Strong Decays Of Heavy Hadrons In HQET

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We discuss the application of the tensor formalism of HQET to the strong decays of heavy hadrons. We treat both meson and baryon decays, and note that all of our results are in agreement with the 'spin-counting' arguments of Isgur and Wise. We briefly discuss the possible extension of the formalism to include $1/m$ corrections.

I. INTRODUCTION

In the past few years, the heavy quark effective theory (HQET) [1–3] has enjoyed much success in treating many aspects of the phenomenology of heavy hadrons [4,5]. The elegant tensor formalism developed by Georgi, and used extensively by others, has proven to be a very powerful tool for applications of HQET. There remains, however, one area that is yet to be treated by this tensor formalism, namely that of the strong decays of these hadrons. Clearly, since these decays, by their very nature, involve non-perturbative QCD, we do not expect HQET to allow us to calculate absolute decay rates. However, it will be useful in examining ratios of decay rates.

The question of the ratios of decay rates has been addressed by Isgur and Wise [6]. In their article, they noted that amplitudes for strong decays of heavy mesons were proportional to sums of products of four Clebsch-Gordan coefficients that arise from recoupling of angular momenta in the parent and daughter hadrons. In fact, their result can be expressed slightly more compactly as a $6-J$ symbol. Their argument makes use of the fact that the heavy quark is a spectator in the decay of the heavy hadron, so that only the light component of the hadron, the so called brown-muck, takes an active part in the decay.

At the hadronic level, a heavy hadron of total spin S decays to one of spin S', with a light hadron of total angular momentum S_h. The daughter hadrons are in a relative L-wave, and conservation of angular momentum gives

$$\mathbf{S_h} + \mathbf{L} = \mathbf{J_h}, \ \ \mathbf{S'_\ell} + \mathbf{S_Q} = \mathbf{S'}, \tag{1}$$

with

$$\mathbf{S'} + \mathbf{J_h} = \mathbf{S}, \tag{2}$$

where S'_ℓ is the spin of the brown muck in the daughter hadron, and S_Q is the spin of the heavy quark. This combination of angular momenta may be represented symbolically as

$$\left[[S_h L]_{J_h} [S'_\ell S_Q]_{S'} \right]_S. \tag{3}$$

On the other hand, one can regard this process as proceeding entirely at the level of the brown muck, since the heavy quark is a spectator in the process, so that

$$\mathbf{S'_\ell} + \mathbf{J_h} = \mathbf{S_\ell}, \ \ \mathbf{S_\ell} + \mathbf{S_Q} = \mathbf{S} \tag{4}$$

or

$$\left[\left[[S_h L]_{J_h} S'_\ell \right]_{S_\ell} S_Q \right]_S. \tag{5}$$

The overlap between these two 'wave functions' or coupling schemes is

$$(-1)^{(S_Q + S'_\ell + J_h + S)} \sqrt{(2S'+1)(2S_\ell+1)} \begin{Bmatrix} S_Q & S'_\ell & S' \\ J_h & S & S_\ell \end{Bmatrix}. \tag{6}$$

This object is proportional to the strong matrix element, and the constants of proportionality are the same for the four decays that are possible between two different multiplets. That is, there exist a single set of proportionality constants for the four decays

$$(S_\ell \pm 1/2) \rightarrow (S'_\ell \pm 1/2) + h. \tag{7}$$

In this article, we show how to use the tensor method to obtain the same information in a manner that we find somewhat more compact than the 'spin-counting' method. In addition, the specific forms of the amplitudes can be obtained using this formalism. Furthermore, the full power of the tensor formalism may then be brought to bear on these processes. For instance, it should be possible to treat the $1/m_Q$ corrections, as well as the radiative ones, to the decay amplitudes.

II. TENSOR FORMALISM

A. General Formalism

In general, we are interested in a matrix element of the form

$$\mathcal{M} = \left\langle \mathcal{X}(p)\mathcal{H}'_Q(v') \,|\, \mathcal{O}_s \,|\, \mathcal{H}_Q(v) \right\rangle, \tag{8}$$

where $\mathcal{X}$ is a light hadron, $\mathcal{H}'_Q$ and $\mathcal{H}_Q$ are heavy hadrons, and $\mathcal{O}_s$ is the operator responsible for the strong decay. The problem in trying to say anything useful about this lies in the fact that $\mathcal{O}_s$ is, in general, a complicated object that is full of non-perturbative QCD dynamics, and about which we know very little. In general, this operator will involve all of the sub-structure of the hadron in a non-trivial way.

Despite this difficulty, we do know that $\mathcal{O}_s$ must be a Lorentz scalar, as well as flavor singlet in all flavors of quarks. In particular, it is flavor singlet in the heavy quark. For the purposes of this discussion, and without any loss of generality, we can write $\mathcal{O}_s$ as

$$\mathcal{O}_s = \sum_i a_i \overline{Q}\Gamma_i Q L_i, \tag{9}$$

where Γ_i is a general Dirac matrix, and is one of 1, γ_μ, $\sigma_{\mu\nu}$, $\gamma_\mu\gamma_5$, γ_5, and L_i contains all of the dynamics involving the brown muck. The a_i are unknown constants. We have written $\mathcal{O}_s$ in this form in order to explicitly display the heavy quark part. Each L_i has the same Lorentz structure as the corresponding Γ_i, to ensure that $\mathcal{O}_s$ is a Lorentz scalar. Note that the L_i are, in principle, many body operators, as the structure of the brown muck is expected to be complicated. While it may be tempting to associate the $\overline{Q}\gamma_\mu Q$ term of $\mathcal{O}_s$, for instance, with 'one-gluon physics', we refrain from making such identifications. This is because we view eqn. (9) simply as a way of parametrizing our ignorance of strong interaction dynamics, and make no interpretations of the physics that could lead to each term.

Since we can represent the heavy hadrons as heavy part $\times$ light part, such as in the Falk representation [7], the matrix element of each term in $\mathcal{O}_s$ factorizes. This means that although the interactions involving the light component are complicated, we can absorb these into a set of unknown form factors (as has been done for weak decays) or unknown coupling constants (for strong decays). All that is left for us to deal with are the heavy quark components, which we know how to treat. Furthermore, we also know how to include corrections due to the finite mass of the heavy quark. However, there are still five terms in $\mathcal{O}_s$.

What helps us further in our treatment of the strong process is the fact that, at leading order in HQET (*i. e.*, in the limit that the mass of the quark Q goes to infinity), the heavy quark will act as a spectator in the decay. In particular, its spin indices are unaffected by the decay (this is the same physics contained in the spin-coupling scheme described above). Thus, the only possible form that can contribute is the scalar contribution, $\Gamma_i = 1$. Beyond leading order we would expect other terms to contribute. This also means that the light part of the matrix element is simplified, as the operator concerned is a Lorentz-scalar. This identification, that only $\Gamma_i = 1$ above can contribute at leading order, is the key to applying the tensor formalism to these decays. All else is now relatively simple, as we know how to 'calculate' matrix elements for any arbitrary Γ, as well as how to include various kinds of corrections.

We close this section by noting that the coupling constants to which we have alluded, which are essentially the matrix elements of the light part of the decay operator, depend only on the brown muck, and are therefore independent of the mass of the heavy quark. Thus, for instance, the same set of coupling constants would be valid for decays of hadrons containing c quarks and for hadrons containing b quarks.

B. Kinematic Questions

HQET, in conjunction with chiral perturbation theory (ChPT), has been applied to the strong single (and double) pion decays of heavy hadrons [8–10]. In that treatment, the requirement that the pion momentum be small, combined with the 'infinite' masses of the parent and daughter hadrons, leads to the fact that the velocity of the heavy daughter hadron is the same as that of the parent. This is in contrast with the weak decays of these states, in which the heavy quark can receive a large momentum change from the emitted virtual W: the velocities of the parent and daughter hadrons are different. In the HQET/ChPT formalism for strong decays, there are therefore two independent kinematic variables, v, the velocity of the heavy hadrons, and p, the momentum of the pion.

In the present formalism, we want to be able to treat the decays of a heavy hadron to another heavy hadron, with the emission of a 'light' hadron that may be any of the infinite tower of excited states. Thus, if the light daughter is sufficiently excited, it can provide the large impulse required to bring about a velocity change in the heavy hadrons. In fact, note that if we were to consider decays of charmed mesons to the a_2, say, the mass of the light daughter hadron is already a sizable fraction (70%, in the case of decays to the ground state charmed mesons) of the mass of the daughter hadron. It therefore appears necessary to make use of full momentum conservation through

$$m_{D^{(j)}} v = m_D v' + p, \tag{10}$$

where v is the velocity of the parent, v' is that of the heavy daughter, and p is the momentum of the light daughter.

We could therefore use two of the three quantities v, v' and p as independent kinematic variables. In this case, since the velocity of the heavy hadron has changed, we are apparently implicitly including $1/m_Q$ (and higher) corrections that are of a purely kinematic nature, and which have no effect on the spin symmetry of HQET. We note, however, that it has become customary to use the physical momentum of the light hadron in examining these decays. This amounts, in essence, to a second choice of kinematics.

We note, however, that since the splittings between states of the heavy spectra are independent of the mass of the heavy quark, so too is p, the momentum of the light hadron produced in the decay. This means that $v - v'$ must scale as $1/m_Q$. For the purposes of 'power counting', it is therefore more convenient to use p as one of the variables, instead of $v - v'$. We have therefore chosen p and v as our kinematic variables. We close by noting that the choice of kinematics will probably become more important for the consideration of $1/m_Q$ corrections.

III. MESON DECAYS

The starting point for this discussion is the representation of the heavy meson states. For concreteness, let us examine decays of excited D^{**} mesons to ground state $D^{(*)}$ mesons. For these, we use the representations constructed by Falk [7].

An excited D meson with total angular momentum J will, in general, be represented by an object linear in a polarization tensor, $\eta^{\mu_1 \cdots \mu_J}(v)$. This polarization tensor is symmetric, transverse and traceless. The latter two properties are expressed by

$$v_{\mu_1}\eta^{\mu_1 \cdots \mu_J}(v) = 0, \ g_{\mu_1 \mu_2}\eta^{\mu_1 \cdots \mu_J}(v) = 0. \tag{11}$$

For a state consisting of a heavy quark Q and a light component with the quantum numbers of an antiquark, the specific representation of any particular state will depend on the angular momentum j of the light component (antiquark) of the state. It is thus more convenient to refer to j than to J, since there will be a degenerate doublet of states with $J = j \pm 1/2$. The full details of the representations can be found in Falk's article [7].

We illustrate the tensor method for calculating strong decay matrix elements by examining two specific sets of decays: the generalization to other cases should be obvious. We begin by looking at decays involving single pions, so that we are interested in the matrix element

$$\mathcal{M} = \left\langle D(v')\pi(p) \left| \overline{c}c \right| D^{(j)}(v) \right\rangle. \tag{12}$$

As identified in the previous section, we are taking the heavy quark operator responsible for the decay as $\overline{c}c$ (and a light scalar operator is understood as multiplying $\overline{c}c$). Note, too, that we are explicitly not using the 'chiral limit' of soft pions, as we allow p to be large. In other words, the velocity in the heavy daughter baryon is not the same as in the parent, and momentum is conserved explicitly through

$$m_{D^{(j)}} v = m_D v' + p. \tag{13}$$

In terms of the trace formalism, the matrix element of interest is

$$\left\langle D(v')\pi(p) \left| \overline{c}c \right| D^{(j)}(v) \right\rangle = \sqrt{M_D M_{D^{(j)}}} \, \text{Tr} \left[\Pi(p)\mathcal{A}_{\mu_1 \ldots \mu_k} \overline{\mathcal{D}}(v') \mathcal{M}_{D^{(j)}}^{\mu_1 \cdots \mu_k}(v) \right], \tag{14}$$

where $\mathcal{D}(v')$ is the matrix representation of the meson D. The matrix $\mathcal{A}_{\mu_1 \ldots \mu_k}$ can only have the form

$$\mathcal{A}_{\mu_1 \ldots \mu_k} = p_{\mu_1} \ldots p_{\mu_k}, \tag{15}$$

while the matrix $\Pi(p)$ must represent the final state pion. The simplest, non-redundant form allowable is

$$\Pi(p) = a\gamma_5, \tag{16}$$

where the constant a is independent of the mass of the heavy quark, by virtue of our chosen normalization. One could also include a term in $\not{p}$, but this is redundant. Thus, the matrix element is

$$\left\langle D(v')\pi(p) \left| \overline{c}c \right| D^{(j)}(v) \right\rangle = a\sqrt{M_D M_{D^{(j)}}} \, \text{Tr} \left[\gamma_5 \overline{\mathcal{D}}(v') \mathcal{M}_{D^{(j)}}^{\mu_1 \cdots \mu_k}(v) \right] p_{\mu_1} \ldots p_{\mu_k}. \tag{17}$$

Due to the spin symmetry of HQET, the decays to the corresponding vector meson D^*, are also described by the same coupling constant a, and the corresponding matrix element is

$$\left\langle D^*(v', \varepsilon)\pi(p) \left| \overline{c}c \right| D^{(j)}(v) \right\rangle = a\sqrt{M_{D^*} M_{D^{(j)}}} \mathrm{Tr}\left[\gamma_5 \overline{\mathcal{D}^*}(v')\mathcal{M}_{D^{(j)}}^{\mu_1\ldots\mu_k}(v)\right] p_{\mu_1}\ldots p_{\mu_k}. \tag{18}$$

Thus, these four decays are all described in terms of a single, unknown, nonperturbative constant a.

We now turn to meson decays that are not as simple. We limit our discussion to decays involving light vector mesons (ρ, for instance), but the generalization to light hadrons of arbitrary spin should be clear. The matrix element for such a decay (still considering decays to the ground state heavy doublet) is

$$\left\langle D^{(*)}(v')\rho(p, \epsilon) \left| \overline{c}c \right| D^{(j)}(v) \right\rangle = \sqrt{M_{D^{(*)}} M_{D^{(j)}}}$$
$$\times \mathrm{Tr}\left[\mathcal{R}(p)\epsilon^{*\nu}\mathcal{A}_{\nu\mu_1\ldots\mu_k}\overline{\mathcal{M}}_{D^{(*)}}(v')\mathcal{M}_{D^{(j)}}^{\mu_1\ldots\mu_k}(v)\right]. \tag{19}$$

The most general form for the matrix $\mathcal{A}_{\nu\mu_1\ldots\mu_k}$ is

$$\mathcal{A}_{\nu\mu_1\ldots\mu_k} = p_{\mu_1}\ldots p_{\mu_{k-1}}\left[av_\nu p_{\mu_k} + b\gamma_\nu p_{\mu_k} + cg_{\nu\mu_k}\right], \tag{20}$$

while $\mathcal{R}(p) = 1$ is the most general, non-redundant form that represents the ρ meson (the polarization vector of the ρ appears explicitly in eqn. (19)). For a decay in which the parent belongs to one of the $(0^-, 1^-)$ or $(0^+, 1^+)$ multiplets, the term $cg_{\nu\mu_k}$ is absent, as there are then no indices on the matrix representation of the parent hadron.

We close this section with a brief discussion of the relationship between the formalism presented here and that of the combined heavy quark effective theory and chiral perturbation theory, for decays involving pions. It is clear that the two approaches are attacking the problem from somewhat different starting points. What we have presented here does not require any constraint on the momentum of the pion produced in the decay, and so may be considered as a formalism that automatically includes all the powers of pion momentum that would arise in a chiral expansion. Note, for instance, that for the D-wave decays of the $(1^+, 2^+)$, we would write down the form that we have written above, while in the chiral approach, operators with two powers of the pion momentum must be explicitly constructed [9,10]. Finally, we note that it is essentially trivial to include two or more pions in any of these decays using this formalism.

IV. BARYON DECAYS

The case of baryon decays may best be subdivided into two separate classes. The first set of decays that we will treat are those in which the heavy daughter hadron is a baryon (such as $\Lambda_b^* \to \Lambda_b \rho$), while in the second class, the heavy daughter hadron will be a meson (such as $\Lambda_b^* \to pB$).

A. Heavy Daughter Baryons

As with the meson decays, our starting point is the representation of the baryon states. We will simply borrow the representations constructed by Falk. We note, however, that we must divide our baryons into two classes, those with 'natural' parity, and those with 'unnatural' parity. This description is determined by the spin and parity of the brown muck, denoted j^P. If $P = (-1)^j$, the baryon is a natural one, while if $P = (-1)^{(j+1)}$, the baryon is unnatural. The need for this division into natural and unnatural baryons will become clear shortly.

Consider the decay $\Sigma_b^{(j)} \to \Lambda_b \pi$, which is described by the matrix element

$$\left\langle \Lambda_b(v')\pi(p) \left| \overline{b}b \right| \Sigma_b^{(j)}(v) \right\rangle = \overline{u}(v')R_{\mu_1\ldots\mu_j}(v)\mathcal{A}^{\mu_1\ldots\mu_j}, \tag{21}$$

where the spinor-tensor $R_{\mu_1\ldots\mu_j}(v)$ represents both states of the doublet, and $\mathcal{A}^{\mu_1\ldots\mu_j}$ contains all of the strong interaction dynamics. Since there is a pion in the decay, and assuming that the Λ_b is the ground state, then the quantity $\mathcal{A}^{\mu_1\ldots\mu_j}$ must be a pseudo-tensor if the $\Sigma_b^{(j)}$ is a natural baryon, or a tensor if it is unnatural. The forms that can be constructed in the two cases are quite different. Let us now examine some more specific examples.

Consider the decay $\Sigma_b^{(*)} \to \Lambda_b \pi$, where $\Sigma_b^{(*)}$ belongs to the $(1/2^+, 3/2^+)$ doublet. The matrix element is

$$\left\langle \Lambda_b(v')\pi(p) \left| \overline{b}b \right| \Sigma_b^{(*)}(v) \right\rangle = \overline{u}(v')R_\mu(v)r^\mu, \tag{22}$$

with r^μ a vector, which can only have the form

$$r_\mu = ap_\mu. \tag{23}$$

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On the other hand, if the $\Sigma_b^{(*)}$ belongs to the $(1/2^-, 3/2^-)$ doublet, then r^μ would be a pseudo-vector, which cannot be constructed from the quantities we have at our disposal. Thus

$$\left\langle \Lambda_b(v')\pi(p) \left| \bar{b}b \right| \Sigma_b^{(1/2^-,3/2^-)}(v) \right\rangle = 0. \tag{24}$$

The generalization of this to parent hadrons of higher spin is easy, since then $\mathcal{A}^{\mu_1\cdots\mu_j}$ of eqn. (21) becomes

$$\mathcal{A}_{\mu_1\ldots\mu_j} = a p_{\mu_1}\ldots p_{\mu_j} \tag{25}$$

for parents of unnatural parity, or

$$\mathcal{A}_{\mu_1\ldots\mu_j} = 0 \tag{26}$$

for parents of natural parity.

In the case of the heavy baryons of natural parity, the amplitudes for decays to the ground state with the emission of a single pion vanish at leading order, and should first be non-zero at order $1/m_Q$. Thus, if these states have no other open channels into which they can decay, they should be quite narrow.

For decays to final states that are not the ground state, such as to the $(1/2^+, 3/2^+)$ multiplet, the decay amplitude is

$$\left\langle \Lambda_b^{(1/2^+,3/2^+)}(v')\pi(p) \left| \bar{b}b \right| \Sigma_b^{(j)}(v) \right\rangle = \overline{R}_\nu(v') R_{\mu_1\ldots\mu_j}(v) \mathcal{A}^{\nu\mu_1\cdots\mu_j}. \tag{27}$$

If the parent hadron has natural parity, then $\mathcal{A}^{\nu\mu_1\cdots\mu_j}$ is a tensor (because the daughter has unnatural parity), and takes the form

$$\mathcal{A}_{\nu\mu_1\ldots\mu_j} = p_{\mu_1}\ldots p_{\mu_{j-1}} \left[a g_{\nu\mu_j} + b p_\nu p_{\mu_j} \right]. \tag{28}$$

For parents of unnatural parity,

$$\mathcal{A}_{\nu\mu_1\ldots\mu_j} = a p_{\mu_1}\ldots p_{\mu_{j-1}} \varepsilon_{\nu\mu_j\alpha\beta} v^\alpha p^\beta. \tag{29}$$

As final examples of the application of the formalism to this kind of decay we consider decays to ρ mesons. The matrix element for decays to the ground state is

$$\left\langle \Lambda_b(v')\rho(p,\epsilon) \left| \bar{b}b \right| \Sigma_b^{(j)}(v) \right\rangle = \overline{u}(v') R_{\mu_1\ldots\mu_j}(v) \epsilon_\nu^* \mathcal{A}^{\nu\mu_1\cdots\mu_j}. \tag{30}$$

If the parent has natural parity, then $\mathcal{A}^{\nu\mu_1\cdots\mu_j}$ is a tensor and takes the form

$$\mathcal{A}_{\nu\mu_1\ldots\mu_j} = p_{\mu_1}\ldots p_{\mu_{j-1}} \left[a g_{\nu\mu_j} + b v_\nu p_{\mu_j} \right]. \tag{31}$$

For a parent of unnatural parity,

$$\mathcal{A}_{\nu\mu_1\ldots\mu_j} = a p_{\mu_1}\ldots p_{\mu_{j-1}} \varepsilon_{\nu\mu_j\alpha\beta} v^\alpha p^\beta. \tag{32}$$

For decays to the $(1/2^+, 3/2^+)$ multiplet, we obtain

$$\left\langle \Lambda_b^{(1/2^+,3/2^+)}(v')\rho(p,\epsilon) \left| \bar{b}b \right| \Sigma_b^{(j)}(v) \right\rangle = \overline{R}_\nu(v') R_{\mu_1\ldots\mu_j}(v) \epsilon_\alpha^* \mathcal{A}^{\nu\alpha\mu_1\cdots\mu_j}. \tag{33}$$

For parents of unnatural parity,

$$\begin{aligned} \mathcal{A}_{\nu\alpha\mu_1\ldots\mu_j} = p_{\mu_1}\ldots p_{\mu_{j-2}} \big\{ &a g_{\nu\mu_j} g_{\alpha\mu_{j-1}} \\ &+ p_\beta \left[b g_{\nu\alpha} g_{\beta\mu_j} + c g_{\nu\mu_j} g_{\alpha\beta} + d g_{\alpha\mu_j} g_{\nu\beta} + e g_{\beta\mu_j} p_\alpha v_\nu \right] \big\}, \end{aligned} \tag{34}$$

while for parents of natural parity,

$$\begin{aligned} \mathcal{A}_{\nu\alpha\mu_1\ldots\mu_j} = p_{\mu_1}\ldots p_{\mu_{j-1}} \big\{ &a \varepsilon_{\mu_j\nu\alpha\beta} p^\beta \\ &+ v^\beta p^\rho \left[b p^\alpha \varepsilon_{\mu_j\nu\beta\rho} + c v^\nu \varepsilon_{\mu_j\alpha\beta\rho} + d p^{\mu_j} \varepsilon_{\nu\alpha\beta\rho} \right] \big\}. \end{aligned} \tag{35}$$

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B. Light Daughter Baryons

For these decays, as with the decays to heavy baryons, it will again be useful to divide the light baryons into natural and unnatural baryons, with a slight modification of the definition. A light baryon of spin J and parity P is considered to be natural if $P = (-1)^{J-1/2}$, unnatural if $P = (-1)^{J+1/2}$. In addition, for states with spin greater than $1/2$, we employ the generalized Rarita-Schwinger fields $u_{\mu_1 \dots \mu_j}(p)$, which satisfy

$$
\begin{aligned}
\not{p} u_{\mu_1 \dots \mu_n}(p) &= m u_{\mu_1 \dots \mu_n}(p), \\
\gamma^{\mu_1} u_{\mu_1 \dots \mu_n}(p) &= 0, \\
p^{\mu_1} u_{\mu_1 \dots \mu_n}(p) &= 0,
\end{aligned}
\tag{36}
$$

where $n = J - 1/2$, and we remind the reader that this object is symmetric in all of its Lorentz indices.

We first consider the decays $\Lambda_b \to B^{(*)} N$, where N is the ground state nucleon, and the parent represents any of the states that belong to one of the $1/2^+$ singlets. The amplitude for the decay is

$$
\left\langle B^{(*)}(v') N(p) \left| \bar{b} b \right| \Lambda_b(v) \right\rangle = \overline{u}(p) \mathcal{A} \overline{\mathcal{M}}_{B^{(*)}}(v') u(v),
\tag{37}
$$

where $\mathcal{A}$ is the most general scalar matrix that can be constructed, and $\mathcal{M}_{B^{(*)}}(v')$ is the matrix representing the $B^{(*)}$ states. Without loss of generality, we can choose $\mathcal{A} = a$, a constant.

We can generalize this for the decay of any excited state, such as $\Lambda_b^{(j)} \to B^{(*)} N$. The amplitude is

$$
\left\langle B^{(*)}(v') N(p) \left| \bar{b} b \right| \Lambda_b^{(j)}(v) \right\rangle = \overline{u}(p) \mathcal{A}^{\mu_1 \cdots \mu_j} \overline{\mathcal{M}}_{B^{(*)}}(v') R_{\mu_1 \dots \mu_j}(v).
\tag{38}
$$

For parents of natural parity, $\mathcal{A}^{\mu_1 \cdots \mu_j}$ is a tensor, and takes the form

$$
\mathcal{A}^{\mu_1 \cdots \mu_j} = p^{\mu_1} \dots p^{\mu_{j-1}} \left[a p^{\mu_j} + b \gamma^{\mu_j} \right].
\tag{39}
$$

For parents of unnatural parity, $\mathcal{A}^{\mu_1 \cdots \mu_j}$ is a pseudotensor, which we can construct very easily (in this case) by using the tensor of the preceding equation, and multiplying it by γ_5. Thus, for parents of unnatural parity,

$$
\mathcal{A}^{\mu_1 \cdots \mu_j} = p^{\mu_1} \dots p^{\mu_{j-1}} \left[a p^{\mu_j} + b \gamma^{\mu_j} \right] \gamma_5.
\tag{40}
$$

Since the use of the γ_5 allows us to go from pseudotensor to tensor for these decays, in what follows we will discuss only the decays of parent baryons with natural parity.

The last set of decays we consider are $\Lambda_b^{(j)} \to B^{(*)} \Delta$. For the amplitude, we write

$$
\left\langle B^{(*)}(v') \Delta(p) \left| \bar{b} b \right| \Lambda_b^{(j)}(v) \right\rangle = \overline{u}_\nu(p) \mathcal{A}^{\nu \mu_1 \cdots \mu_j} \overline{\mathcal{M}}_{B^{(*)}}(v') R_{\mu_1 \dots \mu_j}(v).
\tag{41}
$$

Since the Δ has unnatural parity, $\mathcal{A}^{\nu \mu_1 \cdots \mu_j}$ must be a pseudotensor (for parents of natural parity), and takes the form

$$
\begin{aligned}
\mathcal{A}_{\nu \mu_1 \dots \mu_j} = \Big\{ & p_{\mu_1} \dots p_{\mu_{j-1}} \left[a v_\nu \gamma_{\mu_j} + b g_{\nu \mu_j} + c v_\nu p_{\mu_j} \right] \\
& + d p_{\mu_1} \dots p_{\mu_{j-2}} g_{\nu \mu_{j-1}} \gamma_{\mu_j} \Big\} \gamma_5.
\end{aligned}
\tag{42}
$$

V. DISCUSSION AND CONCLUSION

In the previous sections, we have outlined how the tensor formalism of HQET may be used to examine the strong decays of heavy hadrons. There remain a few points of the formalism that warrant some discussion. First, note that we have not presented any decay rates. Nevertheless, we have examined many cases for these decays, and have found that the ratios of decay rates predicted by Isgur and Wise are indeed obtained.

We have not treated the decays of heavy mesons to a heavy baryon and a light antibaryon. However, the formalism for these decays is very similar to that of the last subsection.

There is one subtlety involved in some of the matrix elements we have shown. Let's examine the case of the $(1/2^-, 3/2^-) \to 1/2^+ \rho$, where the $1/2^+$ is the heavy baryon singlet. In this case, the $6 - J$ symbol becomes

$$
\left\{ \begin{array}{ccc} 1/2 & 0 & 1/2 \\ J_h & S & 1 \end{array} \right\},
$$

which implies that J_h can only have the value 1, regardless of the value of S. However, in our formalism, we have used two independent coupling constants, implying two independent amplitudes. The resolution of this apparent contradiction lies in realizing that for this decay, $J_h = 1$ can be constructed in two different ways, with $L = 0$ or $L = 2$ (for the ρ, $S_h = 1$). Thus, there are indeed two independent amplitudes, corresponding to the two independent partial waves, but the ratio of the two $L = 0$ decay amplitudes is the same as that of the two $L = 2$ decay amplitudes.

In their formalism, Isgur and Wise have pointed out that the total widths for the decays of the two members of a heavy spin multiplet to the two members of another multiplet are identical. In principle, we can obtain this general result in the present formalism, but a proof is beyond the scope of the present article, and is left for possible future work. We note, however, that for all of the cases we have examined explicitly, the sum rule has been found to be valid, as expected.

In subsection IV A, we saw that there were some decays that vanished exactly at this order in the $1/m$ expansion. For such amplitudes, the $1/m$ 'corrections' are therefore the leading terms, and we believe that these corrections should be studied. In addition, it is important to examine the $1/m$ corrections for the non-vanishing amplitudes, as these may lead to large departures from the leading order predictions. This has been done by Falk [9,10] for the D-wave decays of the $(1^+, 2^+)$ D^{**} mesons to the ground states, in the framework of the combined HQET and chiral perturbation theory. It is of some interest to see the kind of contributions that can arise in the present formalism. In particular, an S-wave component is expected for one of these decays.

The coupling constants we have introduced are all independent of the mass of the heavy quark present in the parent and daughter hadrons. By virtue of the heavy flavor symmetry, these coupling constants are therefore valid both for charm and beauty decays. Thus, knowing some charmed decay rates, we could predict the corresponding beauty decay rates. Alternatively, we could attempt to extend this formalism down to strange hadrons, treating the s quark as heavy, to glean some information about what to expect in charm. In this case one would certainly expect $1/m$ corrections to be very important.

Finally, we close on a very speculative note. The key to the formalism presented herein was the identification of the heavy quark current that plays a role in the decay. For the strong decays, this current was identified as being the unit Dirac matrix. It is possible that this idea can be extended to, for example, electromagnetic processes of heavy mesons. In the decay $D^* \to D\gamma$, for instance, it is expected that the photon will couple both to the heavy quark and to the brown muck. We know what to do in the first case, but not in the second. In the second case, however, we may still be able to use the idea that for this part of the current, the heavy quark is a spectator, so that the heavy quark current is again unity, and one is then left with the matrix elements of the light current, which may be parametrized in some way. This has been done, to some extent, by a number of authors [11]. Whether this approach leads to any further development remains to be seen.

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