to vertex corrections to the axial vector currents. The IR finite part of the diagram with electromagnetic vertex correction (Fig. 1b) contains terms which produce the Sudakov suppression after all-order summation. Since the operators which define the correlator T correspond to conserved currents, the sum of all six amplitudes is UV finite. It is also IR finite since there is no net color flowing into or out of the triangle.

Just like in case of the one-loop triangle diagram we calculate Feynman amplitudes corresponding to each two-loop diagram by performing analytically the integrals over the "minus" light cone components of loop momenta. Then, we apply the Borel transformation, and the net result may be schematically written as

$$F_\pi = \Psi \otimes I \otimes \Psi + \Psi \otimes T \otimes \Psi\Psi_g \otimes T_{g1} \otimes \Psi + \Psi \otimes T_{g2} \otimes \Psi_g + \Psi_g \otimes I_{gg} \otimes \Psi_g. \tag{26}$$

The first term comes from the one-loop diagram. The remaining four terms represent the $O(\alpha_s)$ contributions. The T-term corresponds to the vertex correction to the electromagnetic current convoluted with two $\bar{q}q$, *i.e.*, valence meson wave functions. Two other amplitudes, T_{g1} and T_{g2}, are convoluted with the valence wave function on one end and the nonvalence $\bar{q}qg$ wave function on the other one. Finally, the I_{gg}-term corresponds to the nonvalence $\bar{q}qg$ meson wave functions induced both in the initial and final states.

As an example consider the diagram shown in Fig. 1b. The Feynman amplitude is given by

$$(p^+)^3 T(p^2, p'^2) = 12\pi C_F \alpha_s \int \frac{d^4k}{(2\pi)^4} \frac{d^4l}{(2\pi)^4}$$
$$\times \text{Tr}\left[\frac{\gamma^+\gamma_5(\slashed{p}'-\slashed{k})\gamma^\alpha(\slashed{p}'-\slashed{l})\gamma^+(\slashed{p}-\slashed{l})\gamma^\beta(\slashed{p}-\slashed{k})\gamma^+\gamma_5(-\slashed{k})}{(p'-k)^2(p'-l)^2(p-l)^2(p-k)^2 k^2(k-l)^2}\right]. \tag{27}$$

The k^- and l^- integrals pick up poles in the spectator quark $(1/k^2)$ and gluon $(1/(k-l)^2)$ propagators leading to

$$T(p^2, p'^2) = 2(8\pi C_F \alpha_s)(2\sqrt{6})^2 \int \frac{dx d\mathbf{k}_\perp}{16\pi^3} \frac{dy d\mathbf{l}_\perp}{16\pi^3} \frac{y[\mathbf{l}_\perp - \mathbf{q}_\perp] \cdot [\mathbf{l}_\perp + (1-y)\mathbf{q}_\perp]}{\mathbf{l}_\perp^2 [\mathbf{l}_\perp - y\mathbf{q}_\perp]^2}$$

$$\times \left[\frac{1}{p^2 - \frac{\mathbf{k}_\perp^2}{x(1-x)} + i\epsilon} - \frac{1}{p^2 - \frac{\mathbf{k}_\perp^2}{x(1-x)} - \frac{\mathbf{l}_\perp^2}{y(1-y)(1-x)} + i\epsilon} \right]$$

$$\times \left[\frac{1}{p'^2 - \frac{(\mathbf{k}_\perp - x\mathbf{q}_\perp)^2}{x(1-x)} + i\epsilon} - \frac{1}{p'^2 - \frac{(\mathbf{k}_\perp - x\mathbf{q}_\perp)^2}{x(1-x)} - \frac{(\mathbf{l}_\perp - y\mathbf{q}_\perp)^2}{y(1-y)(1-x)} + i\epsilon} \right]. \quad (28)$$

After the Borel transformation, the contribution to F_π from this diagram can be expressed in a form of Eq. (26). Explicitly,

$$F_\pi(Q^2) \to F_\pi^\Gamma(Q^2) = \int \frac{dx d\mathbf{k}_\perp}{16\pi^3} \Psi(x, \mathbf{k}_\perp) T^\Gamma \Psi(x, \mathbf{k}_\perp - x\mathbf{q}_\perp)$$

$$+ \int \frac{dx d\mathbf{k}_\perp}{16\pi^3} \frac{dy d\mathbf{l}_\perp}{16\pi^3} \left[\Psi_g(x, \mathbf{k}_\perp; y, \mathbf{l}_\perp) T_{g1}^\Gamma \Psi(x, \mathbf{k}_\perp - x\mathbf{q}_\perp) + \Psi(x, \mathbf{k}_\perp) T_{g2}^\Gamma \Psi_g(x, \mathbf{k}_\perp - x\mathbf{q}_\perp; y, \mathbf{l}_\perp - y\mathbf{q}_\perp) \right.$$

$$\left. + \Psi_g(x, \mathbf{k}_\perp; y, \mathbf{l}_\perp) I_{gg}^\Gamma \Psi_g(x, \mathbf{k}_\perp - x\mathbf{q}_\perp; y, \mathbf{l}_\perp - y\mathbf{q}_\perp) \right], \quad (29)$$

where

$$T^\Gamma = 8\pi C_F \alpha_s \int \frac{dy d\mathbf{l}_\perp}{16\pi^3} \frac{1}{y(1-y)} \frac{N(y, \mathbf{l}_\perp)}{D(y, \mathbf{l}_\perp) D(y, \mathbf{l}_\perp - y\mathbf{q}_\perp)}, \quad (30)$$

$$T_{g1}^\Gamma = -\frac{\sqrt{8\pi C_F \alpha_s}}{y(1-y)} \frac{N(y, \mathbf{l}_\perp)}{D(y, \mathbf{l}_\perp)}, \quad T_{g2}^\Gamma = -\frac{\sqrt{8\pi C_F \alpha_s}}{y(1-y)} \frac{N(y, \mathbf{l}_\perp)}{D(y, \mathbf{l}_\perp - y\mathbf{q}_\perp)}, \quad I_{gg} = \frac{N(y, \mathbf{l}_\perp)}{y(1-y)}, \quad (31)$$

and we have defined

$$N(y, \mathbf{l}_\perp) \equiv \frac{y[\mathbf{l}_\perp - \mathbf{q}_\perp] \cdot [\mathbf{l}_\perp + (1-y)\mathbf{q}_\perp]}{y(1-y)}, \quad D(y, \mathbf{l}_\perp) \equiv \frac{\mathbf{l}_\perp^2}{y(1-y)}. \quad (32)$$

The valence wave function Ψ is given by Eq. (25) (with $M_\pi = 0$). The contribution from the $\bar{q}qg$ state in the electromagnetic vertex loop is determined by Ψ_g which is given by

$$\Psi_g(x, \mathbf{k}_\perp; y, \mathbf{l}_\perp) = \frac{\sqrt{8\pi \alpha_s C_F}}{D(y, \mathbf{l}_\perp)} \frac{2\sqrt{6}}{f_\pi} \exp\left[\frac{1}{2\beta^2} \left(-\frac{\mathbf{k}_\perp^2}{x(1-x)} - \frac{\mathbf{l}_\perp^2}{y(1-y)(1-x)} \right) \right]. \quad (33)$$

Here y represents the fraction of longitudinal momentum of a quark carried away by the gluon. The spectator quark has longitudinal momentum x. The transverse momentum $\mathbf{l}_\perp$ is the relative momentum between the struck quark and the gluon. Note, that just like the argument of the valence wave function Ψ_2 is determined by the invariant mass of the free $\bar{q}q$ system, the argument of Ψ_g is given by the invariant mass of the free $\bar{q}qg$ system. The contribution proportional to the overlap of valence wave functions is proportional to the vertex form factor, T^Γ. The UV divergence of T^Γ is canceled by a half of the sum of self-energy corrections to two quark lines connected to this vertex. The total contribution to F_π from these two self-energy diagrams can also be cast in the form of Eq. (26) with $I_{gg} = 0$ and is given by

$$F_\pi(Q^2) \to F_\pi^\Sigma(Q^2) = \int \frac{dx d\mathbf{k}_\perp}{16\pi^3} \Psi(x, \mathbf{k}_\perp) T^\Sigma \Psi(x, \mathbf{k}_\perp - x\mathbf{q}_\perp)$$

$$+ \int \frac{dx d\mathbf{k}_\perp}{16\pi^3} \frac{dy d\mathbf{l}_\perp}{16\pi^3} \left[\Psi_g(x, \mathbf{k}_\perp; y, \mathbf{l}_\perp) T_{g1}^\Sigma \Psi(x, \mathbf{k}_\perp - x\mathbf{q}_\perp) + \Psi(x, \mathbf{k}_\perp) T_{g2}^\Sigma \Psi_g(x, \mathbf{k}_\perp - x\mathbf{q}_\perp; \mathbf{l}_\perp - y\mathbf{q}_\perp) \right],$$

$$(34)$$

with

$$T^{\Sigma} = -8\pi C_F \alpha_s \int \frac{dy d\mathbf{l}_\perp}{16\pi^3} \frac{1}{y(1-y)} \left[\frac{y}{D(y,\mathbf{l}_\perp)} + \frac{y}{D(y,\mathbf{l}_\perp - y\mathbf{q}_\perp)} \right], \tag{35}$$

and

$$T_{g1}^{\Sigma} = \frac{\sqrt{8\pi C_F \alpha_s}}{y(1-y)} \frac{y}{D(y,\mathbf{l}_\perp)}, \quad T_{g2}^{\Sigma} = \frac{\sqrt{8\pi C_F \alpha_s}}{y(1-y)} \frac{y}{D(y,\mathbf{l}_\perp - y\mathbf{q}_\perp)}. \tag{36}$$

The remaining UV divergence coming from these two self-energy diagrams cancel the UV divergences resulting from diagrams which dress the axial current vertices. The IR divergence of T^{Γ} corresponding to either $|\mathbf{l}_\perp| \to 0$ or $|\mathbf{l}_\perp - y\mathbf{q}_\perp| \to 0$ is removed by two contributions in Eq. (29) proportional to the $\bar{q}qg$ wave functions. When both denominators in T^{Γ} vanish, $i.e.$, $|\mathbf{l}_\perp| \to 0$ and $|\mathbf{l}_\perp - y\mathbf{q}_\perp| \to 0$, all particles in the loop dressing the electromagnetic vertex go on-shell and this singularity is removed by the contribution proportional to the $\bar{q}qg$ wave functions in both initial and final state. Similarly, the IR divergent contribution to T^{Σ} in Eq. (35) corresponding to either $|\mathbf{l}_\perp| \to 0$ or $|\mathbf{l}_\perp - y\mathbf{q}_\perp| \to 0$ is canceled by mixing with the $\bar{q}qg$ wave functions.

Consider now the third and fourth diagram shown in Fig. 1. After integrating over the "minus" components of the loop momenta they can also be written in the form given by Eq. (26). In particular, in the limit $|\mathbf{q}_\perp| >> \beta$ the dominant contribution comes from the first term in Eq. (26), $i.e.$, the overlap of the valence wave functions and it is given by

$$F_\pi(Q^2) \to F_{\pi,\text{exch}}(Q^2) = \int \frac{dx d\mathbf{k}_\perp}{16\pi^3} \frac{dy d\mathbf{l}_\perp}{16\pi^3} \Psi(y,\mathbf{l}_\perp) T_{\text{exch}}(y,\mathbf{l}_\perp;x,\mathbf{k}_\perp) \Psi(x,\mathbf{k}_\perp - x\mathbf{q}_\perp), \tag{37}$$

where

$$T_{\text{exch}} = \frac{16\pi C_F \alpha_s}{y-x} \left[\frac{x(1-x)\mathbf{l}_\perp^2 + y(1-y)\mathbf{k}_\perp^2 + (yx + (1-y)(1-x))\mathbf{l}_\perp \cdot \mathbf{k}_\perp}{\left(\frac{\mathbf{l}_\perp^2}{y} + \frac{(\mathbf{l}_\perp - \mathbf{k}_\perp)^2}{x-y} - \frac{\mathbf{k}_\perp^2}{x} \right) \left(\frac{\mathbf{l}_\perp^2}{y(1-y)} - \frac{\mathbf{k}_\perp^2}{x(1-x)} \right)} \right]. \tag{38}$$

To leading order in $1/\mathbf{q}_\perp^2$ this would naively give

$$\frac{16\pi C_F \alpha_s}{\mathbf{q}_\perp^2} \int \frac{dx d\mathbf{k}_\perp}{16\pi^3} \frac{dy d\mathbf{l}_\perp}{16\pi^3} \frac{\Psi(y,\mathbf{l}_\perp)\Psi(x,\mathbf{k}_\perp)}{xy}, \tag{39}$$

and therefore the expression for $F_{\pi,\text{exch}}$ becomes identical to the leading order formula, Eq. (3) with the distribution amplitudes given by the asymptotic formula

$$\phi(x) = \int \frac{d\mathbf{k}_\perp}{16\pi^3} \Psi(x,\mathbf{k}_\perp) = \frac{\sqrt{6}}{2} f_\pi x(1-x). \tag{40}$$

The second equality above follows from the normalization condition given in Eq. (13). Of course Eq. (39) does not exactly follow from Eq. (37) since the latter, and therefore also $F_{\pi,\text{exch}}$, is not well defined; T_{exch} has IR divergences coming from two energy denominators in the square brackets in Eq. (38). These singularities correspond to open $\bar{q}q$ and $\bar{q}qg$ intermediate states. These singularities are canceled in Eq. (26) by contributions from $\Psi \otimes T_{1,2g} \otimes \Psi_g$ and $\Psi_g \otimes I_{gg} \otimes \Psi_g$, respectively. From T_{exch} we may however define a $scheme$-dependent hard contribution kernel, $T_{\text{exch}}^{\text{hard}}(\mu)$ and the corresponding hard form factor from one gluon exchange,

$$F_\pi(Q^2) \to F_{\pi,\text{exch}}^{\text{hard}}(Q^2,\mu) = \int \frac{dx d\mathbf{k}_\perp}{16\pi^3} \frac{dy d\mathbf{l}_\perp}{16\pi^3} \Psi(y,\mathbf{l}_\perp) T_{\text{exch}}^{\text{hard}}(y,\mathbf{l}_\perp;x,\mathbf{k}_\perp;\mu) \Psi(x,\mathbf{k}_\perp - x\mathbf{q}_\perp), \tag{41}$$

by cutting-off light cone energy denominators such that Eq. (37) $F_{\pi,\text{exch}}^{\text{hard}}$ becomes IR finite. For example, we may define

$$T_{\text{exch}}^{\text{hard}}(\mu) \equiv \left[\prod_{i=1}^{2} \Theta(D_i) \right] T_{\text{exch}}, \quad T_{\text{exch}}^{IR}(\mu) \equiv T^{exch} - T_{\text{exch}}^{\text{hard}} = \left[1 - \prod_{i=1}^{2} \Theta(D_i) \right] T_{\text{exch}}, \tag{42}$$

where D_i is either one of the two denominators in Eq. (38) and $\Theta(D) \sim 1$, in the momentum region in which $|D| \gtrsim \mu^2$ and $\Theta(D) \to 0$ for $|D| << \mu^2$. The hard contribution from one gluon exchange, $T_{\rm exch}^{\rm hard}$, defined in this way is IR finite but cut-off, μ dependent. The μ-dependent IR singular piece of $T_{\rm exch}$, $T_{\rm exch}^{IR}(\mu)$ when combined with the contributions proportional to the $\bar{q}qg$ wave function produces IR finite but low momentum-dominated contribution. Of course, when this term is combined with the contribution from $T_{\rm exch}^{\rm hard}(\mu)$, the μ-dependence disappears. If we choose the factorization scale μ to be comparable to that setting the width of the soft wave function, $i.e.$ $\mu \sim \beta$ (or $\mu^2 \sim s_0$ for the local duality wave functions) then contribution from nonvalence sectors becomes reduced by the residual contribution from $T_{\rm exch}^{IR} = T_{\rm exch} - T_{\rm exch}^{\rm hard}$, $i.e.$ the IR dominated piece of the valence contribution. Then for $|\mathbf{q}_\perp| >> \mu$ bulk of the form factor will come from the hard gluon exchange given by $T_{\rm exch}^{\rm hard}$. With Θ given by

$$\Theta(D) = \frac{|D|}{|D| + \mu^2},\qquad(43)$$

the effect of the nonvalence contributions is to effectively add a mass term of order μ to the two energy denominators, D_i in Eq. (38) which leads to a similar effect as the gluon mass regulator used in Ref. [17].

III. SUDAKOV SUPPRESSION

Before proceeding to the analysis of the numerical results, let us discuss how the Sudakov suppression from the gluon radiation of the struck quark results in our formalisms. The exchange of the gluon across the electromagnetic vertex, Eq. (28) gives for the quark spectral function, $\rho^{\rm quark}(s,s')$

$$
\begin{aligned}
\rho_{\rm vert}(s,s') = 2(8\pi C_F \alpha_s)(2\sqrt{6})^2 \int \frac{dx d\mathbf{k}_\perp}{16\pi^3} \frac{dy d\mathbf{l}_\perp}{16\pi^3} \frac{y[\mathbf{l}_\perp - \mathbf{q}_\perp]\cdot[\mathbf{l}_\perp + (1-y)\mathbf{q}_\perp]}{\mathbf{l}_\perp^2[\mathbf{l}_\perp - y\mathbf{q}_\perp]^2} \\
\times \left[\delta\left(s - \frac{\mathbf{k}_\perp^2}{x(1-x)}\right) - \delta\left(s - \frac{\mathbf{k}_\perp^2}{x(1-x)} - \frac{\mathbf{l}_\perp^2}{y(1-y)(1-x)}\right)\right] \\
\times \left[\delta\left(s' - \frac{(\mathbf{k}_\perp - x\mathbf{q}_\perp)^2}{x(1-x)}\right) - \delta\left(s' - \frac{(\mathbf{k}_\perp - x\mathbf{q}_\perp)^2}{x(1-x)} - \frac{(\mathbf{l}_\perp - y\mathbf{q}_\perp)^2}{y(1-y)(1-x)}\right)\right].
\end{aligned}
\qquad(44)
$$

We now write the first term in brackets as

$$
\begin{aligned}
[\delta(s - \cdots) - \delta(s - \cdots)] = \delta\left(s - \frac{\mathbf{k}_\perp^2}{x(1-x)}\right)\frac{\frac{\mathbf{l}_\perp^2}{y(1-y)(1-x)}}{\frac{\mathbf{l}_\perp^2}{y(1-y)(1-x)} + \frac{\mathbf{k}_\perp^2}{x(1-x)}} \\
+ \left[\delta\left(s - \frac{\mathbf{k}_\perp^2}{x(1-x)}\right)\frac{\frac{\mathbf{k}_\perp^2}{x(1-x)}}{\frac{\mathbf{l}_\perp^2}{y(1-y)(1-x)} + \frac{\mathbf{k}_\perp^2}{x(1-x)}} - \delta\left(s - \frac{\mathbf{k}_\perp^2}{x(1-x)} - \frac{\mathbf{l}_\perp^2}{y(1-y)(1-x)}\right)\right],
\end{aligned}
\qquad(45)
$$

and similarly for the second term with $s \to s'$ and $\mathbf{k}_\perp \to \mathbf{k}_\perp - x\mathbf{q}_\perp$, $\mathbf{l}_\perp \to \mathbf{l}_\perp - y\mathbf{q}_\perp$. In the first term on the right-hand side in Eq. (45) the $q\bar{q}g$ invariant mass, $\mathbf{l}_\perp^2/(y(1-y)(1-x))$ in now regularized in the IR by the incoming $q\bar{q}$ virtuality, $\mathbf{k}_\perp^2/x(1-x)$. Thus when integrated in Eq. (44) over gluon transverse momentum the first term in Eq. (45) gives an IR finite contribution. The term in the bracket in Eq. (45) is also IR finite. However, the two delta functions give separately IR singular contributions [from integration over $1/\mathbf{l}_\perp^2$]. After integrating over s [with $e^{-s/2\beta^2}$ for the model based on the Borel transformation or $\theta(s_0 - s)$ for that suggested by the local duality], these become proportional to the valence $q\bar{q}$ and $q\bar{q}g$ wave functions, respectively. Combining the IR finite contribution from the pure $\bar{q}q$ sector (first term in the right-hand side of Eq. (45) and a similar one for $s \to s'$) gives

$$
\begin{aligned}
\rho_{\rm vert}^{\rm hard}(s,s') = 2(8\pi C_F \alpha_s)(2\sqrt{6})^2 \int \frac{dx d\mathbf{k}_\perp}{16\pi^3} \frac{dy d\mathbf{l}_\perp}{16\pi^3} \delta\left(s - \frac{\mathbf{k}_\perp^2}{x(1-x)}\right)\delta\left(s' - \frac{(\mathbf{k}_\perp - x\mathbf{q}_\perp)^2}{x(1-x)}\right) \\
\times \frac{y[\mathbf{l}_\perp - \mathbf{q}_\perp]\cdot[\mathbf{l}_\perp + (1-y)\mathbf{q}_\perp]}{\left[\mathbf{l}_\perp^2 + y(1-y)\frac{\mathbf{k}_\perp^2}{x}\right]\left[(\mathbf{l}_\perp - y\mathbf{q}_\perp)^2 + y(1-y)\frac{(\mathbf{k}_\perp - x\mathbf{q}_\perp)^2}{x}\right]}.
\end{aligned}
\qquad(46)
$$

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Treating similarly two self-energy contributions for the struck quark, and adding them to the vertex corrections gives the total IR finite electromagnetic current $q\bar{q}$ matrix element

$$\rho_{\text{em}}^{\text{hard}}(s, s') = S(x, \mathbf{k}_\perp, \mathbf{q}_\perp)(2\sqrt{6})\delta\left(s - \frac{\mathbf{k}_\perp^2}{x(1-x)}\right)(2\sqrt{6})\delta\left(s' - \frac{(\mathbf{k}_\perp - x\mathbf{q}_\perp)^2}{x(1-x)}\right), \tag{47}$$

where the form factor S is given by

$$S(x, \mathbf{k}_\perp, \mathbf{q}_\perp) = 1 - 4\pi C_F \alpha_s \int \frac{dy d\mathbf{l}_\perp}{16\pi^3} y \frac{\left[(2(1-y) + y^2)\mathbf{q}_\perp^2 + y(1-y)\left(\frac{\mathbf{k}_\perp^2}{x} + \frac{(\mathbf{k}_\perp - x\mathbf{q}_\perp)^2}{x}\right)\right]}{\left[\mathbf{l}_\perp^2 + y(1-y)\frac{\mathbf{k}_\perp^2}{x}\right]\left[(\mathbf{l}_\perp - y\mathbf{q}_\perp)^2 + y(1-y)\frac{(\mathbf{k}_\perp - x\mathbf{q}_\perp)^2}{x}\right]}. \tag{48}$$

The contribution from this spectral function to F_π is thus given by

$$F_\pi(Q^2) \rightarrow F_{\pi,\text{em}}^{\text{hard}}(Q^2) = \int [dx d\mathbf{k}_\perp] \Psi(x, \mathbf{k}_\perp) S(x, \mathbf{k}_\perp, \mathbf{q}_\perp) \Psi(x, \mathbf{k}_\perp - x\mathbf{q}_\perp). \tag{49}$$

Due to the wave function suppression, both $\mathbf{k}_\perp^2/x(1-x)$ and $(\mathbf{k} - x\mathbf{q}_\perp)^2/x(1-x)$ are small as compared to $\mathbf{q}_\perp^2$ in the integral in Eq. (48). Thus, the integral for the Sudakov form factor is dominated by $\mathbf{l}_\perp, y \rightarrow 0$ and $\mathbf{l}_\perp - y\mathbf{q}_\perp, y \rightarrow 0$. In these two regions, both transverse $\mathbf{l}_\perp$ and longitudinal y momentum integration lead to $\log(\mathbf{q}_\perp^2)$. To leading order in $(\mathbf{k}_\perp^2/x(1-x))/\mathbf{q}_\perp^2$ and $((\mathbf{k} - x\mathbf{q}_\perp)^2/x(1-x))/\mathbf{q}_\perp^2$, the result is

$$S(x, \mathbf{k}_\perp, \mathbf{q}_\perp) \approx 1 - \frac{C_F \alpha_s}{4\pi} \log^2\left(\frac{x\mathbf{q}_\perp^2}{\mathbf{k}_\perp^2}\right) - \frac{C_F \alpha_s}{4\pi} \log^2\left(\frac{x\mathbf{q}_\perp^2}{(\mathbf{k}_\perp - x\mathbf{q}_\perp)^2}\right). \tag{50}$$

Alternatively, we may define Sudakov form factor introducing factorization scale, μ to regularize the small virtuality of $q\bar{q}g$ intermediate state, just as we did in defining hard contribution from the one gluon exchange diagrams. We thus rewrite the differences of the two δ-functions in Eq. (44) as

$$[\delta(s - \cdots) - \delta(s - \cdots)] = \delta\left(s - \frac{\mathbf{k}_\perp^2}{x(1-x)}\right)\Theta\left(\frac{\mathbf{k}_\perp^2}{x(1-x)}\right)$$
$$+ \left[\delta\left(s - \frac{\mathbf{k}_\perp^2}{x(1-x)}\right)\left(1 - \Theta\left(\frac{\mathbf{k}_\perp^2}{x(1-x)}\right)\right) - \delta\left(s - \frac{\mathbf{k}_\perp^2}{x(1-x)} - \frac{\mathbf{l}_\perp^2}{y(1-y)(1-x)}\right)\right], \tag{51}$$

with Θ defined in Eq. (43), and similarly for the s'-dependent part. The hard contribution which defines perturbative Sudakov form factor comes, as before, from the first term on the right-hand side. Collecting hard contribution from vertex and self energies results in a contribution to F_π given by

$$F_\pi(Q^2) \rightarrow F_{\pi,\text{em}}^{\text{hard}}(Q^2, \mu) = \int \frac{dx d\mathbf{k}_\perp}{16\pi^3} \Psi(x, \mathbf{k}_\perp) S(x, \mathbf{k}_\perp, \mathbf{q}_\perp; \mu) \Psi(x, \mathbf{k}_\perp - x\mathbf{q}_\perp), \tag{52}$$

where $S(x, \mathbf{k}_\perp, \mathbf{q}_\perp; \mu)$ is now given by Eq. (48) with $\mathbf{k}_\perp^2/x$ and $(\mathbf{k}_\perp - x\mathbf{q}_\perp)^2/x$ in the denominator replaced by $(1-x)\mu^2$. For $\mu^2 \sim \beta^2 << \mathbf{q}_\perp^2$ the μ-dependent Sudakov form factor is thus given by

$$S(x, \mathbf{k}_\perp, \mathbf{q}_\perp; \mu) \approx 1 - \frac{C_F \alpha_s}{2\pi} \log^2\left(\frac{\mathbf{q}_\perp^2}{(1-x)\mu^2}\right). \tag{53}$$

The full contribution from the diagram on Fig. 1b including self energy corrections becomes

$$F_\pi(Q^2) \rightarrow F_{\pi,\text{em}}(Q^2, \mu) = \int \frac{dx d\mathbf{k}_\perp}{16\pi^3} \Psi(x, \mathbf{k}_\perp) S(x, \mathbf{k}_\perp, \mathbf{q}_\perp; \mu)]\Psi(x, \mathbf{k}_\perp - x\mathbf{q}_\perp) + \int \frac{dx d\mathbf{k}_\perp}{16\pi^3} \frac{dy d\mathbf{l}_\perp}{16\pi^3}$$
$$\times \left[\hat{\Psi}_g(x, \mathbf{k}_\perp; y, \mathbf{l}_\perp) T_{g1}^{\text{hard}} \Psi(x, \mathbf{k}_\perp - x\mathbf{q}_\perp) + \Psi(x, \mathbf{k}_\perp) T_{g2}^{\text{hard}} \hat{\Psi}_g(x, \mathbf{k}_\perp - x\mathbf{q}_\perp; y, \mathbf{l}_\perp - y\mathbf{q}_\perp)\right.$$
$$\left. + \hat{\Psi}_g(x, \mathbf{k}_\perp; y, \mathbf{l}_\perp) I_{gg}^\Gamma \hat{\Psi}_g(x, \mathbf{k}_\perp - x\mathbf{q}_\perp; y, \mathbf{l}_\perp - y\mathbf{q}_\perp)\right]. \tag{54}$$

Introduction of the factorization scale modifies the T_{gi} amplitudes according to

$$T_{gi} \rightarrow T_{gi}^{\mathrm{hard}} = \frac{\sqrt{8\pi C_F \alpha_s}}{y(1-y)} \left[\frac{y}{2} - N(y, \mathbf{l}_\perp) \frac{\Theta(D_i)}{D_i} \right], \quad (i = 1, 2) \tag{55}$$

and the modification of the nonvalence wave functions $\Psi_g \rightarrow \hat{\Psi}_g$ comes from the subtraction of the IR singular part of the valence sector,

$$\hat{\Psi}_g = \Psi_g - \frac{\sqrt{8\pi \alpha_s C_F}[1 - \Theta(D_i)]}{D_i} \Psi. \tag{56}$$

Even though each individual term in Eq. (54) is μ-dependent the entire sum is μ-independent. For $\mu^2 \sim \beta^2 \sim \langle \mathbf{k}_\perp^2/x(1-x) \rangle << \mathbf{q}_\perp^2$ the second term in Eq. (56) strongly reduces the nonvalence amplitudes, $\hat{\Psi}_g$ and the vertex correction becomes approximately given by the valence hard contribution alone, $i.e.$ $F_{\pi,\mathrm{em}}^{\mathrm{hard}}(Q^2, \mu \sim \beta)$. Also in this case the two expressions for the Sudakov form factor, Eqs. (50) and (53) give almost the same result.

In the ladder approximation Sudakov logarithms exponentiate and S becomes,

$$S \sim \exp\left(-\frac{\alpha_s}{2\pi} C_F \log^2 \frac{Q^2}{(1-x)\mu^2} \right). \tag{57}$$

To get rid of the μ dependence in this case one should sum to all orders in α_s^n contributions from $q\bar{q}g^n$ wave functions whose IR singularities cancel by mixing with nearby Fock sectors. However, as discussed above for the $n = 1$ case, if $\mu \sim \beta$ this residual contribution is expected to be small.

IV. NUMERICAL RESULTS

The numerical results are summarized in Figs. 2-4. They are all given for fixed $\beta = 0.4$ GeV.

In Figs. 2a,b, corresponding to $\alpha_s = 0.25$ and 0.4 respectively, the triangles represent the contribution F_π^0 of the lowest-order diagram Fig. 1a, i.e., the purely soft contribution determined by the Gaussian wave function of Eq. (25). At high Q^2, the function $F_\pi^0(Q^2)$ falls off like $1/Q^4$. For comparison, in Fig. 2a we also show the results of calculation of the soft contribution within the QCD sum rule method [16] (dashed line). The set of circles represents the sum $F_\pi^0 + F_\pi^\Gamma + F_\pi^\Sigma$ of the lowest-order contribution and the $O(\alpha_s)$ correction to the electromagnetic vertex F_π^Γ (29) accompanied by the quark self-energy correction F_π^Σ (34). As shown in Sec.III, the exponentiation of the leading double-logarithmic terms suppresses vertex correction and it can be approximated by $F_{\pi,em}^{\mathrm{hard}}$ given in Eq. (52). Using S from Eq. (57) with $\mu^2 = 2\beta^2$, we obtain the curve depicted by the squares. Finally, the solid line F_π^{tot} combines $F_{\pi,em}^{\mathrm{hard}}$ with the remaining contribution from the one gluon exchange diagrams Figs. 1c,d, the remaining half of the struck quark self-energy diagrams and the spectator's self-energy diagram.

Fig. 3 gives the comparison of the form factor calculation for two model wave functions discussed in Sec.II. The upper and lower dashed lines represent the soft contribution from Eq. (24) for the wave functions given by Eq. (25) and Eq. (18), respectively. The upper and lower solid lines are the respective results for F_π^{tot} (i.e., full calculation to $O(\alpha_s)$ for gluon exchange diagrams and Sudakov-exponentiated vertex correction; $\alpha_s = 0.25$ was used). The overall model dependence is seen to be rather small. As discussed in Sec. II, the one gluon exchange diagrams are responsible for the enhancement of the hard contribution at high Q^2.

Inspecting the results shown in Fig. 2a (2b) we may conclude that for $Q^2 \sim$ few GeV^2 the dominant effect comes from the soft region. The next-order contribution is $\sim 7\%$ ($\sim 10\%$) at $Q^2 \sim 1$ GeV^2 and increases to about 20% (30%) at $Q^2 \sim 4$ GeV^2. A large fraction of the enhancement from one gluon exchange diagrams is however IR dominated. This part of gluon exchange should therefore be considered together with the soft wave function contribution and separated from the hard scattering amplitude represented by hard gluon exchange. This effect is illustrated in Fig. 4. The contributions from hard gluon exchange given by $T_{\mathrm{exch}}^{\mathrm{hard}}$ is plotted together with the remaining contributions from the one gluon exchange diagrams. The dashed line is the result of the asymptotic formula (cf. Eq. (3)) with the asymptotic distribution amplitude given by Eq. (40). The solid line which approaches the asymptotic result from below is the contribution from $T_{\mathrm{exch}}^{\mathrm{hard}}$ with $\mu^2 = 2\beta^2$. The other solid line is the leftover

from the one gluon exchange diagrams. The two sets of points shown by circles and triangles below the asymptotic curve represent the hard gluon exchange contribution for $\mu^2 = 10\beta^2$ and $\mu^2 = 0.2\beta^2$, respectively. The upper circles and triangles give then the remaining one gluon exchange contributions for these two values of the factorization scale, respectively. The net contribution from each of the three sets of points (solid lines, circles and triangles) is μ-independent.

As advocated previously $\mu \sim \beta$ leads to the fastest saturation of the asymptotic form factor by the pure hard gluon exchange *i.e.* as Q^2 increases the choice $\mu \sim \beta$ is optimal for reducing the contribution from nonvalence sectors. At $Q^2 \sim 20\,\mathrm{GeV}^2$ the hard contribution starts to dominate over the IR sensitive part of the one gluon exchange. This is consistent with the results found in Ref. [17] where sensitivity of the form factor to the cut-off imposed on the light cone energy denominators was studied. The discussion on the normalization of the space-like pion form factor data can also be found in the recent analysis with the optimal renormalization scale and scheme dependence [18].

V. CONCLUSIONS

Our main focus in this work was to consistently generate the gluon radiative corrections within an approach motivated by the light cone quantization formalism and QCD sum rules. Just as in the latter approach, our starting objects are the Borel-transformed Green's functions. To make a link with the light-cone quantization, we demonstrated that the action of the Borel transform is analogous to using the Gaussian valence soft wave function which was frequently used in the past [19]. Such a pion wave function can be made to satisfy the standard light cone normalization conditions for the pion decay constant, form factor, etc. The main advantage of the Green's functions method is that applying the Borel transformation to the two-loop Green's functions, we generate both the radiative corrections to the current matrix element and the non-valence $\bar{q}qg$ components of the pion wave function which are absolutely necessary to secure the gauge invariant and infrared-finite results for the total $O(\alpha_s)$ corrections to the pion form factor. In addition, all of these wave functions are readily applicable to the light cone time ordered perturbative expansion of pion form factor beyond the leading twist level in QCD. Further works involving the four-point Green's functions and applications to the virtual Compton scattering are in progress.

Acknowledgment

[1] R. P. Feynman,   *The Photon-Hadron Interaction* (Benjamin, Reading 1972).

[2] S. J. Brodsky and G. R. Farrar, Phys. Rev. Lett. **31**, 1153 (1973).

[3] N.Isgur and C.H.Llewellyn Smith, Phys. Rev. Lett. **52**, 1080 (1984); Phys. Lett. B **217**, 535 (1989); Nucl. Phys. **B317** 526 (1989).

[4] A.V. Radyushkin, Nucl. Phys. **A532**, 141c (1991).

[5] S. J. Brodsky and G. P. Lepage, Phys. Rev. Lett. **53**, 545 (1979); Phys. Lett. B **87**, 359 (1979); G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22**, 2157 (1980).

[6] A. V. Efremov and A. V. Radyushkin,   Theor. Math. Phys.   **42**, 97 (1980);   Phys. Lett.   **B94**, 245 (1980).

[7] V.L. Chernyak and I.R. Zhitnitsky, Phys. Rep. **112**, 173 (1984).

[8] B.L. Ioffe and A.V. Smilga, Phys. Lett. **B 114**, 353 (1982).

[9] V.A. Nesterenko and A.V. Radyushkin, Phys. Lett. **B 115**, 410 (1982).

[10] P.L. Chung, F. Coester and W.N. Polyzou, Phys. Lett. **B 205**, 545 (1988).

[11] F. Schlumpf, Phys. Rev. **D 50**, 6895 (1994).

[12] A. Szczepaniak, C.-R. Ji and S. R. Cotanch, Phys. Rev. **D 49**, 3466 (1994).

[13] A. Szczepaniak, Phys. Rev. D **54**, 1167 (1996).

[14] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. bf B147, 385,448 (1980).

[15] A. V. Radyushkin, Acta Phys. Polon.B **26**, 2067 (1995).

[16] B.L. Ioffe and A.V. Smilga, Nucl. Phys. B **216**, 373 (1983).

[17] C.-R. Ji, A. Pang and A. Szczepaniak, Phys. Rev. **D52**, 4038 (1995).

[18] S.J.Brodsky, C.-R. Ji, A. Pang and D. Robertson, SLAC-PUB-7473, to be published in Phys.Rev.**D**.

[19] S.J.Brodsky, T.Huang and G.P.Lepage, in *Particles and Fields 2*, Proceedings of the Banff Summer Institute, Banff, Alberta, 1981, edited by A.Z. Capri and A.N. Kamal (Plenum, New York,1983) p.143.
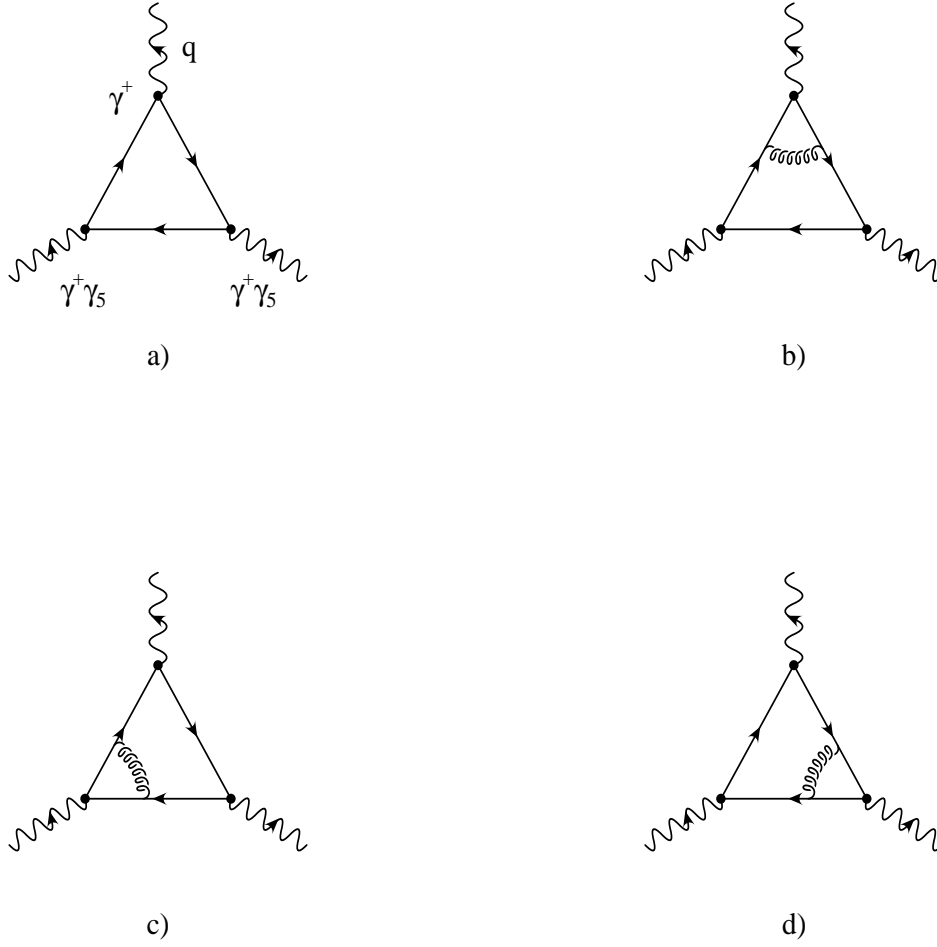
FIG. 1. Perturbative expansion of the three point function used to calculate the form factor.
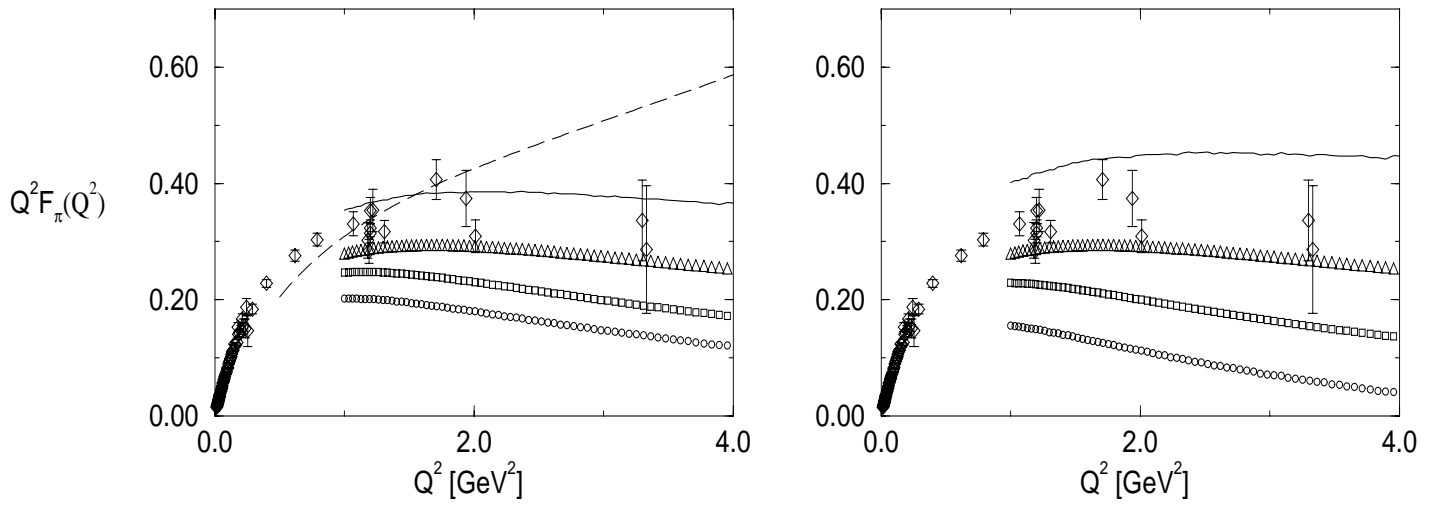

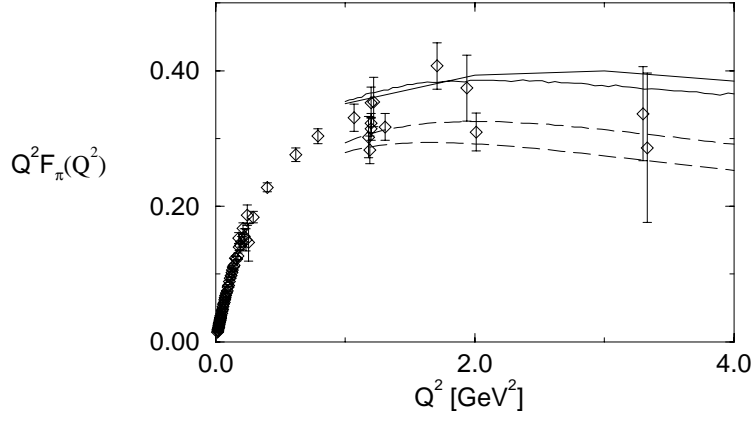FIG. 2. QCD corrections to $Q^2 F_\pi(Q^2)$.

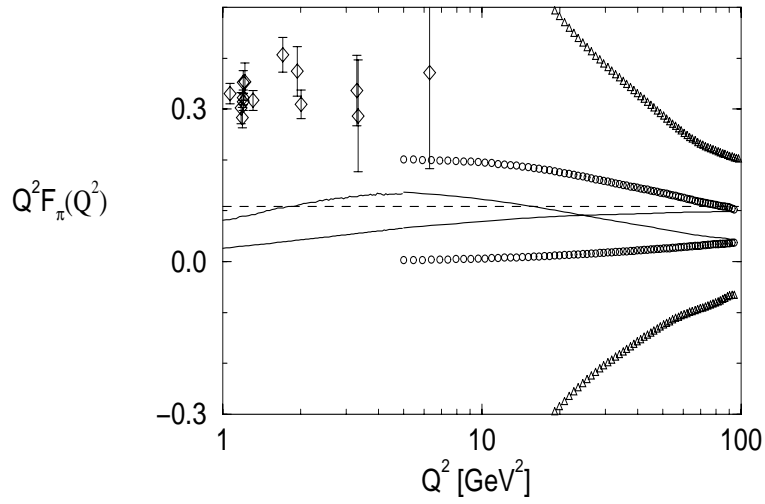FIG. 3. Wave function dependence of QCD corrections to $Q^2 F_\pi(Q^2)$.

FIG. 4. Gluon exchange vs. hard gluon exchange contribution to $Q^2 F_\pi(Q^2)$.