

The Decays $K_L \rightarrow \ell^+ \ell^- \ell'^+ \ell'^-$ Revisited

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Abstract

The double lepton pair decay modes of the K_L meson are analyzed including all contributions of order p^6 in Chiral Perturbation Theory. The experimentally established $e^+ e^- e^+ e^-$ mode and the recently observed $e^+ e^- \mu^+ \mu^-$ mode are discussed in detail.

In this note we reconsider the double lepton pair decays $K_L \rightarrow \ell^+ \ell^- \ell'^+ \ell'^-$ that were studied many years ago by Miyazaki and Takasugi [1]. The two modes of main interest are the $K_L \rightarrow e^+ e^- e^+ e^-$ and the $K_L \rightarrow e^+ e^- \mu^+ \mu^-$ decays. The first proceeds at a rate well determined experimentally [2] with a branching ratio equal to $4.1 \pm 0.8 \times 10^{-8}$, and its interest resides partly in the possibility of determining the form factor associated with the virtual γ couplings to the K_L as well as the interesting interference effect derived from the identical leptons in the final state. Both issues are analyzed here, and the main conclusions are that the form factor effect is small (4%), requiring a substantial experimental improvement over the current error of about 20%, and the interference effect is tiny (0.5%) and most likely beyond experimental access. The second decay has been recently observed for the first time by the E799 collaboration at Fermilab [3]. This experiment quotes a branching ratio of a few parts per billion, a remarkable improvement over the previous upper bound of 4.9×10^{-6} [2]. We discuss in this case the effects of a non-trivial form factor, that according to our result produces an increase of the branching ratio by 30%, which could therefore be tested in the future as the experimental accuracy in this mode improves.

In the following we neglect a contribution to the amplitude due to CP violation. This contribution is of order p^4 but suppressed by the CP violating mixing parameter ϵ which is of order 10^{-3} ($\text{Re}(\epsilon) = 1.6 \times 10^{-3}$, $\phi_\epsilon \simeq 43.5^\circ$). Thus, the CP violating contributions to the widths considered here are suppressed by a factor approximately equal to $|\epsilon| \Lambda_\chi^2 / M_K^2 \sim 1\%$, where $\Lambda_\chi = 4\pi F_\pi$. Thus, in the limit of CP conservation, the $K_L \rightarrow \gamma^* \gamma^*$ amplitude has the most general form:

$$A(K_L \rightarrow \gamma^* \gamma'^*) = \mathcal{F}(t, t') \epsilon^{\mu\nu\rho\sigma} \epsilon_\mu k_\nu \epsilon'_\rho k'_\sigma. \quad (1)$$

Here γ^* and γ'^* are virtual photons with respective invariant mass squared $t = k^2$ and $t' = k'^2$. $\mathcal{F}(t, t')$ is the form factor of order p^6 in the chiral expansion that has a t and t' independent contribution F_1 , due to the π^0 , η and η' poles [4], plus a t and t' dependent contribution $F_2(t, t')$ from one chiral loop plus counterterms. F_1 is entirely fixed up to its sign by the $K_L \rightarrow \gamma\gamma$ decay, and $F_2(t, t')$ is given by [5]:

$$F_2(t, t') = \frac{\alpha_{\text{em}} C_8}{192\pi^3 F_\pi^3} \{ -(a_2 + 2a_4) D(t, t', \mu) + C(\mu) (t + t') \} , \quad (2)$$

where the counterterm has eliminated an UV divergence proportional to $(t + t')$, $F_\pi = 93$ MeV is the pion decay constant, and

$$D(t, t', \mu) = (t + t') \left[\frac{10}{3} - \left(\log \frac{M_K^2}{\mu^2} + \log \frac{M_\pi^2}{\mu^2} \right) \right] + 4 \left[F(M_\pi^2, t) + F(M_K^2, t) + F(M_\pi^2, t') + F(M_K^2, t') \right] , \quad (3)$$

with the chiral logarithms contained in the function $F(m^2, t)$:

$$F(m^2, t) \equiv \left(\left(1 - \frac{y}{4} \right) \sqrt{\frac{y-4}{y}} \log \frac{\sqrt{y} + \sqrt{y-4}}{-\sqrt{y} + \sqrt{y-4}} - 2 \right) m^2, \\ y \equiv \frac{t}{m^2}. \quad (4)$$

The coefficient $C(\mu)$ in the counterterm is determined by the fit to the Dalitz decays as discussed in [5]. There are two scenarios distinguished by the sign of F_1 , where the relative size of the counterterm to the chiral logarithms is different. In one scenario the counterterm (defined at the subtraction scale $\mu = M_\rho$) provides more than 90% of the contribution to the form factor's slope, while in the other that fraction is reduced to about 75%.

Since $\mathcal{F}(0, t) = \mathcal{F}(t, 0) = F(t)$, where $F(t)$ is the form factor of the Dalitz decays studied in [5], we use the results obtained in that reference by fitting the data [6]:

Scenario 1 :

$$F_1 = 0.89 \frac{\alpha_{\text{em}} C_8}{2\pi F_\pi}, \quad a_2 + 2a_4 = -0.3 \pm 0.3, \quad C(\mu = M_\rho) = 14.2 \pm 7.3$$

Scenario 2 :

$$F_1 = -0.89 \frac{\alpha_{\text{em}} C_8}{2\pi F_\pi}, \quad a_2 + 2a_4 = 1.5 \pm 0.3, \quad C(\mu = M_\rho) = -10.3 \pm 7.3 \quad (5)$$

Here, $C_8 = 3.12 \times 10^{-7}$ is the octet coupling in the non-leptonic weak interaction effective Lagrangian of order p^2 .

The decay amplitude has one piece if the final lepton pairs are different, and two pieces if they are identical. In this latter case the two amplitudes are:

$$\begin{aligned}
A_1 &= e^2 F(t, t') \epsilon^{\mu\nu\rho\sigma} \frac{(p_+ + p_-)_\nu (p'_+ + p'_-)_\sigma}{tt'} \\
&\times \bar{u}(p_-) \gamma_\mu v(p_+) \bar{u}(p'_-) \gamma_\rho v(p'_+) .
\end{aligned} \tag{6}$$

$$\begin{aligned}
A_2 &= - e^2 F(s, s') \epsilon^{\mu\nu\rho\sigma} \frac{(p_+ + p'_-)_\nu (p'_+ + p_-)_\sigma}{ss'} \\
&\times \bar{u}(p'_-) \gamma_\mu v(p_+) \bar{u}(p_-) \gamma_\rho v(p'_+) ,
\end{aligned} \tag{7}$$

where p_+ is the momentum of ℓ^+ , etc., and

$$\begin{aligned}
t &= (p_+ + p_-)^2 , & t' &= (p'_+ + p'_-)^2 , \\
s &= (p_+ + p'_-)^2 , & s' &= (p'_+ + p_-)^2 .
\end{aligned} \tag{8}$$

In the case of distinguishable lepton pairs only one appears, say A_1 . The decay width is obtained by summing over the lepton spins and integrating over the four particle phase space. We checked the results obtained in [1] and refer to it for further details.

In the $K_L \rightarrow \ell^+ \ell^- \ell^+ \ell^-$ decay we have:

$$\Gamma(K_L \rightarrow \ell^+ \ell^- \ell^+ \ell^-) = \frac{1}{2} \Gamma_1(K_L \rightarrow \ell^+ \ell^- \ell^+ \ell^-) + \frac{1}{2} \Gamma_2(K_L \rightarrow \ell^+ \ell^- \ell^+ \ell^-) \tag{9}$$

with

$$\begin{aligned}
\Gamma_1 &= \int \sum_{\text{spins}} |A_1 \text{ or } 2|^2 d\Phi , \\
\Gamma_2 &= \int \text{Re}(\sum_{\text{spins}} A_1 A_2^*) d\Phi ,
\end{aligned} \tag{10}$$

where $d\Phi$ represents the four body phase space volume element. On the other hand, in the $K_L \rightarrow e^+ e^- \mu^+ \mu^-$ decay we have instead:

$$\Gamma(K_L \rightarrow e^+ e^- \mu^+ \mu^-) = \Gamma_1(K_L \rightarrow e^+ e^- \mu^+ \mu^-) \tag{11}$$

For convenience we refer to the $K_L \rightarrow \gamma\gamma$ width. Defining $\rho \equiv \Gamma/\Gamma(K_L \rightarrow \gamma\gamma)$ and $\rho_{\text{interference}} \equiv \frac{1}{2} \Gamma_2/\Gamma(K_L \rightarrow \gamma\gamma)$, we obtain the results shown in the table:

Decay mode	Analysis	$\rho_{\text{interference}}$	ρ
$e^+e^-e^+e^-$	Ref [1]	-0.35×10^{-5}	5.89×10^{-5}
	No form factor	-0.036×10^{-5}	6.26×10^{-5}
	With form factor	-0.048×10^{-5}	6.50×10^{-5}
		-0.047×10^{-5}	6.48×10^{-5}
$e^+e^-\mu^+\mu^-$	Ref [1]	0	1.42×10^{-6}
	No form factor	0	1.71×10^{-6}
	With form factor	0	$(2.20 \pm 0.25) \times 10^{-6}$
		0	$(2.18 \pm 0.25) \times 10^{-6}$
$\mu^+\mu^-\mu^+\mu^-$	Ref [1]	-0.051×10^{-9}	0.946×10^{-9}
	No form factor	-0.051×10^{-9}	0.93×10^{-9}
	With Form Factor	-0.077×10^{-9}	$(1.30 \pm 0.15) \times 10^{-9}$
		-0.072×10^{-9}	$(1.35 \pm 0.15) \times 10^{-9}$

TABLE: The results of Ref [1] correspond to a point like form factor. The results of this work with a form factor are given respectively for the two scenarios of equation (5).

In the $e^+e^-e^+e^-$ mode there is a small effect due to the form factor leading to an increase of the width by about 4%. Given the current experimental error of almost 20%, it seems that a test of the form factor can be achieved in the foreseeable future. On the other hand,

we find that the interference term due to the identity of particles is a factor ten smaller than that obtained before [1], and it represents a correction of 0.5%, which seems beyond experimental access. Our prediction is consistent with the experimental rate: $BR(K_L \rightarrow e^+e^-e^+e^-)|_{\text{Theory}} = 3.85 \times 10^{-8}$, and $BR(K_L \rightarrow e^+e^-e^+e^-)|_{\text{Exp.}} = (4.1 \pm 0.8) \times 10^{-8}$. The relative size of the interference effect is larger in the $\mu^+\mu^-\mu^+\mu^-$ mode, but alas, the total branching ratio for this decay is predicted to be about 8×10^{-13} , and clearly outside the scope of future experiments. Finally, the $e^+e^-\mu^+\mu^-$ mode shows a sizeable effect due to the form factor that leads to an increase of the width by about 30%. The recent first-time observation of this decay mode by the E799 collaboration at Fermilab [3] furnishes a first experimental determination of the branching ratio, namely $BR(K_L \rightarrow e^+e^-\mu^+\mu^-)|_{\text{Exp.}} = 2.9_{-2.4}^{+6.7} \times 10^{-9}$. Our result is $BR(K_L \rightarrow e^+e^-\mu^+\mu^-)|_{\text{Theory}} = (1.30 \pm 0.15) \times 10^{-9}$, consistent with the measured value. There is here a strong promise that the reduced error bars resulting from the future experimental efforts will permit to show the effect due to the non-trivial form factor. This is clearly the most interesting mode for further experimental study.

As one would have expected, the analysis of the Dalitz decays is enough to pin down the predictions for the double lepton pair decays, and the two scenarios resulting from that analysis give essentially the same results.

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