Recent Progress in Quantum Hadrodynamics

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Abstract

Quantum hadrodynamics (QHD) is a framework for describing the nuclear many-body problem as a relativistic system of baryons and mesons. Motivation is given for the utility of such an approach and for the importance of basing it on a local, Lorentz-invariant lagrangian density. Calculations of nuclear matter and finite nuclei in both renormalizable and nonrenormalizable, effective QHD models are discussed. Connections are made between the effective and renormalizable models, as well as between relativistic mean-field theory and more sophisticated treatments. Recent work in QHD involving nuclear structure, electroweak interactions in nuclei, relativistic transport theory, nuclear matter under extreme conditions, and the evaluation of loop diagrams is reviewed.

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1. INTRODUCTION

The study of atomic nuclei plays an important role in the development of many-body theories. Early experimental probes of the nucleus were limited to energy scales considerably less than the nucleon mass $M \approx 939 \, \text{MeV}/c^2$, and the nucleus has traditionally been described as a collection of nonrelativistic nucleons interacting through an instantaneous two-body potential, with the dynamics given by the Schrödinger equation. The two-body potential is fitted to the empirical properties of the deuteron and to low-energy nucleon–nucleon (NN) scattering data, and one then attempts to predict the properties of many-nucleon systems. This is a difficult problem, because the NN potential is strong, short ranged ($R \approx 1 \, \text{fm}$), spin dependent, and has a very repulsive central core. Nevertheless, over a period of many years and with the advent of more and more powerful computers, reliable methods have been developed for solving the nonrelativistic nuclear many-body problem.

A new generation of accelerators will allow us to study nuclei at higher energies, at shorter distances, and with greater precision than ever before. For example, electron–nucleus scattering at CEBAF (now known as the Thomas Jefferson National Accelerator Facility) will sample distance scales down to tenths of a Fermi, and ultra-relativistic heavy-ion collisions at RHIC may produce nuclear densities of 10 times equilibrium density and temperatures of several hundred MeV. These experiments will clearly involve physics that goes beyond the Schrödinger equation, such as relativistic motion of the nucleons, dynamical meson exchanges, baryon resonances, modifications of hadron structure in the nucleus, and the dynamics of the quantum vacuum, which will include the production of a quark-gluon plasma. The challenge to theorists is to develop techniques that describe this new physics, while maintaining the important general properties of quantum mechanics, Lorentz covariance, electromagnetic gauge invariance, and microscopic causality.

Since quantum chromodynamics (QCD) of quarks and gluons is the fundamental theory of the strong interaction, it is natural to look to QCD as the means to describe this new physics. While this may be desirable in principle, there are many difficulties in practice, primarily because the QCD coupling is strong at distance scales relevant for the vast majority of nuclear phenomena. Although significant progress has been made in performing strong-coupling lattice calculations, actual QCD predictions at nuclear length scales with the precision of existing (and anticipated) data are not presently available. This situation will probably persist for some time, particularly with regard to many-nucleon systems.

In contrast, a description based on hadronic degrees of freedom is attractive for several reasons. First, these variables are the most efficient at normal densities and low temperatures, and for describing particle absorption and emission, because these are the degrees of freedom actually observed in experiments. Second, hadronic calculations can be calibrated by requiring that they reproduce empirical nuclear properties and scattering observables; we can then extrapolate to the extreme situations mentioned earlier. As an example, accurate microscopic meson-exchange models have been constructed to describe the NN interaction [Na73,Br76,Pa79,Zu81,Ma87,Te87,Ma89,Pl94,St94]. These contain several mesons, the most important of which are the $\pi(0^-, 1)$, $\sigma(0^+, 0)$, $\omega(1^-, 0)$, and $\rho(1^-, 1)$, where the indicated quantum numbers denote spin, parity, and isospin, together with both N and $\Delta(1232)$ degrees of freedom. Moreover, relativistic field theories of hadrons have been successful in describing the bulk and single-particle properties of nuclei and nuclear matter in the mean-

field and Dirac–Brueckner–Hartree–Fock approximations [Se86,Te87,Ma89,Se92].

Our basic goal is therefore to formulate a consistent microscopic treatment of nuclear systems using hadronic (baryon and meson) degrees of freedom. In principle, one could derive the form of the hadronic theory from the underlying QCD lagrangian, using the ideas of modern Effective Field Theory [We90a,We91,We92a,Ge93]. For example, one could define the low-energy effective theory by requiring scattering amplitudes computed using hadrons to "match" the corresponding amplitudes derived from the underlying theory. Unfortunately, unlike certain cases where this matching can be done (the Standard Model of weak interactions, QED descriptions of atomic physics, and interactions in heavy-quark systems) [Ma96], the derivation of a low-energy hadronic theory directly from QCD is intractable at present. Thus we must rely on other properties of QCD to constrain hadronic lagrangians.

One constraint is that the effective hadronic theory should embody the symmetries of QCD: Lorentz invariance, parity conservation, isospin symmetry, and spontaneously broken chiral symmetry.[1] These symmetry constraints severely restrict the forms of the allowed interactions, but they are insufficient to completely specify the low-energy dynamics. Thus we will use existing phenomenology to guide us in the construction of the effective lagrangian, in order to find the relevant degrees of freedom and the most efficient ways to structure the interactions.

For example, meson-exchange models of the NN interaction tell us which mesons and baryons are the most important; these models have been useful both for scattering and for the nuclear matter problem. Moreover, experiments on the electrodisintegration of the deuteron show unambiguously the presence of pion-exchange currents, which arise when the incoming (virtual) photon couples to a pion being exchanged between two nucleons. Medium-energy pion–nucleus and proton–nucleus scattering indicate the importance of baryon resonances, such as the Δ, in nuclear reactions. In addition, a decomposition of the low-energy NN scattering amplitude using Lorentz invariants reveals that the empirical Lorentz scalar, vector, and pseudoscalar amplitudes are much larger than the amplitudes deduced from a nonrelativistic decomposition containing Galilean invariants. These large amplitudes have important consequences for the spin, velocity, and density dependence of the NN interaction and are at the heart of relativistic descriptions of nucleon–nucleus scattering that reproduce spin observables in a very economical fashion [Mc83,Sh83,Cl83,Wa87]. Finally, relativistic mean-field models using classical Lorentz scalar and vector fields show that these are useful degrees of freedom for describing bulk and single-particle nuclear properties and for elucidating the important density dependence in the NN interaction.

A Lorentz-covariant description is important for extrapolation to astrophysical objects and for describing processes at large energy-momentum transfer, as will be observed with the new accelerators. On the other hand, recasting the nuclear many-body problem in nonrelativistic form (*i.e.*, with two-component rather than four-component nucleon spinors) leads to interactions (both two-body and few-body) that are similar to those used in successful calculations of few-nucleon systems. Many years of study within the nonrelativistic framework

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[1]The final two symmetries are only approximate. We also want to maintain the usual gauge invariance in electromagnetic interactions.

have produced a quantitative description of the structure of light nuclei (here both NN and NNN potentials are important) [Wi92,Fr95,Pu95]. Moreover, we have a qualitative understanding of some general features of nuclear structure, such as the importance of the Pauli principle in reducing NN correlations [Wa95], which justifies both the shell model and the single-nucleon optical potential, and the interplay of single-particle and collective degrees of freedom, which determines the shape of the nucleus [Ei87]. Although the equilibrium properties of nuclear matter cannot be precisely reproduced with modern NN potentials [Da83,Da85,Ja92a], the addition of a phenomenological, density-dependent interaction to the free NN potential leads to excellent results for the single-particle structure and charge and mass densities of a large number of nuclei [Go79]. One of the goals of the effective hadronic theory is to provide a deeper understanding of these successes and a more concrete link between these results and the underlying QCD.

We desire a microscopic treatment of the nuclear many-body problem that is consistent with quantum mechanics, special relativity, unitarity, causality, cluster decomposition, and the intrinsic symmetries mentioned earlier. The modern viewpoint is that relativistic quantum field theory based on a local, Lorentz-invariant lagrangian density is simply the most general way to parametrize an S matrix (or other observables) consistent with these desired properties [We95]. Thus there is no reason that relativistic quantum field theory should be reserved for "elementary" particles only. We will refer to relativistic field theories based on hadrons as *quantum hadrodynamics* or QHD [Se86,Se92]. In principle, the field-theoretic formulation allows for the construction of "conserving approximations" [Ba61,Ba62] that maintain the general properties mentioned above. Analyses based on QHD, as defined here, can hopefully provide a correct description of many-baryon systems at large distances and at energy scales that are not too high. One must examine, however, the limitations of hadronic field theory when one attempts to extrapolate away from this regime.

Although spontaneously broken chiral symmetry implies that $\pi\pi$ and πN interactions are weak for small momenta, the NN interaction is too strong to be treated perturbatively in the couplings. (This is also true in nonrelativistic formulations of the nuclear many-body problem.) It is therefore necessary to develop reliable nonperturbative approximations, so that unambiguous comparisons between theory and experiment can be made. The formulation of practical, reliable techniques for finite-density calculations in strong-coupling relativistic quantum field theories is still basically an unsolved problem [Da86,Ka87,Fu89,Ko95d,Ka96a]. The development of such tools in a hadronic field theory is not only useful in its own right, but it may also provide insight into similar approaches for QCD. Nuclear many-body theory has had such influence on other areas of physics in the past.

Historically, QHD models [Wa74,Se79] were confined to the class of renormalizable field theories, which can be characterized by a finite number of coupling constants and masses [Ca82,Co84]. The motivation was that these parameters could be calibrated to observed nuclear properties, and one could then extrapolate into regions of extreme density, temperature, or energy-momentum transfer without the appearance of new, unknown parameters [Wa74]. The hope was that nonrenormalizable and vacuum effects would be small enough that they could be adequately described by the long-distance structure of a renormalizable hadronic theory, through the systematic evaluation of quantum loops [Se92]. These models had several successes, some of which will be described below, but also difficulties in carrying out the renormalization program, as well as concrete indications that renormalizability is too

restrictive. Thus it is important to generalize these models to the more modern viewpoint of nonrenormalizable, effective field theories, which can *still* provide a consistent treatment of the nuclear many-body problem. Reviewing the progress made toward this generalization in the past few years is a central theme of this article.

Another goal of this article is to review the developments in QHD that have transpired since 1992.[2] We begin by discussing simple renormalizable models: QHD–I [Wa74], which contains neutrons, protons, and the isoscalar, Lorentz scalar and vector mesons σ and ω; the linear σ model [Sc57,Ge60,Le72], which contains neutrons, protons, pions, and neutral scalar mesons interacting in a chirally invariant fashion; and the extension of these models to include the isovector $\boldsymbol{\rho}$ meson, which we generically call QHD–II [Se79,Ma82]. These models serve as pedagogical tools for introducing the relativistic mean-field and Dirac–Hartree approximations, and their application to both infinite nuclear matter and the ground states of atomic nuclei. At the relativistic mean-field theory (RMFT) level with classical, isoscalar, Lorentz scalar and vector fields, QHD–I provides a simple picture of the equilibrium properties of nuclear matter. We focus on the new features that arise in a relativistic framework and emphasize the important concepts of Lorentz covariance and self-consistency. When extended to finite nuclei through Dirac–Hartree calculations with a classical, isovector, Lorentz vector field and a few parameters fitted to the properties of nuclear matter, one derives the nuclear shell model and can obtain, through the relativistic impulse approximation (RIA), an efficient description of the scattering of medium-energy nucleons from nuclei. We also show that the bulk and single-particle properties of nuclei provide stringent enough constraints to distinguish between various RMFT models. These successful QHD results are discussed in Section 2.

In Section 3, we examine the linear sigma model. We show how the constraints of linear chiral symmetry and spontaneous symmetry breaking, when realized in the usual fashion, *preclude* a successful description of nuclei at the mean-field level. We then discuss how the exchange of two correlated pions between nucleons produces a strong mid-range attraction in the scalar-isoscalar channel; this allows for an alternative (nonlinear) implementation of the chiral symmetry, with an additional scalar field to simulate correlated two-pion exchange, that produces successful mean-field results. We conclude that the scalar field in QHD–I should be regarded as an *effective* field that includes the pion dynamics that is most important for describing bulk nuclear properties.

In Section 4, we consider the application of effective hadronic theories to the two- and few-nucleon problems. Several meson-exchange models of the NN interaction have been developed and applied to nuclear systems, both relativistically and nonrelativistically. These models focus primarily on the precise reproduction of the two-nucleon data, and they are typically not renormalizable. We discuss the relativistic quasipotential formalism and its successful application to two-nucleon systems, as well as its extension to the study of two-nucleon correlations in nuclear matter. We then examine how the RMFT results are modified when short-range correlations are included and also gain some insight into the relationship between relativistic and nonrelativistic approaches to the nuclear matter problem. The

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[2]We try to be as complete as possible with references from early 1992 through 1995.

successes of these nonrenormalizable models, as well as the introduction of an effective scalar field in Section 3, give strong empirical hints that one must really view QHD as arising from an effective, nonrenormalizable lagrangian that yields an economical description of the underlying theory of QCD in the nuclear domain.

Therefore, in Section 5, we present a formulation of QHD in the modern context of effective field theory. Since a nonrenormalizable QHD lagrangian generally contains an infinite number of unknown constants, one must identify suitable expansion parameters and define a meaningful truncation scheme. Based on the results in Section 2, we argue that although the mean meson fields are large on the scale of nuclear energies (several hundred MeV versus tens of MeV), and one has all the complexities of a strong-coupling theory, the *ratios* of the mean fields to the nucleon mass are small at moderate densities. These ratios thus provide suitable expansion parameters. This observation leads naturally into Section 6, where a wide range of applications of RMFT to nuclear structure throughout the periodic table and to a variety of nuclear phenomena is discussed. The numerous successful calculations since early 1992 are considered within the general context of effective field theory, and the importance of both renormalizable and nonrenormalizable couplings is considered. We also introduce some basic ideas of density functional theory [Dr90] to illustrate how RMFT models can include many-body effects that go beyond the simple Hartree approximation. When combined with the systematics of two-nucleon correlations described in Section 4, these ideas help to explain why RMFT models are so successful.

The RMFT has also been widely applied to relativistic transport theory and heavy-ion reactions through the Vlasov–Uehling–Uhlenbeck equations for the phase-space distribution function. We briefly review the basic theoretical background and discuss developments since 1992 in Section 7. We also consider the extrapolation of RMFT results for the nuclear equation of state to high baryon density and high temperature.

To have a consistent effective field theory, one must also consider loop corrections to the RMFT. These loops contain both familiar many-body contributions [Fe71] and quantum vacuum effects. The underlying vacuum dynamics of QCD is implicitly contained in the parameters of the effective hadronic lagrangian; nevertheless, these vacuum effects are modified in the presence of valence nucleons at finite density. Although chiral perturbation theory [Ga84,Me93a] provides a systematic framework for including pion loops in meson–meson and meson–nucleon scattering amplitudes, the inclusion of vacuum loops that arise in the many-body theory is still an unsolved problem. We review the present status of loop calculations in QHD in Section 8. Finally, Section 9 contains a summary.

The reader is assumed to have a working knowledge of relativistic quantum field theory, canonical quantization, and the use of path integrals at zero temperature. For general background on relativistic field theory, the reader can turn to a number of texts [Bj64,Bj65,Na78,It80,Ra81,Ma84,Ry85,Ka89,St93,Um93,Gr93,We95,We96]; for background on quantum hadrodynamics, two recent reviews exist that cover the basic formulation and developments up to 1992 [Se86,Se92]. A recent text develops many of the theoretical tools in more detail [Wa95].

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2. QUANTUM HADRODYNAMICS (QHD)

The purpose of these next three sections is to develop some insight into the ingredients necessary for a realistic relativistic description of few- and many-body nuclear systems. In this section, we review the basic ideas of QHD and some of its applications. Most of the concepts are developed in [Se86,Se92], which discuss the literature up to 1992, and a recent text provides a more thorough treatment of the theoretical tools and background [Wa95]. We will not repeat all of that material here. Moreover, since we are concentrating on basic results, we postpone until later the discussion of recent progress, where we will try to be as complete as possible with developments and references from early 1992 through 1995. While the literature in this area prior to 1992 is detailed in [Se86,Se92], we would like to single out for special mention the important background papers [Sc51,Jo55,Du56,We67,Mi72].

Quantum hadrodynamics as defined above is a general framework for the relativistic nuclear many-body problem. The detailed dynamics must be specified by choosing a particular lagrangian density. As we will discuss later, there is increasing evidence that QHD must be regarded as an effective field theory, where all types of hadronic couplings satisfying general symmetry requirements are to be included. Nonetheless, the original motivation for using a renormalizable lagrangian to formulate a consistent relativistic nuclear many-body theory based on hadronic degrees of freedom remains valid. Furthermore, as we shall see, there is now convincing empirical evidence that whatever the form of the effective theory, it must be dominated by strong isoscalar, Lorentz scalar and vector interactions.

To introduce the relativistic formalism, we consider a simple model called QHD–I [Wa74], which contains fields for baryons $\left[\psi = \begin{pmatrix} \psi_{\mathrm{p}} \\ \psi_{\mathrm{n}} \end{pmatrix} \right]$ and neutral scalar (ϕ) and vector (V^μ) mesons. The lagrangian density for this model (with $\hbar = c = 1$) is given by [Se86,Se92]

$$\mathcal{L} = \overline{\psi}[\gamma_\mu(i\partial^\mu - g_{\mathrm{v}}V^\mu) - (M - g_{\mathrm{s}}\phi)]\psi + \tfrac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m_{\mathrm{s}}^2\phi^2)$$

$$- \tfrac{1}{3!}\kappa\phi^3 - \tfrac{1}{4!}\lambda\phi^4 - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m_{\mathrm{v}}^2 V_\mu V^\mu + \delta\mathcal{L} \ , \tag{2.1}$$

where $F^{\mu\nu} \equiv \partial^\mu V^\nu - \partial^\nu V^\mu$ and $\delta\mathcal{L}$ contains counterterms. The parameters M, g_{s}, g_{v}, m_{s}, m_{v}, κ, and λ are phenomenological constants that may be determined (in principle) from experimental observables. This lagrangian resembles massive QED with an additional scalar interaction, so the resulting relativistic quantum field theory is *renormalizable* [Bo70]. The inclusion of the scalar self-interactions proportional to ϕ^3 and ϕ^4 make this the most general lagrangian consistent with renormalizability (for these degrees of freedom) [Bo77]. The counterterms in $\delta\mathcal{L}$ are used for renormalization.

The motivation for this model has evolved considerably since it was introduced. As discussed in the Introduction, when the empirical NN scattering amplitude is described in a Lorentz-covariant fashion, it contains large isoscalar, scalar and four-vector pieces [Mc83,Sh83,Cl83,Wa87], and the simplest way to reproduce these is through the exchange of neutral scalar and vector mesons. The neutral scalar and vector components are the most important for describing bulk nuclear properties, which is our main concern here. Other Lorentz components of the NN interaction average essentially to zero in spin-saturated nuclear matter and may be incorporated as refinements to the present model; we will discuss pion dynamics in the following section. The important point is that even in more complete models, the dynamics generated by scalar and vector mesons will remain; thus, it is

important to first understand the consequences of these degrees of freedom for relativistic descriptions of nuclear systems.

The field equations for this model follow from the Euler–Lagrange equations and can be written as

$$(\partial_\mu \partial^\mu + m_s^2)\phi + \tfrac{1}{2}\kappa\phi^2 + \tfrac{1}{6}\lambda\phi^3 = g_s\overline{\psi}\psi \ , \tag{2.2}$$

$$\partial_\nu F^{\nu\mu} + m_v^2 V^\mu = g_v\overline{\psi}\gamma^\mu\psi \ , \tag{2.3}$$

$$[\gamma^\mu(i\partial_\mu - g_v V_\mu) - (M - g_s\phi)]\psi = 0 \ . \tag{2.4}$$

(The counterterms have been suppressed.) Equation (2.2) is a Klein–Gordon equation with a scalar source term and nonlinear scalar self-interactions. Equation (2.3) looks like massive QED with the conserved baryon current

$$B^\mu \equiv (\rho_B, \boldsymbol{\mathcal{B}}) = \overline{\psi}\gamma^\mu\psi, \qquad \partial_\mu B^\mu = 0 \ , \tag{2.5}$$

rather than the (conserved) electromagnetic current as the source. Finally, Eq. (2.4) is the Dirac equation with scalar and vector fields entering in a minimal fashion. These field equations imply that the canonical energy-momentum tensor $T^{\mu\nu}$ is conserved ($\partial_\mu T^{\mu\nu} = \partial_\nu T^{\mu\nu} = 0$).

When quantized, Eqs. (2.2)–(2.4) become *nonlinear quantum field equations*, whose exact solutions (if they exist) are very complicated. In particular, they describe mesons and baryons *that are not point particles*, but rather objects with structure due to the implied (virtual) meson and baryon-antibaryon loops. Here the dynamical input of renormalizability is apparent, since we are assuming that the intrinsic structure (or at least the long-range part of it) can be described using hadronic degrees of freedom. Strictly speaking, the validity of this input and its limitations have yet to be tested conclusively within the framework of QHD; nevertheless, as we shall see, there are now strong indications that this assumption is too optimistic.

We also expect the coupling constants in Eqs. (2.2)–(2.4) to be large, so perturbative solutions are not useful. Fortunately, there is an approximate nonperturbative solution that can serve as a starting point for studying the implications of the lagrangian in Eq. (2.1). Consider a system of B baryons in a large box of volume V at zero temperature. Assume that we are in the rest frame of the matter, so that the baryon flux $\boldsymbol{\mathcal{B}} = 0$. As the baryon density B/V increases, so do the source terms on the right-hand sides of Eqs. (2.2) and (2.3). If the sources are large enough, the meson field operators can be approximated by their expectation values, which are classical fields:

$$\phi \to \langle\phi\rangle \equiv \phi_0, \qquad V^\mu \to \langle V^\mu\rangle \equiv (V_0, 0) \ . \tag{2.6}$$

For our stationary, uniform system, ϕ_0 and V_0 are *constants* that are independent of space and time, and since the matter is at rest, the classical three-vector field $\mathbf{V} = 0$.

It is important to emphasize that the preceding "mean-field" theory (MFT) serves only as a starting point for calculating corrections within the framework of QHD, using Feynman diagrams and path-integral methods, as discussed in [Se86,Se92,Wa95]. We will return later to decide at which densities this starting point is actually useful.

A. The Nuclear Matter Equation of State

When the meson fields in Eq. (2.1) are approximated by the constant classical fields of Eq. (2.6), we arrive at the mean-field lagrangian density

$$\mathcal{L}_{\mathrm{MFT}} = \overline{\psi}[i\partial^\mu\gamma_\mu - g_{\mathrm{v}}V_0\gamma_0 - (M - g_{\mathrm{s}}\phi_0)]\psi - \tfrac{1}{2}m_{\mathrm{s}}^2\phi_0^2 - \tfrac{1}{3!}\kappa\phi_0^3 - \tfrac{1}{4!}\lambda\phi_0^4 + \tfrac{1}{2}m_{\mathrm{v}}^2V_0^2 \ . \qquad (2.7)$$

(The counterterms have been suppressed.) The conserved baryon four-current remains as in Eq. (2.5), and the canonical energy-momentum tensor becomes

$$T_{\mathrm{MFT}}^{\mu\nu} = i\overline{\psi}\gamma^\mu\partial^\nu\psi - (\tfrac{1}{2}m_{\mathrm{v}}^2V_0^2 - \tfrac{1}{2}m_{\mathrm{s}}^2\phi_0^2 - \tfrac{1}{3!}\kappa\phi_0^3 - \tfrac{1}{4!}\lambda\phi_0^4)g^{\mu\nu} \ . \qquad (2.8)$$

As discussed by Freedman [Fr78], there is no need to symmetrize $T^{\mu\nu}$ if we consider only uniform nuclear matter. This follows because the additional terms in the symmetrized tensor enter as a total four-divergence, whose diagonal matrix elements vanish in a uniform system.

Since the meson fields are classical, only the fermion field must be quantized. The Dirac field equation follows from $\mathcal{L}_{\mathrm{MFT}}$:

$$[i\gamma_\mu\partial^\mu - g_{\mathrm{v}}\gamma_0V_0 - (M - g_{\mathrm{s}}\phi_0)]\psi(t,\mathbf{x}) = 0 \ , \qquad (2.9)$$

and since this equation is *linear*, it can be solved exactly. The scalar field ϕ_0 shifts the baryon mass from M to $M^* \equiv M - g_{\mathrm{s}}\phi_0$, while the vector field V_0 shifts the energy spectrum. We look for normal-mode solutions with both positive and negative energies, as is natural for the Dirac equation. These solutions can be used to define quantum field operators ψ and $\psi^\dagger$ in the usual fashion, and by imposing the familiar equal-time anticommutation relations, we can construct the baryon number operator $\hat{B} \equiv \int \mathrm{d}^3x \, (:\overline{\psi}\gamma^0\psi:)$ and the four-momentum operators $\hat{P}^\mu = (\hat{H}, \hat{\mathbf{P}}) \equiv \int \mathrm{d}^3x \, \hat{T}^{0\mu}$, with the results

$$\hat{H} - \langle 0|\hat{H}|0\rangle \equiv \hat{H}_{\mathrm{MFT}} + \delta H \ , \qquad (2.10)$$

$$\hat{H}_{\mathrm{MFT}} = \sum_{\mathbf{k}\lambda}(\mathbf{k}^2 + M^{*2})^{1/2}(A_{\mathbf{k}\lambda}^\dagger A_{\mathbf{k}\lambda} + B_{\mathbf{k}\lambda}^\dagger B_{\mathbf{k}\lambda}) + g_{\mathrm{v}}V_0\hat{B}$$

$$+ (\tfrac{1}{2}m_{\mathrm{s}}^2\phi_0^2 + \tfrac{1}{3!}\kappa\phi_0^3 + \tfrac{1}{4!}\lambda\phi_0^4 - \tfrac{1}{2}m_{\mathrm{v}}^2V_0^2)V \ , \qquad (2.11)$$

$$\delta H = -\sum_{\mathbf{k}\lambda}\left[(\mathbf{k}^2 + M^{*2})^{1/2} - (\mathbf{k}^2 + M^2)^{1/2}\right] \ , \qquad (2.12)$$

$$\hat{B} = \sum_{\mathbf{k}\lambda}(A_{\mathbf{k}\lambda}^\dagger A_{\mathbf{k}\lambda} - B_{\mathbf{k}\lambda}^\dagger B_{\mathbf{k}\lambda}) \ , \qquad (2.13)$$

$$\hat{\mathbf{P}} = \sum_{\mathbf{k}\lambda}\mathbf{k}(A_{\mathbf{k}\lambda}^\dagger A_{\mathbf{k}\lambda} + B_{\mathbf{k}\lambda}^\dagger B_{\mathbf{k}\lambda}) \ . \qquad (2.14)$$

Here $A_{\mathbf{k}\lambda}^\dagger$, $B_{\mathbf{k}\lambda}^\dagger$, $A_{\mathbf{k}\lambda}$, and $B_{\mathbf{k}\lambda}$ are creation and destruction operators for (quasi)baryons and (quasi)antibaryons with shifted mass and energy, and $\hat{B}$ is the "normal-ordered" baryon number operator, which clearly counts the number of baryons minus the number of antibaryons. (The index λ denotes both spin and isospin projections.) The correction term δH arises from placing the operators in $\hat{H}_{\mathrm{MFT}}$ in normal order and includes the contribution to the energy from the filled Dirac sea, where the baryon mass has been shifted by the uniform scalar field ϕ_0 [Se86]. Since all energies are measured relative to the vacuum, we

must subtract the total energy of the Dirac sea in the vacuum state $|0\rangle$, where the baryons have their free mass M. We will return later to discuss this "zero-point energy" correction; for now, let us concentrate on the MFT hamiltonian defined by Eq. (2.11).

Since $\hat{H}_{\mathrm{MFT}}$ is diagonal, this model mean-field problem has been solved *exactly* once the meson fields are specified; their determination is discussed below. The solution retains the essential features of QHD: explicit mesonic degrees of freedom, consistency with relativistic covariance [Fu90], and the incorporation of antiparticles. Since $\hat{B}$ and $\hat{\mathbf{P}}$ are also diagonal, the baryon number and total momentum are constants of the motion, as are their corresponding densities ρ_{B} and $\boldsymbol{\mathcal{P}}$, since the volume is fixed.

For uniform nuclear matter, the ground state is obtained by filling energy levels with spin-isospin degeneracy γ up to the Fermi momentum k_{F}. (The generalization to finite temperature will be discussed at the end of this section.) The Fermi momentum is related to the baryon density $\rho_{\mathrm{B}} \equiv B/V$ by

$$\rho_{\mathrm{B}} = \frac{\gamma}{(2\pi)^3} \int_0^{k_{\mathrm{F}}} \mathrm{d}^3k = \frac{\gamma}{6\pi^2} k_{\mathrm{F}}^3 , \qquad (2.15)$$

where the degeneracy factor is 4 for symmetric ($N = Z$) matter and 2 for pure neutron matter ($Z = 0$). The constant vector field V_0 can be expressed in terms of conserved quantities from the expectation value of the vector meson field equation (2.3):

$$V_0 = \frac{g_{\mathrm{v}}}{m_{\mathrm{v}}^2} \rho_{\mathrm{B}} . \qquad (2.16)$$

The expressions for the energy density and pressure now take the simple forms [Se86]

$$\mathcal{E} = \frac{g_{\mathrm{v}}^2}{2m_{\mathrm{v}}^2} \rho_{\mathrm{B}}^2 + \frac{m_{\mathrm{s}}^2}{2g_{\mathrm{s}}^2}(M - M^*)^2 + \frac{\kappa}{6g_{\mathrm{s}}^3}(M - M^*)^3 + \frac{\lambda}{24g_{\mathrm{s}}^4}(M - M^*)^4$$

$$+ \frac{\gamma}{(2\pi)^3} \int_0^{k_{\mathrm{F}}} \mathrm{d}^3k \; E^*(k) , \qquad (2.17)$$

$$p = \frac{g_{\mathrm{v}}^2}{2m_{\mathrm{v}}^2} \rho_{\mathrm{B}}^2 - \frac{m_{\mathrm{s}}^2}{2g_{\mathrm{s}}^2}(M - M^*)^2 - \frac{\kappa}{6g_{\mathrm{s}}^3}(M - M^*)^3 - \frac{\lambda}{24g_{\mathrm{s}}^4}(M - M^*)^4$$

$$+ \frac{1}{3} \frac{\gamma}{(2\pi)^3} \int_0^{k_{\mathrm{F}}} \mathrm{d}^3k \; \frac{\mathbf{k}^2}{E^*(k)} , \qquad (2.18)$$

where $E^*(k) \equiv (\mathbf{k}^2 + M^{*2})^{1/2}$. The first four terms in Eqs. (2.17) and (2.18) arise from the classical meson fields. The final terms in these equations are those of a relativistic gas of baryons of mass M^*. These expressions give the nuclear matter equation of state at zero temperature in parametric form: $\mathcal{E}(\rho_{\mathrm{B}})$ and $p(\rho_{\mathrm{B}})$.

The constant scalar field ϕ_0, or equivalently, the effective mass M^*, can be determined thermodynamically at the end of the calculation by minimizing $\mathcal{E}(M^*)$ with respect to M^*. This produces the *self-consistency condition*

$$M^* = M - \frac{g_{\mathrm{s}}^2}{m_{\mathrm{s}}^2} \rho_{\mathrm{s}} + \frac{\kappa}{2g_{\mathrm{s}}m_{\mathrm{s}}^2}(M - M^*)^2 + \frac{\lambda}{6g_{\mathrm{s}}^2 m_{\mathrm{s}}^2}(M - M^*)^3 , \qquad (2.19)$$
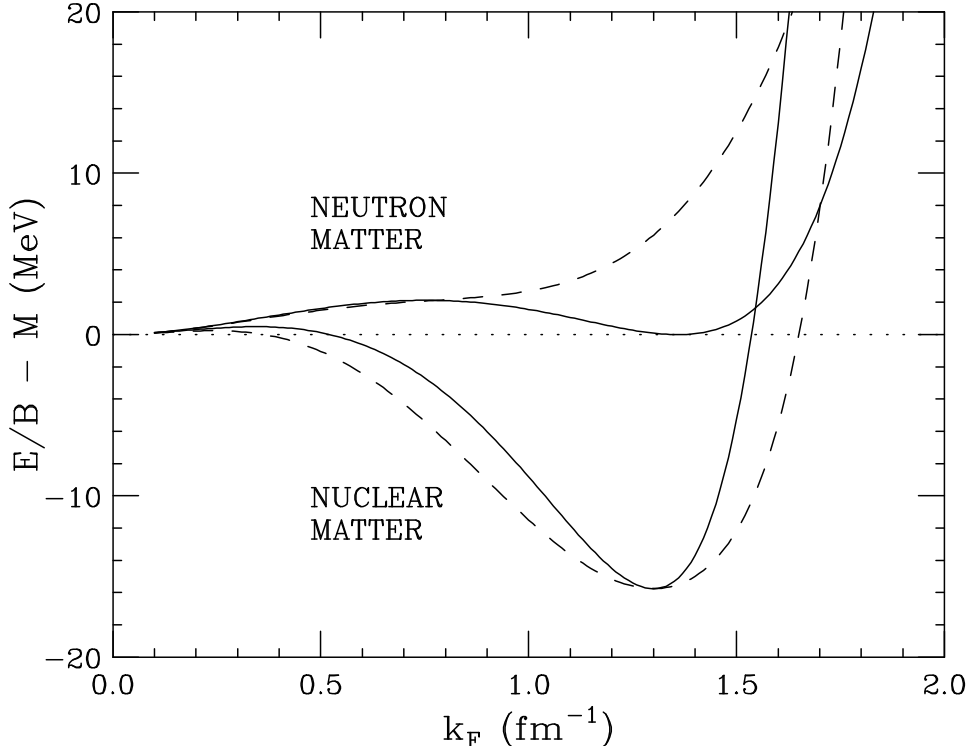
FIG. 1. Saturation curves for nuclear matter. These results are calculated in the relativistic mean-field theory with baryons and neutral scalar and vector mesons (QHD–I). The coupling constants are chosen to fit the value and position of the minimum. The solid curve uses the couplings in Eq. (2.21), while the dashed curve uses the parameter set NLC, as described later in the text. The predictions for neutron matter ($\gamma = 2$) are also shown.

where the scalar density ρ_s is defined by

$$\rho_s \equiv \langle : \overline{\psi}\psi : \rangle = \frac{\gamma}{(2\pi)^3} \int_0^{k_F} d^3k \, \frac{M^*}{E^*(k)} \ . \tag{2.20}$$

Equation (2.19) is equivalent to the MFT scalar field equation for ϕ_0. Note that the scalar density is smaller than the baryon density [Eq. (2.15)] due to the factor $M^*/E^*(k)$, which is an effect of Lorentz contraction. Thus the contribution of rapidly moving baryons to the scalar source is significantly reduced. Most importantly, Eq. (2.19) is a *transcendental self-consistency equation* for M^* that must be solved at each value of k_F. This illustrates the *nonperturbative* nature of the mean-field solution.

To analyze these results, we initially set $\kappa = \lambda = 0$, as in the original version of the model [Wa74]. An examination of the analytic expression (2.17) for the energy density shows that the system is unbound ($\mathcal{E}/\rho_B > M$) at either very low or very high densities. At intermediate densities, the attractive scalar interaction will dominate if the coupling constants are chosen properly. The system then *saturates*. Nuclear matter with an equilibrium Fermi wavenumber $k_F^0 = 1.30 \, \text{fm}^{-1}$ and an energy/nucleon $e_0 \equiv (\mathcal{E}/\rho_B - M) = -15.75 \, \text{MeV}$ is obtained if the
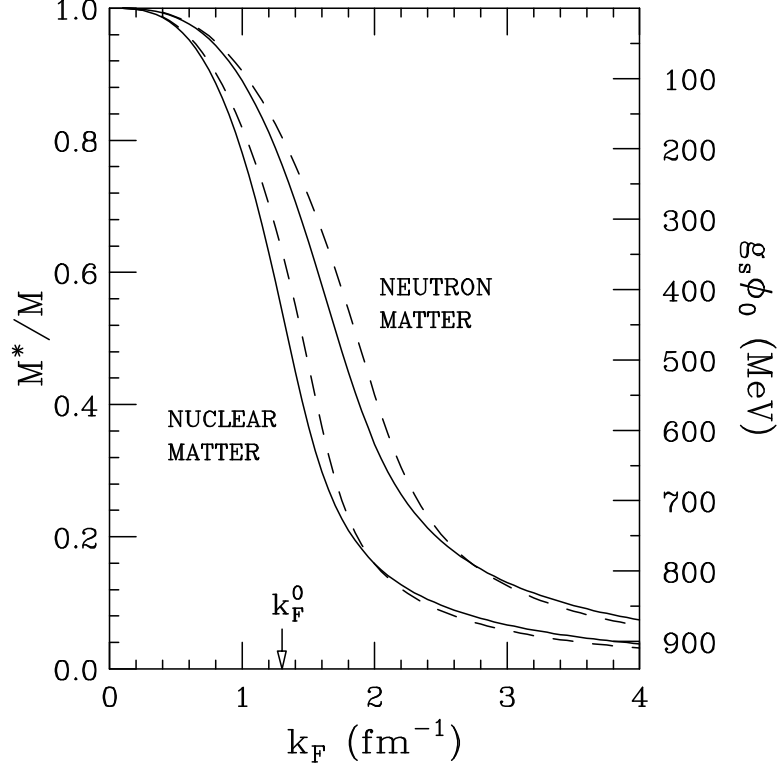
FIG. 2. Effective mass as a function of density for nuclear ($\gamma = 4$) and neutron ($\gamma = 2$) matter based on Fig. 1.

couplings are chosen as[3]

$$C_{\mathrm{s}}^2 \equiv g_{\mathrm{s}}^2 \Big(\frac{M^2}{m_{\mathrm{s}}^2}\Big) = 357.4, \qquad C_{\mathrm{v}}^2 \equiv g_{\mathrm{v}}^2 \Big(\frac{M^2}{m_{\mathrm{v}}^2}\Big) = 273.8 \ . \tag{2.21}$$

In this approximation, the nuclear compression modulus K is $545 \,\mathrm{MeV}$. Note that the meson masses enter only through the *ratios* g_i^2/m_i^2 in Eqs. (2.17), (2.18), and (2.19). (For a more complete discussion of the relevant dimensional coupling parameters and their specification from nuclear matter properties, see [Fu96].) The resulting saturation curve is shown in Fig. 1. In this approximation, the relativistic properties of the scalar and vector fields are responsible for saturation; a Hartree–Fock variational estimate built on the nonrelativistic (Yukawa) potential limit of the interaction shows that such a system is unstable against collapse [Fe71,Wa95].

The solution of the self-consistency condition (2.19) for M^* yields an effective mass that is a decreasing function of the density, as illustrated in Fig. 2. Evidently, M^*/M becomes small at high density and is significantly less than unity at ordinary nuclear densities ($M^*/M = 0.541$ at $k_{\mathrm{F}} = k_{\mathrm{F}}^0$). This is a consequence of the large scalar field $g_{\mathrm{s}}\phi_0$, which is approximately 400 MeV (at $k_{\mathrm{F}} = k_{\mathrm{F}}^0$) and which produces a large attractive contribution

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[3]The values $C_{\mathrm{s}}^2 = 267.1$ and $C_{\mathrm{v}}^2 = 195.9$ used in [Se86] yield $k_{\mathrm{F}}^0 = 1.42 \,\mathrm{fm}^{-1}$.
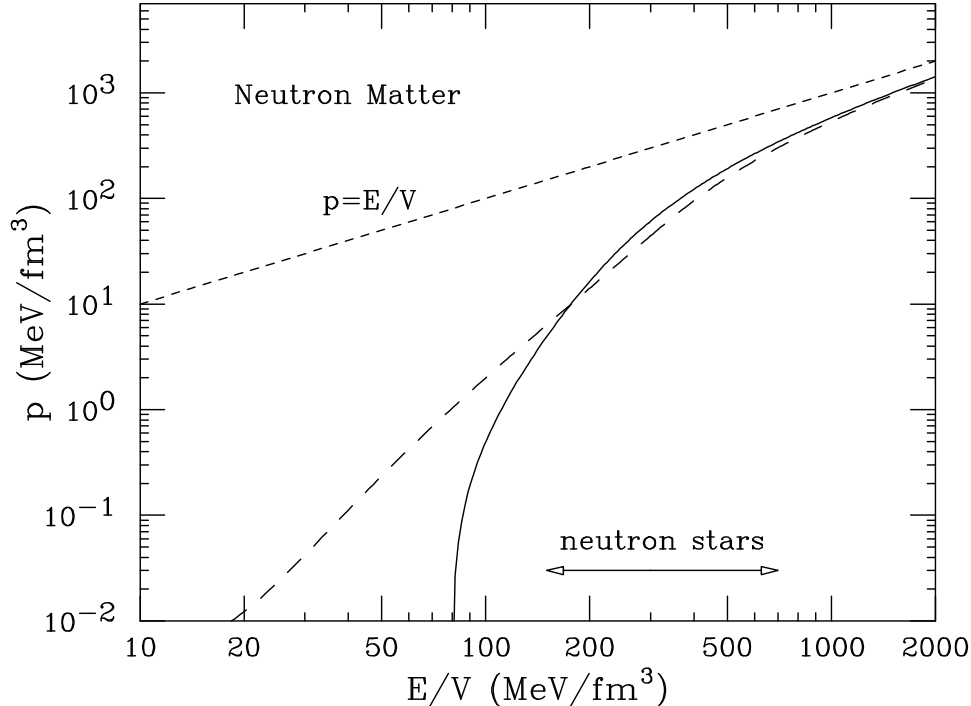
FIG. 3. Predicted equation of state for neutron matter at all densities. The solid and dashed curves show the result for QHD–I based on Fig. 1. The dotted line represents the causal limit $p = \mathcal{E}$. The density regime relevant for neutron stars is also shown.

to the energy/baryon. There is also a large repulsive energy/baryon from the vector field $g_v V_0 \approx 350 \, \text{MeV}$. Thus *the Lorentz structure of the interaction leads to a new energy scale in the problem,* and the small nuclear binding energy ($\approx 16 \, \text{MeV}$) arises from the cancellation between the large scalar attraction and vector repulsion. As the nuclear density increases, M^* decreases, the scalar source ρ_s becomes smaller than the vector source ρ_B, and the attractive forces saturate, producing the minimum in the binding curve.[4] Clearly, because of the sensitive cancellation involved near the equilibrium density, corrections to the MFT must be calculated before the importance of this saturation mechanism can be assessed. Nevertheless, the Lorentz structure of the interaction provides *a new saturation mechanism that is not present in the nonrelativistic potential limit,* as this limit ignores the distinction between Lorentz scalar and vector fields. We will see later that part of this new saturation mechanism can be expressed in terms of repulsive *many-body* forces in a nonrelativistic formulation.

The corresponding curves for neutron matter obtained by setting $\gamma = 2$ are also shown in Figs. 1 and 2, and the equation of state (pressure *vs.* energy density) for neutron matter at all densities is given in Fig. 3. At high densities, the system approaches the "causal limit" $p = \mathcal{E}$ representing the stiffest possible equation of state. Thus we have a simple, two-parameter model that is consistent with the equilibrium point of normal nuclear matter

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[4]Figure 15 in [Se86] shows the decrease in ρ_s relative to ρ_B for $0 < k_F \leq k_F^0$.

and that allows for a covariant, causal extrapolation to any density.

Nevertheless, the two-parameter (linear) model is fit only to the equilibrium point of nuclear matter; the values of M^* and K at equilibrium are predictions. For example, for a given Fermi wavenumber k_F^0 and energy/nucleon e_0 at equilibrium, M^* at equilibrium must satisfy [Fu96]

$$(e_0 + M + \sqrt{(k_F^0)^2 + M^{*2}})\rho_B^0 - (M - M^*)\rho_s^0 - \frac{2\gamma}{(2\pi)^3}\int_0^{k_F^0} d^3k\, E^*(k) = 0 \ . \qquad (2.22)$$

As will be shown in the next subsection, the properties of finite nuclei place significant constraints on both M^* and K, and the linear model predicts too small a value for the former and too large a value for the latter. Moreover, the absence of isovector mesons in QHD–I leads to a bulk symmetry energy that is too small, and consequently, the repulsive forces in neutron matter are underestimated. This can be corrected by introducing a mean field for the isovector $\boldsymbol{\rho}$ meson, as we describe shortly. Thus, for comparison, we also show in Figs. 1 through 3 the more realistic MFT results obtained in a nonlinear model (NLC), with parameters fit to the properties of nuclei and given in Table I, below. The important point is that the additional parameters allow for small adjustments of the nuclear properties near equilibrium, *but the basic features implied by the large scalar and vector fields remain intact* [Wa74,Se86,Bo89,Bo91,Fu96].

B. Finite Nuclei

We now generalize the results of the preceding subsection to study atomic nuclei. We continue to work in the mean-field approximation to QHD–I, but since the system now has finite spatial extent, these fields are spatially dependent.[5] If we initially restrict consideration to spherically symmetric nuclei, the meson fields depend only on the radius, and since the baryon current is conserved, the spatial part of the vector field $\mathbf{V}$ again vanishes [Se86]. Thus the mean-field QHD–I lagrangian of Eq. (2.7) becomes

$$\mathcal{L}_{\mathrm{MFT}}^{(I)} = \overline{\psi}[i\gamma_\mu\partial^\mu - g_v\gamma_0 V_0 - (M - g_s\phi_0)]\psi - \tfrac{1}{2}[(\boldsymbol{\nabla}\phi_0)^2 + m_s^2\phi_0^2]$$
$$- \tfrac{1}{3!}\kappa\phi_0^3 - \tfrac{1}{4!}\lambda\phi_0^4 + \tfrac{1}{2}[(\boldsymbol{\nabla}V_0)^2 + m_v^2 V_0^2] \ , \qquad (2.23)$$

and the Dirac equation for the baryon field is

$$\{i\gamma_\mu\partial^\mu - g_v\gamma_0 V_0(r) - [M - g_s\phi_0(r)]\}\psi(x) = 0 \ . \qquad (2.24)$$

Appropriate values for the scalar and vector couplings (g_s and g_v), masses (m_s and m_v), and nonlinear parameters (κ and λ) will be given below.

Although the baryon field is still an operator, the meson fields are classical; hence Eq. (2.24) is linear, and we may again seek normal-mode solutions of the form $\psi(x) = \psi(\mathbf{x})\,e^{-iEt}$. This leads to the eigenvalue equation

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[5]As usual in discussions of nuclear structure, calculations will be carried out in a frame where the nucleus is at rest.

$$h\psi(\mathbf{x}) \equiv \{-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + g_{\mathrm{v}}V_0(r) + \beta[M - g_{\mathrm{s}}\phi_0(r)]\}\psi(\mathbf{x}) = E\psi(\mathbf{x}) , \qquad (2.25)$$

which defines the single-particle Dirac hamiltonian h, with $\boldsymbol{\alpha}$ and β the usual Dirac matrices. Equation (2.25) has both positive- and negative-energy solutions $\mathcal{U}(\mathbf{x})$ and $\mathcal{V}(\mathbf{x})$, which allow the field operators to be constructed in the Schrödinger picture. The positive-energy spinors can be written as

$$\mathcal{U}_\alpha(\mathbf{x}) \equiv \mathcal{U}_{nkmt}(\mathbf{x}) = \begin{pmatrix} i[G_{nkt}(r)/r]\Phi_{km} \\ -[F_{nkt}(r)/r]\Phi_{-km} \end{pmatrix} \zeta_t , \qquad (2.26)$$

where n is the principal quantum number, Φ_{km} is a spin-1/2 spherical harmonic [Ed57], and ζ_t is a two-component isospinor labeled by the isospin projection t. (We take $t = \frac{1}{2}$ for protons and $t = -\frac{1}{2}$ for neutrons.) The phase choice in Eq. (2.26) produces real bound-state wave functions F and G for real potentials ϕ_0 and V_0, and the normalization is given by

$$\int_0^\infty \mathrm{d}r(|G_\alpha(r)|^2 + |F_\alpha(r)|^2) = 1 , \qquad (2.27)$$

which ensures unit probability to find each nucleon somewhere in space.

The classical meson field equations follow from Eq. (2.23) and resemble Eqs. (2.2) and (2.3) restricted to static, spherically symmetric fields. With the general form for the spinors in Eq. (2.26), we can evaluate the nuclear densities, which serve as source terms in the meson field equations. Assume that the nuclear ground state consists of filled shells up to some value of n and k, which may be different for protons and neutrons; this is appropriate for doubly magic nuclei. In addition, assume that all bilinear products of baryon operators are normal ordered, which removes contributions from the negative-energy spinors $\mathcal{V}_\alpha(\mathbf{x})$. This amounts to neglecting the filled Dirac sea of baryons and defines the mean-field approximation. The contributions from the Dirac sea to nuclear matter will be considered in Section 5.

With these assumptions, the local baryon (ρ_{B}) and scalar (ρ_{s}) densities become

$$\begin{pmatrix} \rho_{\mathrm{B}}(\mathbf{x}) \\ \rho_{\mathrm{s}}(\mathbf{x}) \end{pmatrix} = \sum_\alpha^{\mathrm{occ}} \overline{\mathcal{U}}_\alpha(\mathbf{x}) \begin{pmatrix} \gamma^0 \\ 1 \end{pmatrix} \mathcal{U}_\alpha(\mathbf{x}) = \sum_a^{\mathrm{occ}} \left(\frac{2j_a + 1}{4\pi r^2} \right) (|G_a(r)|^2 \pm |F_a(r)|^2) , \qquad (2.28)$$

which holds for filled shells, as appropriate for spherically symmetric nuclei. The remaining quantum numbers are denoted by $\{\alpha\} = \{a; m\} \equiv \{n, k, t; m\}$, and the nonzero integer k determines j and ℓ through $k = (2j + 1)(\ell - j)$. Notice that since the shells are filled, the sources are spherically symmetric.

The sources produce the meson fields, which satisfy static Klein–Gordon equations:

$$\frac{\mathrm{d}^2}{\mathrm{d}r^2}\phi_0(r) + \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r}\phi_0(r) - m_{\mathrm{s}}^2\phi_0(r) - \frac{\kappa}{2}\phi_0^2(r) - \frac{\lambda}{6}\phi_0^3(r) = -g_{\mathrm{s}}\rho_{\mathrm{s}}(r) , \qquad (2.29)$$

$$\frac{\mathrm{d}^2}{\mathrm{d}r^2}V_0(r) + \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r}V_0(r) - m_{\mathrm{v}}^2V_0(r) = -g_{\mathrm{v}}\rho_{\mathrm{B}}(r) . \qquad (2.30)$$

The equations for the baryon wave functions follow upon substituting Eq. (2.26) into Eq. (2.25), which produces

$$\frac{\mathrm{d}}{\mathrm{d}r}G_a(r) + \frac{k}{r}G_a(r) - [E_a - g_{\mathrm{v}}V_0(r) + M - g_{\mathrm{s}}\phi_0(r)]F_a(r) = 0 , \qquad (2.31)$$

$$\frac{\mathrm{d}}{\mathrm{d}r}F_a(r) - \frac{k}{r}F_a(r) + [E_a - g_{\mathrm{v}}V_0(r) - M + g_{\mathrm{s}}\phi_0(r)]G_a(r) = 0 . \qquad (2.32)$$

Thus the spherical nuclear ground state is described by coupled, ordinary differential equations that may be solved by an iterative procedure, as discussed in [Ho81,Fu87]. They contain all information about the static ground-state nucleus in this approximation.

The mean-field hamiltonian can be computed just as for infinite matter, and after normal ordering, the ground-state energy is given by

$$E = \int \mathrm{d}^3x \left\{ \frac{1}{2}[(\boldsymbol{\nabla}\phi_0)^2 + m_{\mathrm{s}}^2\phi_0^2] - \frac{1}{2}[(\boldsymbol{\nabla}V_0)^2 + m_{\mathrm{v}}^2V_0^2] + \frac{\kappa}{3!}\phi_0^3 + \frac{\lambda}{4!}\phi_0^4 \right.$$
$$\left. + \sum_\alpha^{\mathrm{occ}} \mathcal{U}_\alpha^\dagger(\mathbf{x})[-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta(M - g_{\mathrm{s}}\phi_0) + g_{\mathrm{v}}V_0]\mathcal{U}_\alpha(\mathbf{x}) \right\} . \qquad (2.33)$$

Here the meson fields are functions of the radial coordinate. Notice that if we interpret this expression as an *energy functional* for the Dirac–Hartree ground state, extremization with respect to the meson fields reproduces the field equations (2.29) and (2.30), with the densities from Eq. (2.28). Moreover, extremization with respect to the baryon wave functions $\mathcal{U}_\alpha^\dagger(\mathbf{x})$, subject to the constraint

$$\int \mathrm{d}^3x \; \mathcal{U}_\alpha^\dagger(\mathbf{x})\mathcal{U}_\alpha(\mathbf{x}) = 1 \qquad (2.34)$$

for all occupied states (which is enforced by Lagrange multipliers E_α), leads to the Dirac equation (2.25). This alternative derivation of the Dirac–Hartree equations from an energy functional is useful for extensions of the simple model discussed above [Fu96].

Once the solutions to the Dirac–Hartree equations have been found, the ground-state energy can be computed by using the Dirac equation (2.25) to introduce the eigenvalues E_a and by partially integrating the meson terms to introduce the densities. In the end, the total energy of the system is given by

$$E = \sum_a^{\mathrm{occ}} E_a(2j_a + 1) - \frac{1}{2}\int \mathrm{d}^3x[-g_{\mathrm{s}}\phi_0(r)\rho_{\mathrm{s}}(r) + g_{\mathrm{v}}V_0(r)\rho_{\mathrm{B}}(r)] - \frac{1}{12}\int \mathrm{d}^3x[\kappa\phi_0^3(r) + \tfrac{1}{2}\lambda\phi_0^4(r)] .$$
$$(2.35)$$

Before discussing the Dirac–Hartree solutions, let us generalize the equations to include some additional degrees of freedom and couplings. Although the isoscalar meson fields are the most important for describing general properties of nuclear matter, a quantitative comparison with actual nuclei requires the introduction of some additional dynamics.

For example, it is necessary to include the electromagnetic interaction to account for the Coulomb repulsion between protons. Moreover, since hadronic interactions exhibit an almost exact SU(2) isospin symmetry, the nucleons can couple to isovector mesons in addition to the isoscalar (neutral) mesons of QHD–I. These isovector mesons, for example the $\boldsymbol{\rho}$ and $\boldsymbol{\pi}$, come in three charge states $(+, 0, -)$ and couple differently to the proton and neutron.

TABLE I. Dirac–Hartree Parameter Sets. Note that $m_{\rm s}$ and κ are in MeV.

Set	$g_{\rm s}^2$	$g_{\rm v}^2$	g_ρ^2	$m_{\rm s}$	κ	λ
L2	109.63	190.43	65.23	520.	0	0
NLB	94.01	158.48	73.00	510.0	800	10
NLC	95.11	148.93	74.99	500.8	5000	-200

Thus they affect the nuclear symmetry energy, which arises when there are unequal numbers of neutrons and protons.

The construction of a renormalizable lagrangian containing charged, massive vector fields is somewhat complicated and is discussed at length in Abers and Lee [Ab73]; applications to the present model can be found in [Se86]. For our purposes, we require only the classical contributions from these fields, and in this case, the lagrangian simplifies considerably. In particular, since the nuclear ground state has well-defined charge, only the neutral rho meson field (denoted by b_0) enters, and since the ground state is assumed to have well-defined parity and spherical symmetry, there is no classical pion field. Thus the mean-field lagrangian for this extended model, which we call QHD–II, is given by

$$\mathcal{L}_{\rm MFT}^{\rm (II)} = \overline{\psi}[i\gamma_\mu\partial^\mu - g_{\rm v}\gamma_0 V_0 - g_\rho\tfrac{1}{2}\tau_3\gamma_0 b_0 - e\tfrac{1}{2}(1+\tau_3)\gamma_0 A_0 - (M - g_{\rm s}\phi_0)]\psi$$
$$- \tfrac{1}{2}[(\boldsymbol{\nabla}\phi_0)^2 + m_{\rm s}^2\phi_0^2] - \tfrac{1}{3!}\kappa\phi_0^3 - \tfrac{1}{4!}\lambda\phi_0^4 + \tfrac{1}{2}[(\boldsymbol{\nabla}V_0)^2 + m_{\rm v}^2 V_0^2]$$
$$+ \tfrac{1}{2}(\boldsymbol{\nabla}A_0)^2 + \tfrac{1}{2}[(\boldsymbol{\nabla}b_0)^2 + m_\rho^2 b_0^2] \ . \tag{2.36}$$

Here A_0 is the Coulomb potential, e is the proton electromagnetic charge, g_ρ is the rho-nucleon coupling constant, and τ_i are the usual isospin Pauli matrices. For now, all of the boson fields are assumed to be functions of the radial coordinate only.

The Dirac–Hartree equations for this extended model can be derived just as before. The Dirac equations for the baryon wave functions now contain b_0 and A_0, and because of the structure of the τ_3 matrix, b_0 couples with opposite sign to protons and neutrons, and A_0 couples only to the protons. In addition to the source terms in Eq. (2.28), which sum over both proton and neutron occupied states, the source term for the ρ meson involves the *difference* between proton and neutron densities, while the Coulomb source involves only protons. These different types of couplings allow for a more accurate reproduction of real nuclei, where the proton and neutron wave functions are not identical. The full set of equations are presented in [Se86] and are used to compute the results discussed below.

1. Spherical Nuclei

The solutions of the preceding equations depend on the parameters $g_{\rm s}$, $g_{\rm v}$, $m_{\rm s}$, and g_ρ (when the ρ meson is included); κ and λ will be set to zero in this subsection. We take the experimental values $M = 939\,{\rm MeV}$, $m_{\rm v} = m_\omega = 783\,{\rm MeV}$, $m_\rho = 770\,{\rm MeV}$, and $e^2/4\pi = \alpha = 1/137.036$ (which determines the Coulomb potential) as fixed. The free parameters can be chosen by requiring that when the Dirac–Hartree equations are solved in the limit of infinite nuclear matter, the empirical equilibrium density ($\rho_{\rm B}^0 = 0.1484\ {\rm fm}^{-3}$),
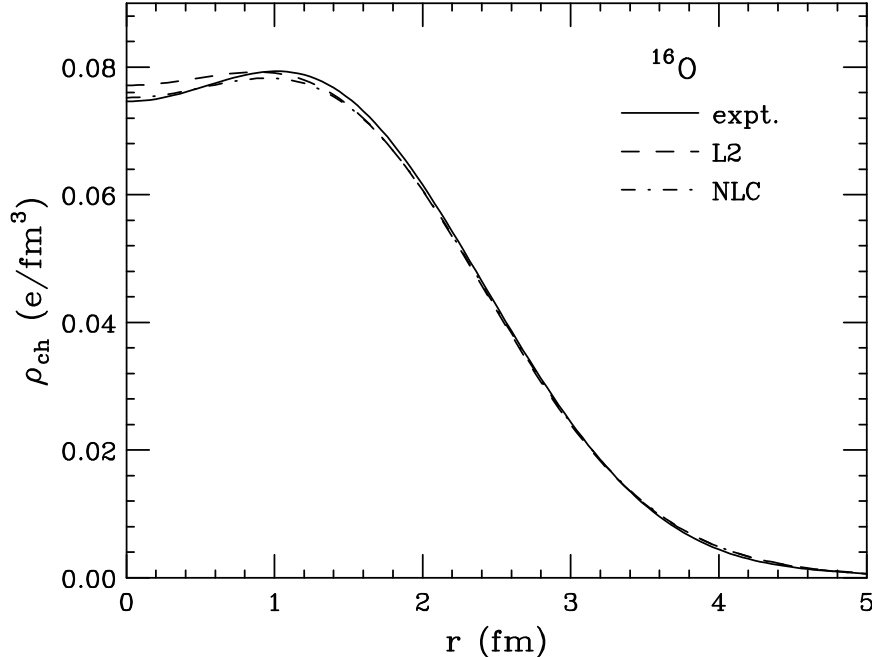
FIG. 4. Charge density distribution for ^{16}O. The experimental curve is from [De87]. The Dirac–Hartree calculations for parameter set L2 yield the long-dashed curve, while those from set NLC yield the dot-dashed curve.

energy/nucleon ($e_0 = -15.75$ MeV), and bulk symmetry energy (35 MeV) are reproduced.[6] The empirical equilibrium density is determined here from the density in the interior of ^{208}Pb and corresponds to $k_{\mathrm{F}}^0 = 1.30\,\mathrm{fm}^{-1}$. We also fit the empirical rms charge radius of ^{40}Ca ($r_{\mathrm{rms}} = 3.482$ fm), which is determined primarily by m_{s}. This procedure produces the parameters in the row labeled L2 in Table I, which are taken from [Ho81]. This parameter set yields the same values for C_{s}^2 and C_{v}^2 as in Eq. (2.21), so that $M^*/M = 0.541$ and $K \approx 545\,\mathrm{MeV}$ at equilibrium.

Once the parameters have been specified, the properties of all closed-shell nuclei are determined in this approximation. For example, Figs. 4 through 6 show the Dirac–Hartree charge densities of ^{16}O, ^{40}Ca, and ^{208}Pb compared with the empirical distributions determined from electron scattering [De87]. The empirical proton charge form factor has been folded with the calculated "point proton" density to determine the charge density.

In Fig. 7, the predicted energy levels in ^{208}Pb are compared with experimental values derived from neighboring nuclei [Bo69,Ra79]. The relativistic calculations clearly reveal a shell structure; the level orderings and major shell closures of the nuclear shell model are correctly reproduced. This successful result arises from the spin-orbit interaction that occurs naturally when a Dirac particle moves in large, spatially varying classical scalar and

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[6]The number of significant digits in the empirical input values is not intended to indicate how accurately these quantities are known. We are merely reporting the precise values used in [Ho81] to determine the model parameters.
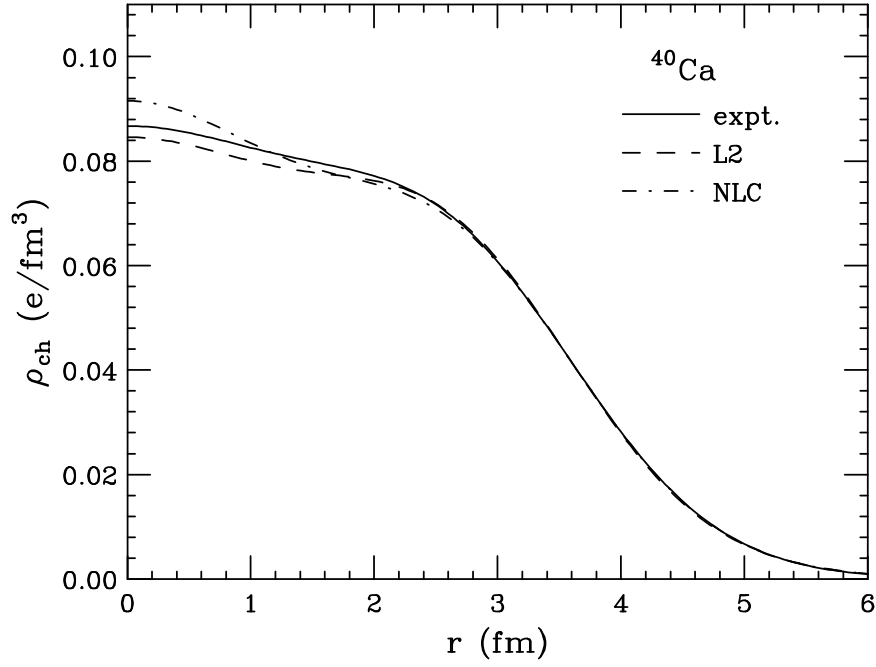
FIG. 5. Charge density distribution for ^{40}Ca. The experimental curve is from [De87]. The Dirac–Hartree calculations for parameter set L2 yield the long-dashed curve, while those from set NLC yield the dot-dashed curve.
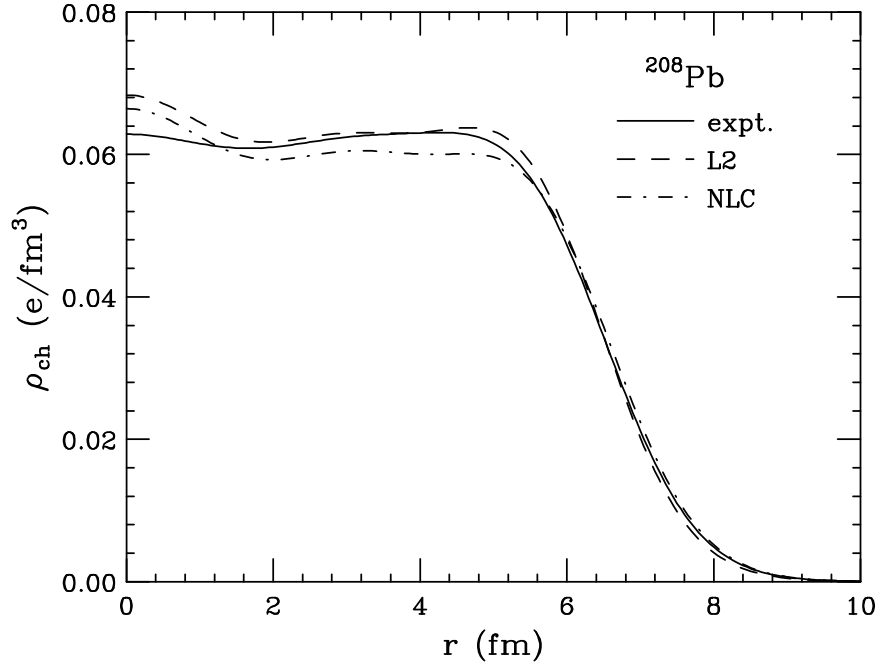

FIG. 6. Charge density distribution for ^{208}Pb. The solid curve is from [De87]. Dirac–Hartree results are indicated by the long-dashed curve (set L2) and the dot-dashed curve (set NLC).
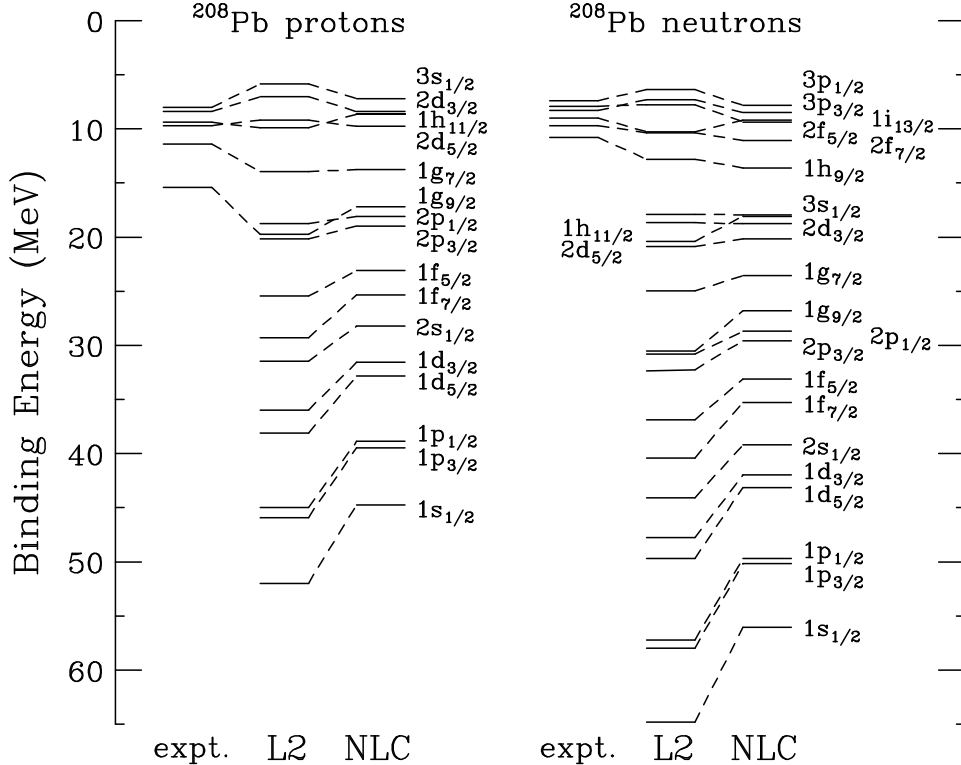
FIG. 7. Predicted spectrum for occupied single-particle levels in ^{208}Pb. Experimental values are taken from neighboring nuclei.

vector fields [Fu36,Kr73,Kr74,Mi74,Mi75]. Note that whereas $g_s\phi_0$ and $g_v V_0$ tend to *cancel* in the central potential that determines saturation, they *add* constructively in the spin-orbit potential. We emphasize that no parameters are adjusted specifically to produce the spin-orbit interaction, as is usually the case in nonrelativistic calculations. Thus, with a minimal number of phenomenological parameters determined from bulk nuclear properties, *one derives the level structure of the nuclear shell model.*

We emphasize that in this relativistic model of nuclear structure, the calculation of the ground state is *self-consistent.* The scalar and vector fields follow directly from the scalar and baryon densities, which are in turn determined by the solutions to the Dirac equation (2.25) in the classical fields. Moreover, this relativistic shell model arises from a simple approximation to the underlying QHD lagrangian. Thus one has a consistent many-body framework to systematically investigate corrections.

2. Deformed Nuclei

To study the systematics of this relativistic model of nuclear structure, we extend the preceding equations to deal with deformed, axially symmetric nuclei. This allows us to calculate not only the ground states of nuclei with fully closed shells, but also those for even-even nuclei between closed shells. We will concentrate here on nuclei with $12 \leq B \leq 40$, which includes the $1p$ and $2s$–$1d$ shells [Fe71]. The restriction to azimuthal and reflection-symmetric deformations is reasonable for light, even-even nuclei.

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These assumed symmetries of the ground state, together with the assumption of well-defined parity, imply that the nonvanishing meson fields are the same ones that appear in spherical nuclei [Pr87]. Thus the Dirac–Hartree equations are essentially the same as those written earlier, except that all fields now depend on both a radial and an angular coordinate [for example, $\phi_0(r, \theta)$], and all the differential equations become partial differential equations. The source densities are still computed as in Eq. (2.28), but they now depend on r and θ. There are several methods for solving the resulting set of coupled partial differential equations, and the interested reader is directed to the literature for a discussion [Le86,Pa87,Fu87,Zh91a]. The equilibrium deformation is obtained by choosing the occupied single-particle states to minimize the energy.

The results of Furnstahl, Price, and Walker [Fu87] for quadrupole moments in the $2s$–$1d$ shell show how this observable can constrain the properties of nuclear matter near equilibrium density. These authors use the full complement of parameters in the present model and constrain them to produce nuclear saturation at $k_{\mathrm{F}}^0 = 1.30\,\mathrm{fm}^{-1}$ with a binding energy of 15.75 MeV and a bulk symmetry energy of 35 MeV, as well as the correct rms radius for ^{40}Ca, just as for the calculations of spherical nuclei in the preceding subsection. An infinite number of parameter sets will satisfy these constraints, and two examples (NLB and NLC) are shown in Table I. Both of these sets produce roughly equal values for M^* at nuclear matter equilibrium ($M^*/M = 0.61$ for NLB and $M^*/M = 0.63$ for NLC), so they generate similar spin-orbit splittings and deformations. As seen in Figs. 4–7, NLC accurately reproduces the nuclear charge densities and splittings for spherical nuclei (results for NLB are similar). Moreover, the agreement with experimental quadrupole moments is excellent, particularly the systematic trends and the oscillation between oblate and prolate shapes around $B = 32$ (see Fig. 7 in [Fu87]). Thus the successful description of spherical nuclei in this relativistic model can be extended to reproduce the observed systematics of light deformed nuclei *with the same parameters*.

These authors also observe, however, that if one uses the set L2 to compute the quadrupole moments, the smaller value of $M^*/M = 0.54$ at equilibrium leads to a prediction of *spherical* shapes for ^{12}C, ^{28}Si, and ^{32}S, which are inconsistent with B(E2; $0^+ \rightarrow 2^+$) values derived from experiment [Le75]. Thus, the sensitive dependence of the deformation on the level density near the Fermi surface (which is determined essentially by the inverse of M^*) allows one to conclude that the small adjustments in equilibrium properties afforded by the nonlinear scalar couplings κ and λ are significant. A similar analysis of the $2s$–$1d$ shell binding energies (see Fig. 10 in [Fu87]) shows that compression moduli greater than roughly 350 MeV are too large, which is true of sets L2 ($K \approx 545\,\mathrm{MeV}$) and NLB ($K \approx 420\,\mathrm{MeV}$). Note that the favored parameter set NLC (with $K \approx 225\,\mathrm{MeV}$) has a negative value for the quartic coupling λ, which may lead to problems for large values of the scalar field, since the energy is no longer bounded from below. As we will see later, however, the mean-field energy is actually valid only as an expansion in powers of the fields, and thus global questions about stability are not particularly relevant.

C. Nucleon–Nucleus Scattering

The scattering of medium-energy nucleons from nuclei can provide information about both nuclear structure and the NN interaction. Since the NN interaction has complex spin, isospin, momentum, and density dependence, nucleon–nucleus scattering exhibits a wide variety of phenomena. As a starting point for describing these phenomena, we use the Dirac–Hartree description of the nucleus, together with the relativistic impulse approximation (RIA), which assumes that the interaction between the projectile and target nucleons has the same form as the interaction between two nucleons in free space. This interaction is used to produce a nucleon–nucleus optical potential that incorporates the leading term in a multiple-scattering series.

Although the simple QHD models discussed above are useful for studying the average properties of the nuclear interaction, they are less useful for describing the detailed quantitative features (such as spin dependence) of the full NN scattering amplitude. These quantitative features are important for any reasonable description of the nucleon–nucleus scattering observables. The RIA allows us to combine the empirical free-space scattering amplitude with a relativistic calculation of the nuclear ground state.

The RIA as originally formulated [Mc83,Sh83,Cl83] involves two basic procedures. First, the experimental NN scattering amplitude is represented by a particular set of five Lorentz-covariant functions [Mc83a] that multiply the so-called "Fermi invariant" Dirac matrices. The Lorentz covariant functions are then folded with the Dirac–Hartree target densities to produce a first-order optical potential for use in the Dirac equation for the projectile [Ho91]. Here we briefly summarize the formalism and the results; a more complete discussion is given in [Se86].

The constraints of Lorentz covariance, parity conservation, isospin invariance, and that free nucleons are on their mass shell imply that the invariant NN scattering operator $\mathcal{F}$ can be written in terms of five complex functions for pp scattering and five for pn scattering. In the original RIA, $\mathcal{F}$ was taken as

$$\mathcal{F} = \mathcal{F}^S + \mathcal{F}^V \gamma_{(0)}^\mu \gamma_{(1)\mu} + \mathcal{F}^P \gamma_{(0)}^5 \gamma_{(1)}^5 + \mathcal{F}^T \sigma_{(0)}^{\mu\nu} \sigma_{(1)\mu\nu} + \mathcal{F}^A \gamma_{(0)}^5 \gamma_{(0)}^\mu \gamma_{(1)}^5 \gamma_{(1)\mu} \ , \qquad (2.37)$$

where the subscripts (0) and (1) refer to the incident and struck nucleons, respectively. Each amplitude $\mathcal{F}^L$ is a complex function of the Lorentz invariants t (four-momentum transfer squared) and s (total four-momentum squared), or equivalently, of the momentum transfer q and incident energy E. It is found empirically that the amplitudes $\mathcal{F}^S$, $\mathcal{F}^V$, and $\mathcal{F}^P$ are much larger than any amplitudes obtained in a nonrelativistic decomposition, which uses Galilean-invariant operators.

The RIA optical potential $U_{\text{opt}}(q, E)$ is defined as

$$U_{\text{opt}}(q, E) = -\frac{4\pi i p}{M} \langle \Psi | \sum_{n=1}^A \mathrm{e}^{i\mathbf{q}\cdot\mathbf{x}(n)} \mathcal{F}(q, E; n) | \Psi \rangle \ , \qquad (2.38)$$

where $\mathcal{F}$ is the scattering operator of Eq. (2.37), p is the magnitude of the projectile three-momentum in the nucleon–*nucleus* c.m. frame (where the scattering observables are calculated), $|\Psi\rangle$ is the A-particle nuclear ground state, and the sum runs over all nucleons in the

target. $\mathcal{F}$ is a function of the momentum transfer q and collision energy E, which we take to be the proton–nucleus c.m. energy; this amounts to neglecting nuclear recoil.

With these simplifications, the Dirac optical potential is local, and only diagonal nuclear densities are needed. For a spin-zero nucleus, the only nonzero densities are the baryon and scalar densities of Eq. (2.28), plus a tensor term computed by inserting σ^{0i} between the spinors in Eq. (2.28). Thus the optical potential takes the form

$$U_{\text{opt}} = U^S + \gamma^0 U^V - 2i\boldsymbol{\alpha} \cdot \hat{r} U^T \ , \tag{2.39}$$

where $U^L \equiv U^L(r; E)$ for each component. The tensor contribution U^T is small and is neglected in what follows. The RIA optical potential then has only scalar and vector contributions, and the Dirac equation for the projectile has precisely the same form as in Eq. (2.25), with U^V replacing $g_{\text{v}}V_0$ and U^S replacing $(-g_{\text{s}}\phi_0)$:

$$h\,\mathcal{U}_0(\mathbf{x}) = \left\{ -i\boldsymbol{\alpha} \cdot \boldsymbol{\nabla} + U^V(r; E) + \beta[M + U^S(r; E)] \right\}\mathcal{U}_0(\mathbf{x}) = E\,\mathcal{U}_0(\mathbf{x}) \ . \tag{2.40}$$

In practice, one includes in U^V the Coulomb potential computed from the empirical nuclear charge density; other electromagnetic contributions arising from the proton anomalous magnetic moment are of similar size to the tensor term U^T.

Since representative RIA results have been shown many times in the literature (see, for example, Figs. 25–28 in [Se86]), we will not reproduce the figures here.[7] The target densities in these calculations are taken from the results for spherical nuclei discussed above, *with no further adjustment of parameters*, and the RIA calculations agree remarkably well with the data. Moreover, when compared with nonrelativistic impulse-approximation calculations, the relativistic results are superior, particularly for the spin observables. Although the nonrelativistic results improve when higher-order corrections are included [Co90,Ra90], one concludes that the important spin- and density-dependent effects are already contained in the relativistic impulse-approximation framework [Pi84,Hy85,Lu87].

The spin dynamics is inherent in the relativistic formalism and arises naturally from the large Lorentz scalar and vector potentials in the Dirac equation (2.40). This is precisely the same spin dynamics that produces the observed spin-orbit splittings in the bound single-particle levels. *Thus the relativistic Hartree calculations provide a minimal unifying theoretical basis for both the nuclear shell model and medium-energy proton–nucleus scattering—two essential aspects of nuclear physics.*

D. Nuclear Excited States

We now turn to the calculation of nuclear response functions and the properties of nuclear excited states, as described in the random-phase approximation (RPA) built on the MFT ground state. These excitations arise from the consistent linear response of the ground state, in which the nucleons move coherently in varying classical meson fields that are in turn

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[7]Computer codes for performing RIA calculations are also readily available [Ho91].

determined by oscillatory nuclear sources. Many studies of the relativistic nuclear response have been carried out for both infinite matter and finite nuclei, beginning with the pioneering work of Chin [Ch77], and here we will focus on two basic issues. The first is the importance of consistency, which implies that the particle-hole interaction in the excited states must be the same (and use the same parameter values) as the interaction in the ground state. This ensures that the occupied and unoccupied single-particle states are indeed orthogonal. Second, the relativistic response involves not only the familiar positive-energy particle-hole configurations, but also configurations that mix positive- and negative-energy states. These new configurations are crucial for the conservation of the electromagnetic current and the separation of the "spurious" $J^\pi = 1^-$ state. This emphasizes that the Dirac single-particle basis is complete only when both positive- and negative-energy states are included.

The calculation of the linear response is basically the same as in nonrelativistic many-body theory [Fe71]. The principal idea is to compute the particle-hole (polarization) propagator and to extract the collective excitation energies and transition amplitudes from the poles and residues of this propagator. Several methods have been developed and applied to finite nuclei [Fu85,Fu85a,Ni86,We87,We88,Fu88a,Bl88,Sh89,Da90,Ho90,Pi90,Pr92a]. Initial calculations were restricted primarily to isoscalar excitations, because fitting bulk nuclear properties constrains only the isoscalar particle-hole interaction significantly, and because the isovector response depends critically on pion dynamics, with the associated complexity that we discuss later.

An excellent discussion of the role of consistency is contained in the work of Dawson and Furnstahl [Da90], where results for the low-lying, negative-parity, isoscalar states in ^{12}C, ^{16}O, and ^{40}Ca are compared to several empirical levels that might be reasonably described as particle-hole excitations. The full RPA eigenvalues agree favorably with the empirical values, which is a nontrivial result because of the large cancellations between scalar and vector contributions. Moreover, it is found that the negative-energy states play an important role in determining the RPA spectrum, particularly for the spurious 1^- state. Lorentz covariance implies that in a consistent RPA calculation, this state should appear at zero excitation energy, which occurs only when the full Dirac basis is maintained.

Moreover, RPA calculations by Furnstahl [Fu89b] of the ratio of the transition charge density to the longitudinal current for a particular $(3^-, 0)$ excitation in ^{16}O show that if only the positive-energy (particle-hole) configurations are retained, the electromagnetic current is not conserved. In contrast, with the full RPA calculation, including the contributions from negative-energy states, current conservation is restored. The conclusion is that it is *essential* to include all states in the Dirac basis to maintain the conservation of the current.

E. Nuclear Matter at Finite Temperature

The preceding discussion of the nuclear matter equation of state was restricted to zero temperature. The extension to finite temperature is straightforward in the MFT, since the hamiltonian is diagonal and the mean-field thermodynamic potential Ω can be calculated

exactly.[8] The results for the scalar density, baryon density, energy density, and pressure are given by (here $\kappa = \lambda = g_\rho = 0$) [Se86]

$$\rho_{\rm s} = \frac{\gamma}{(2\pi)^3} \int {\rm d}^3k \, \frac{M^*}{E^*(k)} \, (n_k + \overline{n}_k) \; , \tag{2.41}$$

$$\rho_{\rm B} = \frac{\gamma}{(2\pi)^3} \int {\rm d}^3k \, (n_k - \overline{n}_k) \; , \tag{2.42}$$

$$\mathcal{E} = \frac{g_{\rm v}^2}{2m_{\rm v}^2} \rho_{\rm B}^2 + \frac{m_{\rm s}^2}{2g_{\rm s}^2} (M - M^*)^2 + \frac{\gamma}{(2\pi)^3} \int {\rm d}^3k \, E^*(k)(n_k + \overline{n}_k) \; , \tag{2.43}$$

$$p = \frac{g_{\rm v}^2}{2m_{\rm v}^2} \rho_{\rm B}^2 - \frac{m_{\rm s}^2}{2g_{\rm s}^2} (M - M^*)^2 + \frac{1}{3} \frac{\gamma}{(2\pi)^3} \int {\rm d}^3k \, \frac{\mathbf{k}^2}{E^*(k)} \, (n_k + \overline{n}_k) \; , \tag{2.44}$$

where the baryon and antibaryon distribution functions are

$$\begin{pmatrix} n_k(T, \nu) \\ \overline{n}_k(T, \nu) \end{pmatrix} \equiv \left\{ 1 + {\rm e}^{[E^*(k) \mp \nu]/T} \right\}^{-1} \; , \tag{2.45}$$

and the reduced chemical potential is $\nu \equiv \mu - g_{\rm v} V_0$. (We set Boltzmann's constant $k_{\rm B} = 1$.) The appropriate value of M^* is determined by minimizing the thermodynamic potential with respect to that parameter:

$$\left(\frac{\partial \Omega}{\partial M^*} \right)_{\mu, V, T} = 0 \; . \tag{2.46}$$

The nuclear matter equation of state at all densities and temperatures for this hadronic MFT model (QHD–I) is shown in [Se92,Wa95]. The model has also been combined with a simple description of quark-gluon matter to describe the hadron/quark phase transition as a function of temperature and density [Se86]. A more extensive examination of the equation of state, as well as a discussion of the covariance of the finite-temperature results, is contained in [Fu90].

3. PION DYNAMICS AND CHIRAL SYMMETRY

The relativistic neutral scalar and vector fields are the most important for determining the bulk properties of nuclear systems. Nevertheless, the lightest and most accessible meson is the pion, whose interactions with nucleons and nuclei have been extensively studied at the meson factories. It is therefore impossible to formulate a complete and quantitative hadronic theory without including pionic degrees of freedom. Here we briefly review some important aspects of pion dynamics in QHD. More complete discussions, together with references to the original literature, can be found in [Se86,Se92a,Wa95].

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[8]We neglect the zero-point corrections from the Dirac sea in this section (see [Fr78] and [Fu91]), as well as thermal contributions from the massive isoscalar mesons.

In the limit of massless u and d quarks, QCD possesses a global, chiral $SU(2)_L \times SU(2)_R$ symmetry. Chiral transformations may be written in terms of a set of vector (V) and axial-vector (A) generators, which produce corresponding isospin rotations. The symmetry is spontaneously broken, leading to the existence of pseudoscalar Goldstone bosons (pions). The vector (isospin) symmetry, which forms an $SU(2)_V$ subgroup of the original chiral group, remains unbroken. Our effective hadronic theories should respect these underlying symmetries, which have important consequences for the way mesons interact with themselves and with each other; a thorough discussion is contained in [Wa95]. In nature, electromagnetic interactions and finite quark masses imply that these symmetries are only approximate, but the symmetry-violating terms can be added as small perturbations.

A. The Linear Sigma Model

The simplest model illustrating these ideas is the linear sigma model [Sc57,Ge60,Le72], which contains a pseudoscalar (γ_5) coupling between pions and nucleons, plus an auxiliary scalar field (denoted here by s) to implement the symmetry. A small symmetry-violating (SV) term is included to generate a finite pion mass. We will also include a massive, neutral, isoscalar vector field to supply a repulsive interaction, as in QHD–I.

By demanding that the theory be Lorentz covariant, parity invariant, isospin and chiral invariant, and renormalizable, one is led to the form

$$\mathcal{L}_{\sigma\omega} = \mathcal{L}_{\text{chiral}} + \mathcal{L}_{\text{SV}} , \tag{3.1}$$

$$\mathcal{L}_{\text{chiral}} = \overline{\psi}[\gamma_\mu(i\partial^\mu - g_{\text{v}}V^\mu) - g_\pi(s + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi})]\psi + \tfrac{1}{2}(\partial_\mu s\partial^\mu s + \partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi})$$

$$-\tfrac{1}{4}\widetilde{\lambda}(s^2 + \boldsymbol{\pi}^2 - v^2)^2 - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m_{\text{v}}^2 V_\mu V^\mu + \delta\mathcal{L} , \tag{3.2}$$

$$\mathcal{L}_{\text{SV}} = \epsilon s . \tag{3.3}$$

Here ψ, $\boldsymbol{\pi}$, and s are the nucleon, isovector pion, and neutral scalar meson fields, respectively, and g_π is the pion–nucleon coupling constant. The parameters $\widetilde{\lambda}$ and v describe the strength of the meson self-interactions, and ϵ is a small chiral-symmetry-violating parameter related to the pion mass; the exact chiral limit is obtained by setting $\epsilon = 0$. The form of the meson self-interactions allow for spontaneous symmetry breaking, which is used to give the nucleon a finite mass.

The lagrangian (3.2) is invariant under global vector and axial-vector isospin transformations, which imply (by Noether's theorem) the conserved isovector currents[9]

$$\mathbf{T}^\mu = \tfrac{1}{2}\overline{\psi}\gamma^\mu\boldsymbol{\tau}\psi + \boldsymbol{\pi} \times \partial^\mu\boldsymbol{\pi} , \tag{3.4}$$

$$\mathbf{A}^\mu = \tfrac{1}{2}\overline{\psi}\gamma_5\gamma^\mu\boldsymbol{\tau}\psi + \boldsymbol{\pi}\partial^\mu s - s\partial^\mu\boldsymbol{\pi} \tag{3.5}$$

in the chiral limit $\epsilon = 0$. When $\epsilon \neq 0$, we obtain instead the PCAC relation

$$\partial_\mu\mathbf{A}^\mu = \epsilon\boldsymbol{\pi} , \tag{3.6}$$

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[9]The (isoscalar) baryon current $B^\mu = \overline{\psi}\gamma^\mu\psi$ is also conserved.

which follows from the field equations. Note that the chiral symmetry is realized *linearly*, which means that under a general chiral transformation, neutrons mix with protons, and the scalar mixes with the pions. (The vector field is a chiral scalar.) Moreover, the linear symmetry requires that the scalar and pion couple to the nucleon with equal strength $g_\pi \approx 13.4$. In applications, one often relaxes this condition (with no justification) and sets $g_s = g_\pi/g_A$, where the axial coupling $g_A \approx 1.26$; this *ad hoc* procedure allows the Goldberger–Treiman relation to be satisfied at the tree level [We67,Le72].

The baryon mass is generated by spontaneous symmetry breaking [Le72,Se86], which implies that the scalar field s has a nonzero vacuum expectation value s_0, and that the pion is a massless Goldstone boson (in the limit $\epsilon = 0$). In terms of the shifted scalar field

$$\sigma \equiv s_0 - s \tag{3.7}$$

and the physical masses defined by

$$M = g_\pi s_0 \ , \qquad \epsilon = \frac{M}{g_\pi} m_\pi^2 \ , \qquad \widetilde{\lambda} = \frac{m_\sigma^2 - m_\pi^2}{2M^2} g_\pi^2 \ , \tag{3.8}$$

the lagrangian $\mathcal{L}_{\sigma\omega}$ reads

$$\begin{aligned}
\mathcal{L}_{\sigma\omega} = \ &\overline{\psi}[i\gamma_\mu\partial^\mu - g_v\gamma_\mu V^\mu - (M - g_\pi\sigma) - ig_\pi\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}]\psi + \tfrac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) \\
&- \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m_v^2 V_\mu V^\mu + \tfrac{1}{2}(\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} - m_\pi^2\boldsymbol{\pi}^2) \\
&+ g_\pi\frac{m_\sigma^2 - m_\pi^2}{2M}\sigma(\sigma^2 + \boldsymbol{\pi}^2) - g_\pi^2\frac{m_\sigma^2 - m_\pi^2}{8M^2}(\sigma^2 + \boldsymbol{\pi}^2)^2 \ ,
\end{aligned} \tag{3.9}$$

where the counterterms $\delta\mathcal{L}$ will here and henceforth be suppressed. Note that the explicit chiral symmetry violation is now contained entirely in the parameter m_π^2, the square of the pion mass. Moreover, if we consider the (renormalized) parameters M, m_π, m_v, and g_π as experimentally known, then apart from the vector meson coupling g_v, there is only one free parameter in this model, namely, the chiral scalar mass m_σ. Both the signs and magnitudes of the nonlinear meson interactions proportional to σ^3 and σ^4 are determined by the chiral symmetry and its spontaneous breaking in the vacuum.

A vector-isovector $\boldsymbol{\rho}$ meson can be added in a renormalizable manner to produce QHD–II, as discussed in [Se86].[10] At the mean-field level of interest here, the rho meson enters as in Eq. (2.36). Given the similarity between the lagrangians in Eqs. (3.9) and (2.1), it is only natural (but incorrect!) to identify the σ field with the scalar field ϕ studied earlier. Since the pion field vanishes at the mean-field level (we assume our systems possess the same symmetries as those in Section 2), the dynamical equations are exactly the same as before, except that the nonlinear scalar interactions are now determined by the symmetry:

$$\frac{\kappa_{\sigma\omega}}{M} = -\frac{3g_\pi}{M^2}(m_\sigma^2 - m_\pi^2) \ , \qquad \lambda_{\sigma\omega} = \frac{3g_\pi^2}{M^2}(m_\sigma^2 - m_\pi^2) \ . \tag{3.10}$$

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[10]Including the isovector $\boldsymbol{\rho}$ meson destroys the chiral symmetry. A fully chiral, renormalizable model (QHD–III) containing both the $\boldsymbol{\rho}$ and $\mathbf{a}_1$ mesons is derived in [Se92b]. See also [Ko93a,Fo95a].

Unfortunately, the identification of σ with ϕ leads to serious difficulties. First, even though there are still two free parameters in symmetric nuclear matter, it is impossible to reproduce the empirical equilibrium point, as documented long ago by Kerman and Miller [Ke74]. (Their result is reproduced in Fig. 5 of [Fu96].) The basic problem is that the nonlinear scalar interactions are large and have the wrong signs. [Compare $\kappa_{\sigma\omega} < 0$ and $\lambda_{\sigma\omega} > 0$ in Eq. (3.10) with the favored NLC values in Table I.] This problem was solved in [Bo83,Sa85], where interactions between the scalar and vector fields were introduced by making the replacement

$$\tfrac{1}{2}m_{\mathrm{v}}^2 V_\mu V^\mu \longrightarrow \tfrac{1}{2}\eta^2 g_{\mathrm{v}}^2 V_\mu V^\mu (s^2 + \boldsymbol{\pi}^2) \tag{3.11}$$

in Eq. (3.2). Now the vector field also acquires its mass through spontaneous symmetry breaking, when the scalar field is shifted as in Eq. (3.7). This results in the replacement

$$\tfrac{1}{2}m_{\mathrm{v}}^2 V_\mu V^\mu \longrightarrow \frac{1}{2} m_{\mathrm{v}}^2 V_\mu V^\mu - \frac{g_\pi m_{\mathrm{v}}^2}{M} V_\mu V^\mu \sigma + \frac{1}{2} \frac{g_\pi^2 m_{\mathrm{v}}^2}{M^2} V_\mu V^\mu (\sigma^2 + \boldsymbol{\pi}^2) \tag{3.12}$$

in Eq. (3.9). The signs of the scalar–vector cubic and quartic interactions are *opposite* to those in Eq. (3.9), which now allows the nuclear matter equilibrium point to be reproduced.[11]

Nevertheless, an extensive mean-field analysis shows that it is impossible to generate realistic results for nuclear densities, single-particle spectra, and total binding energies within this framework [Fu93,Fu93a,Fu96]. The primary problem is still the existence of scalar nonlinearities with incorrect systematics; in addition, the large value of the (known) scalar coupling leads to a scalar mass that is much larger than that required by the phenomenology ($m_\sigma \approx 800\,\mathrm{MeV}$ compared to a desired value of $m_\sigma \approx 500\,\mathrm{MeV}$).

We conclude that the standard form of spontaneous chiral symmetry breaking, implemented in a model with a linear realization of the symmetry, *cannot* produce successful nuclear phenomenology at the mean-field level when we identify the chiral scalar σ with the scalar in QHD–I.[12] Note that the interaction potential (the term proportional to $\tilde{\lambda}$) in Eq. (3.2) is the only form consistent with both chiral symmetry and renormalizability. This failure of the sigma model is therefore evidence, even at the mean-field level, that the simultaneous constraints of linear chiral symmetry and renormalizability are too restrictive.

A different approach to the chiral symmetry is more compatible with observed nuclear properties and with successful relativistic mean-field models. Here one takes the chiral scalar mass m_σ to be *large* to eliminate the unphysical scalar nonlinearities and then generates the mid-range attractive force between nucleons *dynamically* through correlated two-pion

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[11]The new scalar–vector interactions destroy the renormalizability of the model, but we will overlook that for the moment.

[12]It has recently been shown [He94a,Ca96] that realistic nuclear systematics can be achieved in a linear realization by using a *logarithmic*, nonrenormalizable scalar potential to generate the spontaneous symmetry breaking.

exchange.[13] Numerous calculations [Ja75,Du77,Du80,Li89,Li90,Ki94] have shown that the exchange of two interacting pions in the scalar-isoscalar channel produces a strong attractive force with a range comparable to that of a scalar meson with a mass of 500 to 600 MeV. In principle, this approach can be realized within the linear models discussed above, by rejecting the mean-field approximation (which is inadequate) and by retaining contributions from correlated two-pion exchange between nucleons. This clearly introduces much greater complexity, since one must first construct a boson-exchange kernel containing correlated two-pion exchange and then allow this kernel to act to all orders (for example, in a ladder approximation) to determine the NN interaction and the resulting nuclear matter energy density [Ma89]. This calculation is further complicated by the need to maintain chiral symmetry at finite density, which is difficult to do when one uses the non-derivative ("pseudoscalar" or PS) πN coupling implied by the linear realization of the symmetry [Ma82,Ho82,Wa95]. These complexities motivate us to look for a more efficient way to implement the chiral symmetry.

B. A Nonlinear Realization of Chiral Symmetry

The chiral dynamics of pions and the successful mean-field picture described in Section 2 can be combined if the chiral symmetry is realized in a *nonlinear* fashion. In a nonlinear realization, chiral transformations mix the nucleon and pion fields, and a scalar-isoscalar field is unchanged. There are many ways to implement a nonlinear realization; here we follow the approach of Weinberg [We67] and discuss a different method in Section 5.

First notice that a chiral rotation matrix can be written as

$$\exp(\tfrac{i}{2}\boldsymbol{\omega}\cdot\boldsymbol{\tau}\gamma_5) = \mathbf{1}\cos(\omega/2) + i\hat{n}\cdot\boldsymbol{\tau}\gamma_5\sin(\omega/2) \ , \qquad (3.13)$$

where $\mathbf{1}$ denotes the unit matrix in spin-isospin space and $\boldsymbol{\omega} \equiv \hat{n}\omega$ is real. The key observation is that the linear terms in σ and $\boldsymbol{\pi}$ in Eq. (3.9) can be written as a chiral rotation matrix:

$$M - g_\pi\sigma + ig_\pi\boldsymbol{\tau}\cdot\boldsymbol{\pi}\gamma_5 = [(M - g_\pi\sigma)^2 + g_\pi^2\boldsymbol{\pi}^2]^{1/2}\exp(i\boldsymbol{\theta}\cdot\boldsymbol{\tau}\gamma_5)$$

$$= [(M - g_\pi\sigma)^2 + g_\pi^2\boldsymbol{\pi}^2]^{1/2}[\mathbf{1}\cos\theta + i\hat{n}\cdot\boldsymbol{\tau}\gamma_5\sin\theta] \ , \qquad (3.14)$$

where $\boldsymbol{\pi} = \hat{n}\pi$ and $\boldsymbol{\theta} = \hat{n}\theta$. It now follows immediately that

$$\cos\theta = \frac{M - g_\pi\sigma}{[(M - g_\pi\sigma)^2 + g_\pi^2\boldsymbol{\pi}^2]^{1/2}} \ , \qquad \sin\theta = \frac{g_\pi\pi}{[(M - g_\pi\sigma)^2 + g_\pi^2\boldsymbol{\pi}^2]^{1/2}} \ . \qquad (3.15)$$

One can then eliminate the linear pion–nucleon interaction in the lagrangian $\mathcal{L}_{\sigma\omega}$ by defining a new baryon field N as a unitary transformation of the old baryon field ψ:

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[13]One can see that the scalar self-interactions become small by writing the lagrangian (3.9) in terms of the re-scaled field $\chi \equiv m_\sigma\sigma$. Note, however, that the $\sigma\boldsymbol{\pi}^2$ coupling becomes large when m_σ becomes large.

$$N \equiv \exp(\tfrac{i}{2}\boldsymbol{\theta} \cdot \boldsymbol{\tau}\gamma_5)\psi = \sqrt{\tfrac{1}{2}(1 + \cos\theta)}\,(\mathbf{1} + i\boldsymbol{\xi} \cdot \boldsymbol{\tau}\gamma_5)\psi \ , \tag{3.16}$$

where

$$\boldsymbol{\xi} \equiv \hat{n}\frac{\sin\theta}{1 + \cos\theta} \ , \tag{3.17}$$

and by defining new meson fields $\boldsymbol{\pi}'$ and σ' as

$$\boldsymbol{\xi} \equiv \frac{g_\pi}{2M}\,\boldsymbol{\pi}' \equiv \frac{f}{m_\pi}\,\boldsymbol{\pi}' \ , \qquad M - g_\pi\sigma' \equiv \frac{M - g_\pi\sigma}{\cos\theta} \ . \tag{3.18}$$

Although the algebra is tedious, it is straightforward [Wa95] to rewrite $\mathcal{L}_{\sigma\omega}$ in terms of the new fields as

$$\begin{aligned}
\mathcal{L}_{\sigma\omega} = \overline{N}\Big\{ & i\gamma_\mu\partial^\mu - g_{\mathrm{v}}\gamma_\mu V^\mu - (M - g_\pi\sigma') \\
& + \frac{1}{1 + (f/m_\pi)^2\boldsymbol{\pi}'^2}\left[(f/m_\pi)\gamma^\mu\gamma_5\boldsymbol{\tau} \cdot \partial_\mu\boldsymbol{\pi}' - (f/m_\pi)^2\gamma^\mu\boldsymbol{\tau} \cdot \boldsymbol{\pi}' \times \partial_\mu\boldsymbol{\pi}'\right]\Big\}N \\
& + \tfrac{1}{2}(\partial_\mu\sigma'\partial^\mu\sigma' - m_\sigma^2\sigma'^2) + \tfrac{1}{2}R[R\,\partial_\mu\boldsymbol{\pi}' \cdot \partial^\mu\boldsymbol{\pi}' - m_\pi^2\boldsymbol{\pi}'^2] \\
& + (m_\sigma^2 - m_\pi^2)[(f/m_\pi)\sigma'^3 - \tfrac{1}{2}(f/m_\pi)^2\sigma'^4] - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m_{\mathrm{v}}^2V_\mu V^\mu \ , \tag{3.19}
\end{aligned}$$

where we have defined the ratio

$$R \equiv R(\sigma', \boldsymbol{\pi}') \equiv \frac{1 - 2(f/m_\pi)\sigma'}{1 + (f/m_\pi)^2\boldsymbol{\pi}'^2} \ . \tag{3.20}$$

This somewhat imposing lagrangian has several important advantages. First, the linear PS coupling between the nucleon and pion has been replaced by a derivative or pseudovector (PV) coupling and a so-called "sea gull" term, where the nucleon couples to two pions simultaneously. In fact, there are an infinite number of such couplings, when one expands the prefactor $[1 + (f/m_\pi)^2\boldsymbol{\pi}'^2]^{-1}$ as a power series in $\boldsymbol{\pi}'^2$. Nevertheless, all of these couplings involve at least one derivative acting on the pion field; thus, in the limit of vanishing pion momenta, the pions and nucleons *decouple*, as required by the chiral soft-pion theorems that follow from the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ current algebra and PCAC [Sa69,Do92].

Second, the new pseudovector coupling constant is

$$f^2 = g_\pi^2\left(\frac{m_\pi}{2M}\right)^2 \approx 1.0 \ , \tag{3.21}$$

which is much smaller than the pseudoscalar coupling constant $g_\pi^2/4\pi \approx 14.4$. Moreover, the explicit derivative couplings eliminate the sensitive cancellations between Feynman diagrams that are necessary in the linear realization. The large coupling and sensitive cancellations show that the linear realization is an *inefficient* way to implement the symmetry.

As noted earlier, m_σ should be taken to be large, to avoid the unwanted nonlinear interactions. If desired, m_σ can be kept finite, so that it plays the role of a regulator that maintains the renormalizability of the model [Ma82], or m_σ can be taken to infinity, so that the chiral scalar field *decouples*, resulting in the effective nonlinear model of Weinberg

[We67,Wa95].[14] If the process $\pi + \pi \to \pi + \pi$ is investigated in this model with a heavy σ, and the chiral-invariant Born amplitude is unitarized, one observes a broad, low-mass, near-resonant amplitude in the $(0^+, 0)$ channel, even though the chiral σ has a large mass. When this model $\pi\pi$ scattering amplitude is included in the two-pion-exchange part of the NN interaction, the result is a dynamically generated, broad, low-mass ($\approx 600\,\text{MeV}$) peak that resembles the exchange of a light scalar meson [Li89,Li90].

This strong scalar-isoscalar two-pion exchange can be simulated by introducing a low-mass, *effective* scalar field ϕ coupled directly to the nucleon, and scalar self-interactions can be added to include a density dependence in the mid-range NN attractive force. We emphasize that the purpose of the effective field is to parametrize the NN attraction, so that we can avoid the complicated calculation of scalar-isoscalar pion loops. All scalar propagation will be restricted to *spacelike* momenta, and thus scalar particles are always virtual.[15] At the mean-field level, the classical scalar field is an efficient way to incorporate the effects of pion exchange that are the most important for describing bulk nuclear properties.

If we now take $m_\sigma \to \infty$, the resulting nonlinear, chiral, effective lagrangian is (primes on the fields are omitted)

$$
\begin{aligned}
\mathcal{L}_{\text{eff}} = \overline{N}\Big\{ & \gamma_\mu(i\partial^\mu - g_{\text{v}}V^\mu) - (M - g_{\text{s}}\phi) \\
& + \frac{1}{1 + (f/m_\pi)^2 \boldsymbol{\pi}^2}\left[(f/m_\pi)\gamma^\mu\gamma_5\boldsymbol{\tau}\cdot\partial_\mu\boldsymbol{\pi} - (f/m_\pi)^2\gamma^\mu\boldsymbol{\tau}\cdot\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi}\right]\Big\}N \\
& + \tfrac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m_{\text{s}}^2\phi^2) - \tfrac{1}{3!}\kappa\phi^3 - \tfrac{1}{4!}\lambda\phi^4 - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m_{\text{v}}^2 V_\mu V^\mu \\
& + \frac{1}{2(1 + (f/m_\pi)^2\boldsymbol{\pi}^2)}\left[\frac{1}{1 + (f/m_\pi)^2\boldsymbol{\pi}^2}\,\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} - m_\pi^2\boldsymbol{\pi}^2\right].
\end{aligned}
\tag{3.22}
$$

The explicit symmetry-violating term proportional to m_π^2 has been included, and since $f/m_\pi = g_\pi/2M$ from eq. (3.21), the exact chiral limit ($m_\pi \to 0$) is sensible.

We retain the notation N for the baryon field to remind us that it transforms nonlinearly, in contrast to the field ψ used earlier. (Linear transformations are independent of the pion field, while nonlinear transformations depend on the pion field.) Although the nonlinear transformation law implied by Eq. (3.16) is complicated, it has been shown [We67,Co69] that under a general chiral transformation,

$$
N \longrightarrow N' = h(x)N \,,
\tag{3.23}
$$

---

[14]In contrast to the lagrangian (3.9), if one rewrites the *nonlinear* lagrangian (3.19) in terms of $\chi \equiv m_\sigma\sigma'$, it is easy to see that *all* interactions involving the scalar field vanish in the $m_\sigma \to \infty$ limit.

[15]The effective scalar-isoscalar field is introduced to simplify the description of the attractive NN interaction. Thus it plays a role in problems with $B \geq 1$ and cannot be directly interpreted as an on-shell particle. The idea of introducing virtual degrees of freedom to describe fermionic interactions is not revolutionary; phonon fields in metals and the Ginzburg–Landau field in quantum liquids have played such a role for many years with considerable success [Fe71].

where $h(x) \in \mathrm{SU}(2)_V$ is an *isospin* rotation matrix that depends *locally* on the pion field. Thus the inclusion of a Yukawa coupling

$$\mathcal{L}_\mathrm{Y} = g_\mathrm{s} \overline{N} N \phi \tag{3.24}$$

(as well as scalar self-couplings) in $\mathcal{L}_\mathrm{eff}$ leaves the chiral symmetry intact, because the light scalar is an isoscalar, and the nucleon field transforms as in Eq. (3.23) [Wa95].[16] Moreover, since the pion mean field vanishes, the chiral mean-field theory obtained from $\mathcal{L}_\mathrm{eff}$ produces *precisely* the same field equations and energy density as in Section 2. Finally, although the coupling strength g_s of the light scalar is comparable to g_π (as verified by explicit calculation of the correlated two-pion exchange [Li89,Li90]), we no longer require $g_\mathrm{s} = g_\pi$ and are free to adjust g_s within a reasonable range. The nonlinear scalar interactions, which parametrize the density dependence of the correlated two-pion exchange, also contain adjustable parameters.

Thus, through these somewhat lengthy arguments, we conclude that *the mean-field QHD–I model studied earlier is consistent with chiral symmetry*, provided we think in terms of a nonlinear realization of the symmetry. The importance of the resulting scalar-isoscalar mean field and optical potential in producing a successful nuclear phenomenology was illustrated in Section 2. Although the one-pion-exchange interaction (and its iteration) produces a relatively small contribution in nuclear matter due to its spin dependence, the correlated two-pion-exchange contribution has a spin-independent, isoscalar part that is large and that must be included from the outset.[17] The discussion in Section 2 shows that the scalar-isoscalar field in QHD–I is an efficient and successful way to incorporate these important pionic effects. *A profound change has occurred, however, because in contrast to the original proposal of QHD–I as a renormalizable field theory, we are now forced to consider the scalar field as an effective degree of freedom and the lagrangian in Eq. (3.22) as a nonrenormalizable effective lagrangian.* We will consider this more general strategy for QHD in Section 5.

C. The Δ Isobar

The dominant phenomenological features of the low-energy πN interaction are that low-momentum pions interact weakly with nucleons (they decouple as $q_\mu \to 0$), and that the interaction is dominated by the first pion–nucleon resonance, the $\Delta(1232)$. This resonance represents the first excited state of the baryon, with $(J^\pi, T) = (\frac{3}{2}^+, \frac{3}{2})$. It is essential to

---

[16]Note that one *cannot* add a Yukawa interaction $\overline{\psi} \psi \phi$ with an effective scalar field to the linear sigma model [Eq. (3.9)], as this destroys the chiral symmetry.

[17]In the few-nucleon problem, the situation is reversed. Because of the more complicated spin dependence of the wave function, matrix elements for one-pion exchange no longer average to zero. Moreover, the existence of an almost-bound state at essentially zero energy in the $^1S_0$ NN channel [Fe71] implies that there is a nearly exact cancellation between the attractive (scalar) and repulsive (vector) parts of the NN force in few-body systems. Thus, the pion-exchange "tensor force" plays a more prominent role.

include this degree of freedom in the theory to generate realistic results for pion–nucleus interactions and for few-nucleon systems. It is impossible to put a field with these quantum numbers into a renormalizable lagrangian, but it has been shown that the Δ degree of freedom can be produced *dynamically* within the model. The most efficient way to incorporate the Δ is as an effective degree of freedom.

Here we simply summarize the results, as the arguments and references to the original literature are detailed in [Se86,Se92a,Wa95]. The sum of πN ladder diagrams with nucleon exchange can be investigated within the framework of the chiral πN theory discussed above. Partial-wave dispersion relations can be used, with the one-baryon-exchange mechanism considered as the driving term, and the resulting integral equations solved with the N/D method [Li91]. This is a relativistic extension of nonrelativistic Chew–Low theory. As in the Chew–Low theory, a resonance is found in the $(\frac{3}{2}^{+}, \frac{3}{2})$ channel. Evidently, the box diagram in the ladder sum involves a loop integral, which obtains significant contributions from large loop momenta or short distances; thus, the predicted *position* of the resonance is sensitive to the approximations made. In contrast, the predicted resonance *width* is much less sensitive. As a result of the work summarized in [Se92a,Wa95], it is clear that the first excited state of the nucleon, the $\Delta(1232)$ with $(\frac{3}{2}^{+}, \frac{3}{2})$, which is the dominant feature of low-energy pion–nucleus interactions, is generated *dynamically* in QHD. Thus it can also be included as an effective degree of freedom in a nonrenormalizable hadronic lagrangian [Bi82,De92a,We93,He96,Ta96].

4. FEW-NUCLEON SYSTEMS

The purpose of this section is to present some additional evidence of the successes of hadronic field theory by considering the few-nucleon problem. As before, we want to focus on basic ideas and phenomenology that will guide us in the construction of an effective field theory for the many-body problem. We begin by summarizing modern models of the NN interaction and show that when applied to two-nucleon systems, accurate results are obtained both for electromagnetic observables and for threshold pion production. We also consider the extension of the two-nucleon problem into nuclear matter, by discussing the relativistic generalization of the independent-pair approximation [Fe71,Wa95], which is often called Dirac–Brueckner–Hartree–Fock (DBHF) theory. This will allow us to compare nuclear matter calculations involving two-nucleon correlations to the mean-field results discussed earlier and also to understand the relationship between the relativistic two-nucleon interaction and nonrelativistic many-body forces.

A. The Nucleon–Nucleon Interaction

Models of the NN interaction have been studied seriously for nearly 50 years. An excellent historical review is contained in [Ma89]. Modern meson-exchange models provide excellent fits to experimental observables up to and somewhat beyond pion-production threshold, which is at a laboratory kinetic energy of roughly 300 MeV. Once the models are calibrated to the NN data, they can be used to study both the interaction of few-nucleon systems

with experimental probes and the nuclear many-body problem. As mentioned in the Introduction, to achieve accurate reproduction of the data, several mesons are needed [the most important are the $\pi(0^-, 1)$, $\sigma(0^+, 0)$, $\omega(1^-, 0)$, and $\rho(1^-, 1)$, although the $\eta(0^-, 0)$ and $\delta(0^+, 1)$ are often included], and many models contain both N and Δ degrees of freedom. There are models based entirely on boson exchange [Zu81,Ma87,Te87,Ma89,Gr90,Pl94], as well as models that are supplemented by dispersion relations [Br76,Pa79] or by Regge theory [Na73,St94]. (A comparison of the different models is contained in [Ma94e].) These models are not renormalizable and contain form factors at the meson–nucleon vertices that are not usually expanded in powers of momentum, as one would do in a strict implementation of effective field theory [We91,We92a,Or92,Va93,Or96]. (In other words, the interactions are *nonlocal.*) The range parameters of the vertex functions are treated as adjustable, and they are important for producing realistic results for observables.

The NN system is usually studied with a *quasipotential* approach, in which one starts with the two-nucleon Bethe–Salpeter equation and then reduces it to a three-dimensional integral equation by defining an approximate two-nucleon propagator. In free space, the Bethe–Salpeter equation can be written schematically as [Bj65]

$$T = K + i \int KGGT \ , \tag{4.1}$$

where K is the full two-body scattering kernel and G is the exact baryon propagator. (For illustration, we consider only nucleons here.) The four-dimensional integral implied in Eq. (4.1) can be reduced to a three-dimensional integral by defining a unitarized, two-particle propagator g_0 and a quasipotential U:

$$T = U + \int Ug_0T \ , \tag{4.2}$$

$$U = K + \int K(iGG - g_0)U \ . \tag{4.3}$$

These two equations are equivalent to (4.1), and if one solves both of them, the results should agree with the Bethe–Salpeter results for any choice of g_0. In practice, however, only Eq. (4.2) is retained, under the assumption that the corrections from (4.3) are small; moreover, the quasipotential U is approximated by keeping just the one-boson-exchange "ladder" kernel (which we call V), or the ladder terms plus some "box" and "crossed-box" amplitudes [Ma89].

To study the electromagnetic response of the two-nucleon system, the quasipotential is fitted to reproduce the NN phase shifts and the properties of the deuteron, and then the coupling of the electromagnetic current is introduced. Since the quasipotential can be defined in an infinite number ways, it is important to test the sensitivity by utilizing different approaches; currently, calculations using three different quasipotentials are available [Hu90,De93,Va95]. Calculations of the deuteron charge and magnetic form factors are in good agreement with experiment out to three-momentum transfers of roughly $q^2 \approx 80 \, \text{fm}^{-2}$, provided that meson-exchange currents are included. Moreover, calculated results for the tensor polarization (T_{20}) of the deuteron agree with experiment out to $q^2 \approx 20 \, \text{fm}^{-2}$. Significant effort has been spent in deriving meson-exchange currents that guarantee electromagnetic gauge invariance, even when phenomenological form factors are used at the vertices

[Gr87,Co94a]. Nevertheless, there are still some open questions regarding the consistent inclusion of realistic off-shell nucleon form factors and the model dependence of the transverse parts of the meson-exchange currents.

Meson-exchange currents are also important for describing pion production near threshold in the reaction $pp \to pp\pi^0$. A precise measurement of the total cross section for this process has recently been performed [Me90,Me92], but calculations that include only single-nucleon mechanisms severely underestimate the measured values [Ko66,Sc69]. In contrast, the inclusion of a scalar-meson exchange current (and smaller contributions from other mesons) increases the cross section by about a factor of five and leads to excellent agreement with the data [Le92,Me92,Ho94b]. The results are insensitive to changes in the potential that generates the two-nucleon wave function and to different choices for the phenomenological meson–nucleon form factors. Although some questions have recently been raised about possible competing mechanisms, alternative calculations cannot reproduce the measured cross section [Co96].

B. Two-Nucleon Correlations in Nuclear Matter

The extension of the quasipotential approach to nuclear matter is known as Dirac–Brueckner–Hartree–Fock theory [An83,Ho84a,Br84,Ho87,Te87,Am92]. The scattering matrix T in Eq. (4.2) is replaced by a reaction matrix Γ, which is determined by the quasipotential equation

$$\Gamma = V + \int V g \Gamma \; , \tag{4.4}$$

where V is the (ladder-approximated) quasipotential and g is a unitarized, two-nucleon propagator that includes interactions with the surrounding medium [Ho84a,Se86,Ho87]. In practice, one solves for matrix elements of Γ, so that the driving term on the right-hand side involves matrix elements of V; the new ingredient is that the Dirac wave functions must be determined *self-consistently*, since the nucleons have large scalar and vector self-energies, analogous to the mean fields studied earlier.

If one writes the nucleon self-energy $\Sigma(k)$ in (the rest frame of) nuclear matter in terms of scalar (Σ^s), timelike vector (Σ^0), and three-vector (Σ^v) parts [Se86]:

$$\Sigma(k) = \Sigma^s(k) - \gamma^0 \Sigma^0(k) + \boldsymbol{\gamma} \cdot \mathbf{k} \, \Sigma^v(k) \; , \tag{4.5}$$

then the self-energy is determined by summing effective direct and exchange interactions between nucleon pairs, which may be written schematically as

$$\Sigma(k) = \sum_{E_p \leq E_F} \left[\langle \mathbf{kp} | \Gamma | \mathbf{kp} \rangle - \langle \mathbf{kp} | \Gamma | \mathbf{pk} \rangle \right] \; . \tag{4.6}$$

This is indeed a self-consistency condition, since the self-energy determines the Dirac wavefunctions and the reaction-matrix elements that appear on the right-hand side, and it is in turn determined by these quantities. The Σ^s and Σ^0 components are analogous to the mean fields $(-g_s\phi_0)$ and $(-g_v V_0)$ studied earlier; Σ^v is a new contribution that arises because exchange diagrams are included.

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The numerous approximations and procedures that go into solving the DBHF equations are discussed at length in [Ho84a,Br84,Se86,Ho87,Te87,Am92]. The most important approximation is the use of a quasipotential, which implies that the four-dimensional integral in the box diagram that forms the fundamental unit of the ladder sum is reduced to a three-dimensional integral. There are many possible reductions that can be identified as the relativistic extension of the nonrelativistic Bethe–Goldstone equation [Wa95]; unfortunately, numerical results for the nuclear matter binding energy are sensitive to the reduction used. Even after this reduction, calculated results are sensitive to the high-momentum part of the loop integrals, implying important contributions from baryon transitions to states lying well above the Fermi surface. Phenomenological form factors inserted at the vertices significantly reduce this sensitivity, but questions about the off-mass-shell and density dependence of these form factors remain to be studied. Moreover, since the self-energy Σ enters explicitly in the self-consistent baryon spinors, the binding energy of nuclear matter is sensitive to the self-consistency condition [Le89a]. At present, the construction of a self-consistency condition that leads to a conserving approximation when relativistic ladder diagrams are summed is still a controversial topic [Po88,De91,Hu93].

It must be emphasized that many of the conventional approximations used in DBHF theory have no counterparts in nonrelativistic Brueckner–Goldstone theory, and thus the sensitivity to these approximations has never been systematically tested. Nevertheless, it is still possible to make three important *qualitative* statements about the effects of relativistic two-nucleon correlations in nuclear matter:

1. Although the correlation corrections produce changes in the RMFT nuclear matter binding energy that are of the same order as the binding energy itself, *the corrections to the large* RMFT *scalar and vector self-energies are small* [Ho87]. Thus the DBHF self-energies are essentially the same size as the scalar and vector mean fields studied earlier. (Full DBHF calculations yield typical results of $0.55 \lesssim M^*/M \lesssim 0.65$ at equilibrium density [Ma89,Br90a,De91,Am92].) Moreover, the momentum dependence (or state dependence) of Σ^s and Σ^0 is small; these self-energies are essentially constant for occupied states in the Fermi sea. Finally, the new term Σ^v is small. The conclusion is that the successful RMFT picture presented above involving large, constant, Lorentz scalar and vector fields persists when two-nucleon correlations are included.[18]

2. The depletion of the Fermi sea due to correlation effects is considerably smaller in the relativistic framework than in the nonrelativistic framework [Ja90].

3. The self-consistency condition (4.6) introduces a density dependence into the effective interaction Γ that goes beyond what is included in nonrelativistic Brueckner–Goldstone theory. Because of this extra density dependence, it is possible to *simultaneously* fit both the NN phase shifts and the nuclear matter equilibrium point at the two-hole-line level [Ma89,Br90a,De91]. Although an accurate calculation of the nuclear matter

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[18]The large scalar and vector self-energies are also consistent with recent analyses based on QCD sum rules [Co91a,Co92a,Fu92a,Ji93a,Ji94a].

equilibrium point is difficult, due to the sensitive cancellations, and there are still open questions about three-hole-line corrections, true many-body forces, and the role of the quantum vacuum, this successful result *cannot be obtained* in a nonrelativistic framework with two-body potentials [Da83,Da85]. Further investigations are needed to make this result more conclusive.

C. Relation to Nonrelativistic Calculations

To close this section, we briefly examine why the DBHF theory produces more favorable results for nuclear matter saturation at the two-hole-line level than corresponding nonrelativistic Brueckner–Goldstone calculations. It is well known that nonrelativistic calculations based on realistic NN potentials predict equilibrium points that have either the correct density but too little binding energy, or the correct binding energy at too high a density; this produces the familiar "Coester line". What is needed is an additional, density-dependent repulsive mechanism, and the new saturation mechanism discussed in Section 2 is precisely of this type. Because of Lorentz covariance and self-consistency, as the nuclear density increases, the nucleon effective mass M^* decreases, and the nucleon velocities increase. This weakens the attractive force, and the net result is an increased repulsion.

Although this additional repulsion is often described as a purely "relativistic" effect, it is easy to see that in a nonrelativistic framework, some of it can be attributed to three- and many-body forces [Br87,Fo95]. Rather than work with a complicated DBHF interaction, the basic point can be illustrated using the RMFT model of Section 2; for suitable choices of couplings and masses, simple scalar and vector exchange resemble quite closely the scalar and vector components of the DBHF interaction Γ. (We are not concerned here with numerous details like pi- and rho-exchange diagrams, the pion-exchange "tensor force", etc. We will also set the nonlinear scalar couplings κ and λ to zero to simplify the equations.)

The basic idea is to expand the RMFT nuclear matter energy density of Eq. (2.17) in powers of the Fermi momentum $k_{\rm F}$, and then to group terms together to isolate various powers of $\rho_{\rm B}$. One can then identify contributions from increasing powers of $\rho_{\rm B}$ as arising from two-body forces, three-body forces, etc., if one attempted to reproduce the same energy/nucleon with nonrelativistic potentials. We begin with the expansion of the self-consistency condition (2.19), which is given through order $k_{\rm F}^{11}$ by [Se86,Fo95]

$$M^* = M - \frac{g_{\rm s}^2 \rho_{\rm B}}{m_{\rm s}^2} \left[1 - \frac{3k_{\rm F}^2}{10M^2} + \frac{9k_{\rm F}^4}{56M^4} - \frac{5k_{\rm F}^6}{48M^6} + \frac{105k_{\rm F}^8}{1408M^8} \right.$$
$$\left. - \frac{3}{5} \frac{g_{\rm s}^2}{m_{\rm s}^2} \rho_{\rm B} \left(\frac{k_{\rm F}^2}{M^3} - \frac{48k_{\rm F}^4}{35M^5} \right) - \frac{9}{10} \left(\frac{g_{\rm s}^2}{m_{\rm s}^2} \right)^2 \left(\frac{\rho_{\rm B}^2 k_{\rm F}^2}{M^4} \right) + \cdots \right] . \qquad (4.7)$$

Here $\rho_{\rm B} = \gamma k_{\rm F}^3/6\pi^2$ as usual.

Substitution of this result into Eq. (2.17) and expansion through order $k_{\rm F}^{11}$ yields, for the energy/nucleon,

$$\mathcal{E}/\rho_{\mathrm{B}} = M + \left[\frac{3k_{\mathrm{F}}^2}{10M} - \frac{3k_{\mathrm{F}}^4}{56M^3} + \frac{k_{\mathrm{F}}^6}{48M^5} - \frac{15k_{\mathrm{F}}^8}{1408M^7} + \frac{21k_{\mathrm{F}}^{10}}{3328M^9} + \cdots \right]$$

$$+ \frac{g_{\mathrm{v}}^2}{2m_{\mathrm{v}}^2}\rho_{\mathrm{B}} - \frac{g_{\mathrm{s}}^2}{2m_{\mathrm{s}}^2}\rho_{\mathrm{B}} + \frac{g_{\mathrm{s}}^2}{m_{\mathrm{s}}^2}\frac{\rho_{\mathrm{B}}}{M}\left[\frac{3k_{\mathrm{F}}^2}{10M} - \frac{36k_{\mathrm{F}}^4}{175M^3} + \frac{16k_{\mathrm{F}}^6}{105M^5} - \frac{64k_{\mathrm{F}}^8}{539M^7} + \cdots \right]$$

$$+ \left(\frac{g_{\mathrm{s}}^2\rho_{\mathrm{B}}}{m_{\mathrm{s}}^2M} \right)^2 \left[\frac{3k_{\mathrm{F}}^2}{10M} - \frac{351k_{\mathrm{F}}^4}{700M^3} + \cdots \right] + \left(\frac{g_{\mathrm{s}}^2\rho_{\mathrm{B}}}{m_{\mathrm{s}}^2M} \right)^3 \left[\frac{3k_{\mathrm{F}}^2}{10M} - \cdots \right] . \qquad (4.8)$$

The first term is the baryon rest mass, followed by the nonrelativistic Fermi-gas energy and the first few relativistic corrections, which are essentially negligible at equilibrium density. (These terms are the ones usually used to justify a nonrelativistic treatment of the nuclear matter problem [Ne85].) The next two terms (proportional to ρ_{B}) give the nonrelativistic limit of the potential energy coming from the vector and scalar mesons. The following term in brackets (with overall factor ρ_{B}) is a relativistic correction to the scalar potential energy that arises from the Lorentz contraction factor in the scalar density, evaluated for nucleons of mass M. The final two terms (with overall factors of ρ_{B}^2 and ρ_{B}^3) are also corrections to the scalar potential energy. These arise from the self-consistency condition (4.7) on M^*; self-consistency implies $M^* < M$, which increases the velocities of the nucleons and thus also increases their energies. These repulsive contributions are a signature of the velocity dependence inherent in a Lorentz scalar interaction. Nevertheless, the leading contributions to these terms (in powers of ρ_{B}) can be reproduced in a nonrelativistic calculation by including repulsive three-nucleon and four-nucleon potentials [Br87,Fo95].

There are several relevant points to be noted:

- Important "relativistic" effects in the RMFT of nuclear matter are equivalent to many-body forces in a nonrelativistic framework.

- Although only the three- and four-body contributions are shown in Eq. (4.8), *all* of the leading terms at each power of ρ_{B} arising from the self-consistency condition are *repulsive*. Together with the Lorentz contraction contribution, they provide the new saturation mechanism discussed earlier.

- Since the scalar and vector mesons have relatively large masses (compared to the pion), the nonrelativistic many-body potentials used to generate these terms will be short-ranged [Co95a].

- With typical values for the RMFT parameters, one finds $g_{\mathrm{s}}^2\rho_{\mathrm{B}}/m_{\mathrm{s}}^2M \approx 0.5$ at equilibrium density. It follows that the three- and four-body repulsive terms in Eq. (4.8) are roughly 5 MeV and 2.5 MeV, respectively, which are significant on the scale of the nuclear matter binding energy.

- These relativistic (or many-body, if you prefer) contributions are inherent in the DBHF framework and allow a two-hole-line calculation to (essentially) reproduce the equilibrium point of nuclear matter, with an NN interaction fitted to phase shifts (see Fig. 10.13 in [Ma89]). Although, to our knowledge, a relativistic three-hole-line calculation of nuclear matter has never been performed, one might hope that the smaller

depletion of the Fermi sea (as noted above) would lead to smaller three-body corrections than in the nonrelativistic case, implying that the two-hole-line results are fairly robust. Relativistic three-body-cluster effects remain to be investigated.

5. EFFECTIVE FIELD THEORY (EFT)

A. Introduction

Quantum chromodynamics (QCD) is generally accepted as the underlying theory of the strong interaction. As we have seen, however, at energies relevant for most nuclear phenomena, hadrons are convenient and efficient degrees of freedom. In particular, meson-exchange models of the NN interaction accurately describe low-energy properties of the two-nucleon system, and relativistic mean-field theory based on QHD–I and QHD–II provides a realistic description of the bulk and single-particle properties of nuclei.

Unfortunately, renormalizable QHD models have encountered difficulties due to large effects from loop integrals that incorporate the dynamics of the quantum vacuum [Co87, Pe87,Fu88,Fu89,We90,Li90a]. On the other hand, the "modern" approach to renormalization [Le89,Po92,Ge94,We95], which makes sense of effective, "cutoff" theories[19] with low-energy, composite degrees of freedom, provides an alternative. When composite degrees of freedom are used, the structure of the particles is described with increasing detail by including more and more *nonrenormalizable* interactions in a derivative expansion [Le89]. Moreover, nonrenormalizable interaction terms between the boson fields allow us to describe the short-distance behavior of the underlying theory of QCD. So, based on the successful phenomenology we have seen so far, we would like to generalize the description using the modern viewpoint of effective field theory (EFT).

The strategy underlying EFT relies on two basic observations. First, one argues that relativistic quantum field theory is simply the most general way to parametrize an S matrix (or other observables) consistent with analyticity, unitarity, causality, cluster decomposition, and symmetries (*e.g.*, Lorentz covariance, chiral symmetry, . . .) [We95]. With this view, there is no reason that relativistic quantum field theory should be reserved for "elementary" particles only.

Second, one observes that in most problems in physics, the relevant phenomena are confined to a specific length scale, and thus it is not necessary to explicitly include dynamics at significantly shorter length scales [Ge93]. This implies that, at least formally, one can construct an effective field theory to be used at a given length scale by "integrating out" heavier degrees of freedom corresponding to shorter length scales; the effects of these heavier degrees of freedom will be implicitly contained in various coupling parameters in the low-energy, effective theory [Le89,Ka95a,Ma96]. By fitting these parameters to experimental data, one can derive relationships between different observables within the dynamical regime of interest.

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[19]Here "cutoff" means a regulator that maintains the appropriate symmetries.

Note that this strategy is *opposite* to the strategy of renormalizable field theory, where one argues that the *fixed* number of unknown parameters can be determined at any convenient length scale and then extrapolated to any other length scale using the equations of the "renormalization group". Indeed, the effective theory is expected to contain *numerous* couplings of nonrenormalizable form when one integrates out the heavier degrees of freedom. These nonrenormalizable couplings incorporate the "compositeness" of the low-energy degrees of freedom through their implicit dependence on short-distance physics.

These considerations imply, of course, that the low-energy effective theory will generally contain an infinite number of interaction terms, and thus one needs an *organizing principle* to make sensible calculations. First, one must find a suitable expansion parameter (or parameters) that is small in the region of interest. Second, one assumes "naturalness", which means that all of the unknown couplings in the theory, when written in appropriate dimensionless form (as discussed below), are of order unity. Thus one can estimate contributions from various terms by counting powers of the expansion parameter(s) and then truncate the lagrangian at the desired level of accuracy.

Although the resulting framework is apparently structured less rigidly than the more familiar edifice of renormalizable quantum field theory, EFT nevertheless has its own "rules of the game": First, one cannot simply limit calculations to the tree level; loops can and must be included, as this is the only way to correctly incorporate unitarity. Second, unknown parameters are to be determined either by explicitly integrating out the short-distance physics and "matching" the low-energy parameters to the results (if the underlying theory is tractable) or by fitting to experiment (where, hopefully, the number of parameters is fewer than the number of data to be described).

In the case of the nuclear many-body problem, it is still impossible to compute the desired low-energy parameters by working directly with QCD. Nevertheless, important constraints on the effective hadronic lagrangian are established by maintaining the symmetries of QCD. These include Lorentz invariance, parity conservation, isospin invariance, chiral symmetry, and electromagnetic gauge invariance. These symmetries constrain the lagrangian most directly by restricting the form of possible interaction terms; if one is forced to fit unknown parameters to the data, one must include in the effective lagrangian all (non-redundant) terms that are consistent with the underlying symmetries. Moreover, the nature of the symmetries may also dictate the appropriate low-energy degrees of freedom, or give relationships between some of the unknown parameters.

Redundancy may arise because there is significant freedom in choosing the generalized coordinates of the effective lagrangian. In contrast to renormalizable theories, where there is (usually) a preferred choice of field coordinates in which the lagrangian is manifestly renormalizable, the choice of field variables in an effective theory is motivated by the desire to make the description of the interactions as efficient as possible. (This is similar to the situation in classical lagrangian mechanics.) If a point transformation of the fields (subject to some mild constraints [Co69]) allows one to eliminate certain interaction terms, they can be considered redundant [Ba88,Ge91,Ba94b]. The goal is then to find the best set of generalized coordinates (fields), so that the inevitable truncations are as accurate as possible.

A well-known application of EFT is chiral perturbation theory (ChPT), in which one observes that the spontaneous breaking of chiral symmetry in QCD implies that Goldstone bosons (pions, ...) are the relevant low-energy degrees of freedom [Le94]. Chiral symme-

try also implies that the pion interactions can be grouped order-by-order in the number of derivatives, so that there is a systematic expansion at low energies in powers of external momenta (and m_π^2) [Ga84]. Moreover, because the loop expansion also proceeds in powers of momenta, one can systematically compute loop corrections. ChPT has been reasonably successful in describing scattering in the $B = 0$ and $B = 1$ sectors of low-energy QCD [Do92,Me93a]. Studies of two- and many-nucleon systems have been initiated and are currently under active investigation [Or92,Va93,Ly93,Or96,Ka96]. However, the prospects for extending ChPT to many-body calculations at finite density are unclear at present.

Thus we are motivated to consider alternatives. How can we exploit the ideas of EFT to develop a systematic, nonrenormalizable approach to nuclear structure? Let us enumerate the relevant concepts:

1. Pions can be included by applying the framework of ChPT.

2. Nucleon fields are necessary, since these are the observed fermionic degrees of freedom at low energies; that is, nucleons carry the conserved baryon number B. Moreover, nucleon compositeness is retained even at the "tree" level, if we include nonrenormalizable interaction terms, as noted earlier.

3. QCD constraints will be imposed through *symmetries*; all allowed (non-redundant) terms must be included.

4. Since redefinitions of the field variables do not affect observables, we must strive for the most efficient variables and parametrizations of the interactions.

5. We must identify suitable expansion parameters. Based on our earlier discussion, we know that the nuclear mean fields (or self-energies) $\Phi \equiv g_s \phi_0$ and $W \equiv g_v V_0$ are roughly several hundred MeV at ordinary densities. Thus we take as expansion parameters the ratios Φ/M and W/M, which are small at normal densities, and also the ratios of gradients of the fields to M^2, which are small in nuclei ($|\boldsymbol{\nabla}\Phi|/M^2 \approx |\boldsymbol{\nabla}W|/M^2 \lesssim 0.1$) [Fu95]. If we then assume that the unknown coefficients are natural, we can truncate the effective lagrangian. Of course, after the calculations are finished and the parameters have been fitted to empirical data, we must check that the naturalness criterion is satisfied.

6. What about non-Goldstone bosons, like ω, ρ, and our effective scalar field ϕ? We argue that these are known to be useful degrees of freedom, since the mid-range NN interaction can be efficiently described by the exchange of these mesons [Ma87,Ma89]. Moreover, the coupling constants in ChPT at $O(E^4)$ in the meson sector can be reproduced by a meson-resonance lagrangian applied at tree level, with the vector mesons playing the leading role [Me88,Do89,Ec89,Ec89a]; this is consistent with the well-known hypothesis of vector-meson dominance [Sa69]. Finally, we have already noted that two-pion exchange in the scalar-isoscalar channel can be efficiently simulated by a low-mass scalar field.[20] Even though both the nucleon and scalar are composite ob-

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[20]Recent results indicate that a low-mass scalar plays a similar role in πN scattering [Me96].

jects, recent work shows that the leading term [of $O(\Phi^2/M)$] in the interaction energy of a nucleon in a classical scalar field is *model independent* and determined solely by Lorentz covariance [Wa95a,Bi95].

7. An accurate description of pion interactions with few-body systems and nuclei requires that the Δ resonance be included. This can be done most efficiently by introducing the Δ as another effective degree of freedom [Pe68,Bi82,De92a,We93,Ta96]. For simplicity, and because we are concerned primarily with bulk and isoscalar properties of nuclei, we will omit Δ interactions in the models discussed below.

Thus our strategy will be to construct an EFT lagrangian containing nucleons, pions, and low-mass scalar and vector mesons. Heavier particles (with masses greater than roughly 1 GeV) will be integrated out.

B. Naive Dimensional Analysis

There are still two important points to be addressed. First, we must understand how to extract the dimensional scales of each term in the lagrangian, so that the remaining dimensionless constants can be checked for naturalness. A naive dimensional analysis (NDA) for assigning a coefficient of the appropriate size to any term in the effective lagrangian has been proposed by Manohar and Georgi [Ma84a,Ge93a]. This allows for a determination of both the dimensional scales associated with each term and for the inclusion of an overall dimensionless constant that can be used to adjust the strength. The basic assumption of naturalness is that once the appropriate dimensional scales have been extracted, the overall dimensionless coefficients should all be of order unity. The NDA rules for a given term in the lagrangian density are:

(1) Include a factor of $1/f_\pi$ for each strongly interacting field.

(2) Assign an overall factor of $f_\pi^2 M^2$.

(3) Multiply by factors of $1/M$ to achieve dimension $(\text{mass})^4$.

(4) Include appropriate counting factors (such as $1/n!$ for ϕ^n).

Here $f_\pi \approx 93\,\text{MeV}$ is the pion-decay constant, and the nucleon mass M is taken as the generic large-momentum cutoff scale, which characterizes the mass scale of physics beyond Goldstone bosons. In some cases [Fu96b], it is more appropriate to use the (non-Goldstone) meson masses rather than M, but we usually do not distinguish here between the two.

As noted by Georgi [Ge93a], rule (1) simply assumes that the amplitude for producing any strongly interacting particle is proportional to the amplitude f_π for emitting a Goldstone boson. This is a reasonable assumption, since f_π is the only natural scale. Thus, by dividing each field by f_π, we should arrive at a factor of $O(1)$. Rule (2) can be understood as an overall normalization factor that arises from the standard way of writing the mass terms of non-Goldstone bosons. For example, one may write the mass term of a scalar-isoscalar field $\phi(x)$ as

$$\frac{1}{2} m_\mathrm{s}^2 \phi^2 = \frac{1}{2} f_\pi^2 M^2 \frac{m_\mathrm{s}^2}{M^2} \frac{\phi^2}{f_\pi^2} \ , \tag{5.1}$$

where the scalar mass m_s is treated as roughly the same size as M. By applying rule (1) and extracting the overall factor of $f_\pi^2 M^2$, the remaining ratios are of $O(1)$. Since all terms will have the same overall scale factor $f_\pi^2 M^2$, higher-order terms or terms with gradients of fields will be suppressed by powers of $1/M$ relative to the leading mass terms, as a result of "integrating out" physics above the scale M. (A simple example is the low-momentum expansion of a tree-level propagator for a heavy meson of mass m_H, which leads to terms with powers of ∂^2/m_H^2.) It is exactly because of these $1/M$ suppression factors and dimensional analysis that one arrives at rule (3). The origin of the combinatorial factors in rule (4) is discussed in [Fu96a].

Applying these rules to a generic term in the effective lagrangian involving the isoscalar fields and the nucleon field leads to (generalization to include the pion, rho, and photon is straightforward) [Fr96,Fu96a]

$$\mathcal{L} \sim g \frac{1}{m!} \frac{1}{n!} \left(\frac{\overline{\psi} \Gamma \psi}{f_\pi^2 M} \right)^\ell \left(\frac{\phi}{f_\pi} \right)^m \left(\frac{V}{f_\pi} \right)^n \left(\frac{\partial \text{ or } m_\pi}{M} \right)^p f_\pi^2 M^2 \ , \tag{5.2}$$

where ψ is a baryon field, Γ is any Dirac matrix, derivatives are denoted generically by ∂, and we have allowed for the possibility of chiral-symmetry-violating terms that contain the small parameter m_π/M. The product of all the dimensional factors then sets the scale in terms of the pion-decay constant f_π and the nucleon mass M. The overall coupling constant g is dimensionless and of $O(1)$ if naturalness holds.

These scaling rules imply that a general potential for the scalar meson can be expanded as

$$V_\mathrm{S} = m_\mathrm{s}^2 \phi^2 \left(\frac{1}{2} + \frac{\kappa_3}{3!} \frac{g_\mathrm{s} \phi}{M} + \frac{\kappa_4}{4!} \frac{g_\mathrm{s}^2 \phi^2}{M^2} + \cdots \right) \ . \tag{5.3}$$

Here we have included a factor of $1/f_\pi$ for each power of ϕ; these factors are then eliminated in favor of $g_\mathrm{s} \approx M/f_\pi$, which is basically the Goldberger–Treiman relation [Wa95]. Factorial counting factors are also included, since the NDA rules are actually meant to apply to the tree-level scattering amplitude generated by the corresponding vertex [We90a,Fu96a].

The "naturalness" assumption states that after the dimensional factors and appropriate counting factors are extracted, the overall dimensionless coefficients [g in Eq. (5.2) and the κ_3, κ_4, ... in Eq. (5.3)] should be of order unity. It should be clear, however, that the preceding arguments are not a *proof* of naturalness, since we know of no physical law that forbids large coefficients from appearing. Nevertheless, without such an assumption, it is basically impossible to construct an effective lagrangian with any predictive power.[21] Until

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[21]The assumption of renormalizability also leads to a finite number of parameters and well-defined predictions, but does so by imposing *unnatural* restrictions on the lagrangian, namely, that many parameters are identically zero in the absence of relevant symmetry arguments.

one can derive the effective hadronic lagrangian from QCD, the naturalness assumption must be checked by fitting to experimental data.

The second point to be addressed is that we are actually fitting the QHD parameters based on calculations of finite-density observables (rather than scattering observables). One could, of course, fit the parameters to scattering observables, and then perform corresponding calculations of nuclear properties, but we must nevertheless understand how fitting parameters at the mean-field level is to be interpreted in the context of EFT. For this interpretation, we rely on the ideas of density functional theory, which we discuss in Section 6.

C. Nonlinear Chiral Symmetry Revisited

Before applying the ideas of EFT to the construction of a model lagrangian, we illustrate some basic features of nonlinear realizations of chiral symmetry by building on our earlier results. Return to the lagrangian of the chiral $\sigma\omega$ model, which (after spontaneous symmetry breaking) is given in Eq. (3.9). Recall that the nucleon mass M is related to the vacuum expectation value of the original scalar field (which we denote by s_0) through

$$s_0 = \frac{M}{g_\pi} \ . \tag{5.4}$$

We now introduce an $SU(2)$ matrix $\underline{U}$ defined in terms of the pion fields by

$$\underline{U} \equiv \exp\left(\frac{i}{s_0}\,\boldsymbol{\tau}\cdot\boldsymbol{\pi}\right) \ , \tag{5.5}$$

as well as left- and right-handed baryon fields

$$\psi_L \equiv \frac{1}{2}\,(1 - \gamma_5)\psi \ , \quad \psi_R \equiv \frac{1}{2}\,(1 + \gamma_5)\psi \ , \tag{5.6}$$

where ψ is the baryon field in Eq. (3.9). Consider now the following phenomenological, nonlinear generalization $\mathcal{L}$ of Eq. (3.9):[22]

$$\mathcal{L} = i\Big[\overline{\psi}_R\gamma_\mu(\partial^\mu + ig_{\rm v}V^\mu)\psi_R + \overline{\psi}_L\gamma_\mu(\partial^\mu + ig_{\rm v}V^\mu)\psi_L\Big] - g_\pi s_0\Big(1 - \frac{\sigma}{s_0}\Big)\Big[\overline{\psi}_R\underline{U}^\dagger\psi_L + \overline{\psi}_L\underline{U}\psi_R\Big]$$

$$+ \tfrac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma) + \tfrac{1}{4}s_0^2\,{\rm tr}\,(\partial_\mu\underline{U}\partial^\mu\underline{U}^\dagger) - \mathcal{V}(\underline{U}, \partial_\mu\underline{U}; \sigma) + \tfrac{1}{4}m_\pi^2 s_0^2\,{\rm tr}\,(\underline{U} + \underline{U}^\dagger - 2)$$

$$- \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m_{\rm v}^2 V_\mu V^\mu \ . \tag{5.7}$$

Here $\mathcal{V}$ is a meson potential built from the indicated fields and their derivatives. For $m_\pi^2 = 0$, the new lagrangian is invariant under chiral $SU(2)_{\rm L} \times SU(2)_{\rm R}$ transformations of the form

$$\psi_L \to \underline{L}\psi_L \ , \quad \psi_R \to \underline{R}\psi_R \ , \quad \underline{U} \to \underline{L}\,\underline{U}\,\underline{R}^\dagger \ , \tag{5.8}$$

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[22]At this point, we free ourselves from the constraint of renormalizability.

where $\underline{L}$ and $\underline{R}$ are independent, *global* $SU(2)$ matrices, as long as $\mathcal{V}$ is chosen to be invariant. (The σ and V^μ fields are unchanged.)

It is now a simple exercise to show that in the limit $s_0 \to \infty$, in which the nucleon mass M becomes very large, the two lagrangians are identical:

$$\mathcal{L} \longrightarrow \mathcal{L}_{\sigma\omega} \ , \qquad s_0 \to \infty \ , \tag{5.9}$$

provided only that the potential in $\mathcal{L}$ is chosen to take the simple form

$$\mathcal{V}(\underline{U}, \partial_\mu \underline{U}; \sigma) \equiv \frac{1}{2} m_\sigma^2 \sigma^2 + O\Big(\frac{1}{s_0}\Big) \ . \tag{5.10}$$

Inspection of Eq. (3.9) shows that the meson interactions in the $\sigma\omega$ model reduce to this form in the indicated limit. More generally, different choices for $\mathcal{V}$ lead to different descriptions of the spontaneous breaking of chiral symmetry, all of which preserve the $SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}}$ invariance of the lagrangian.

To proceed further with $\mathcal{L}$ of (5.7), we implement the following clever change of variables, which leaves the particle content unchanged [Do92]:

$$\underline{U} \equiv \underline{\xi}\underline{\xi} \ , \quad N_L \equiv \xi^\dagger \psi_L \ , \quad N_R \equiv \xi \psi_R \ . \tag{5.11}$$

This change of variables takes the "square root" of $\underline{U}$ and mixes the pions into the nucleons in a manner reminiscent of the chiral transformation in Eq. (3.16). Straightforward algebra now shows that the part of $\mathcal{L}$ containing the fermion fields becomes (we leave the remainder of $\mathcal{L}$ in terms of $\underline{U}$)

$$\mathcal{L}_{\mathrm{fermion}} = \overline{N}\Big[i\gamma^\mu(\partial_\mu + i\underline{v}_\mu + ig_{\mathrm{v}} V_\mu) + \gamma^\mu \gamma_5 \underline{a}_\mu - M + g_\pi \sigma\Big] N \ , \tag{5.12}$$

where

$$\underline{v}_\mu \equiv -\frac{i}{2}(\underline{\xi}^\dagger \partial_\mu \underline{\xi} + \xi \partial_\mu \xi^\dagger) \ , \quad \underline{a}_\mu \equiv -\frac{i}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger) \ . \tag{5.13}$$

The new form of the lagrangian is invariant under the following nonlinear chiral transformation [Do92]

$$\underline{\xi}(x) \to \underline{L}\underline{\xi}(x)\underline{h}^\dagger(x) = \underline{h}(x)\underline{\xi}(x)\underline{R}^\dagger \ , \tag{5.14}$$

$$N(x) \to \underline{h}(x)N(x) \ , \tag{5.15}$$

where $\underline{h}(x)$ is a *local* $SU(2)$ isospin transformation that is defined by Eq. (5.14) and that depends on the pion field through ξ. This transformation implies

$$\underline{a}_\mu \to \underline{h}\underline{a}_\mu \underline{h}^\dagger \ , \qquad \underline{v}_\mu \to \underline{h}\underline{v}_\mu \underline{h}^\dagger - i\underline{h}\partial_\mu \underline{h}^\dagger \ . \tag{5.16}$$

By using the relation $\partial_\mu(\underline{h}^\dagger \underline{h}) = 0$, one also obtains

$$(\partial_\mu + i\underline{v}_\mu)N \to \underline{h}[(\partial_\mu + i\underline{v}_\mu)N] \ , \qquad \underline{U} \to \underline{L}\underline{U}\underline{R}^\dagger \ , \tag{5.17}$$

which means that the indicated derivative of the nucleon field transforms *covariantly* [that is, the same way as the nucleon field in Eq. (5.15)], and the pion matrix $\underline{U}$ transforms *globally*. Thus the remaining parts of the lagrangian $\mathcal{L}$ remain unchanged under the transformation (except for the symmetry-violating term proportional to m_π^2).

This new realization of the chiral symmetry has the following important properties:

- Parity is conserved.

- The fermions (baryons) appear in isospin multiplets.

- A baryon mass term is allowed in the lagrangian.

- The Goldstone bosons (pions) arising from the spontaneous chiral symmetry breaking enter through the chiral matrices U and ξ.

- The pions are coupled to the nucleons with derivative couplings [Eqs. (5.12) and (5.13)]; hence one reproduces all the soft-pion results implied by chiral symmetry.

- The isospin transformation $\underline{h}(x)$, which is an element of the *unbroken* $SU(2)_{\mathrm{V}}$ subgroup of the full $SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}}$ group, is *local*: it depends on x because the pion fields contained in ξ depend on x.

- The scalar field σ (which still appears in the theory) and vector field V^μ are chiral scalars. They can therefore be removed without destroying the invariance of the lagrangian, or alternatively, additional chiral scalars can be included.

- One can rewrite the preceding results in conventional form by identifying

$$s_0 = f_\pi \ . \tag{5.18}$$

In the elegant papers [Ca69,Co69], it is shown that *any* nonlinear realization of $SU(2)_L \times SU(2)_R$ with these properties can be brought into the form of Eqs. (5.14) and (5.15).

D. A Nonlinear Chiral Model with Vector-Meson Dominance

We now show how the ideas of EFT can be combined with the hadronic phenomenology discussed in Sections 2–4 by constructing a nonrenormalizable, effective lagrangian that realizes chiral symmetry in a nonlinear fashion [Fu95,Fu96a]. Vector mesons are included in a manner consistent with both chiral symmetry and vector-meson dominance (VMD), and a light scalar field is introduced as before to simulate the exchange of two correlated pions between nucleons. Although the dynamics described by these non-Goldstone bosons could also be generated through pion-loop diagrams, the advantage is that we can now avoid (at least initially) the evaluation of complicated loop integrals in studying the many-body problem. Moreover, the low-energy electromagnetic structure of the nucleon is described *within the theory* using VMD, so that *ad hoc* form factors are not needed, as in the calculations of Section 2.B.

The general framework for nonlinear realizations of chiral symmetry is stated very compactly in the original work of Callan, Coleman, Wess, and Zumino (CCWZ) [Ca69,We96], and here we paraphrase their discussion. (The results of the previous subsection illustrate a specific example of these ideas.) We assume that G is a compact, connected, semisimple Lie group that has a continuous subgroup H. We denote by V_i and A_ℓ a complete, orthonormal set of generators of G, such that V_i are the generators of H. In some neighborhood of the identity of G, every group element $g \in G$ can be uniquely decomposed as

$$g = \mathrm{e}^{\alpha \cdot A}\, \mathrm{e}^{\beta \cdot V} \ , \qquad\qquad (5.19)$$

where $\alpha \cdot A = \sum_\ell \alpha_\ell A_\ell$, $\beta \cdot V = \sum_i \beta_i V_i$, and α_ℓ and β_i are real constants.

A nonlinear realization of G that becomes a linear representation when $g \in H$ is given on the local field variables (ω, ψ) by the transformation

$$(\omega, \psi) \longrightarrow g(\omega, \psi) = (\omega', \psi') \ , \qquad\qquad (5.20)$$

where

$$g\, \mathrm{e}^{\omega \cdot A} = \mathrm{e}^{\omega' \cdot A}\, \mathrm{e}^{u' \cdot V} \ , \quad \psi' = D(\mathrm{e}^{u' \cdot V})\psi \ , \qquad\qquad (5.21)$$

$\omega' = \omega'(\omega; g)$, and $u' = u'(\omega; g)$. Here ω represents the Goldstone boson fields, ψ denotes the other fields in the theory, and ω' is generally a nonlinear function of ω. $D(h)$, with $h \in H$, is a linear, unitary representation of the unbroken subgroup H, which is assumed to be written in fully reduced form. In [Co69] it is proved that any nonlinear realization of G that is linear on H can be brought into the preceding form through a suitable redefinition of field variables.

Unlike a *linear* realization of the symmetry, in which all particles must be assigned to a representation of the full group G, the non-Goldstone fields ψ in a *nonlinear* realization are in representations of the unbroken subgroup H only, just as in the example in the previous subsection (where the unbroken group is the $SU(2)_\mathrm{V}$ of isospin). Information about the structure of the full group G is encoded in the Goldstone fields ω and the local transformation $D(\mathrm{e}^{u' \cdot V})$, with $\mathrm{e}^{u' \cdot V} \in H$. Thus a nonlinear realization eliminates the necessity of assigning particles to chiral multiplets, as in the linear σ model and its extensions [Se92b]. These assignments are additional dynamical input that go beyond the assumption of group invariance alone [Co69].

We now proceed to construct our model following the strategy outlined at the end of Section 5.A. We include fields for the nucleon, pion, rho meson, and omega meson, together with an effective scalar-isoscalar field to simulate two-pion exchange; the Δ will be omitted here for brevity, but its inclusion is discussed in [Ta96]. The Goldstone pion fields $\pi^a(x)$, with $a = 1, 2,$ and 3, form an isovector, which can be considered as the phase of a chiral rotation matrix:

$$\underline{\xi}(x) \equiv \exp(i\,\underline{\pi}(x)/f_\pi) \ , \quad \underline{\pi}(x) \equiv \frac{1}{2}\, \boldsymbol{\tau} \cdot \boldsymbol{\pi}(x) \ . \qquad\qquad (5.22)$$

Here the τ^a are Pauli matrices and $f_\pi \approx 93\,\mathrm{MeV}$ is the pion decay constant. The isospinor nucleon field is represented by a column matrix

$$\underline{N}(x) = \begin{pmatrix} p(x) \\ n(x) \end{pmatrix} \ , \qquad\qquad (5.23)$$

where $p(x)$ and $n(x)$ are the proton and neutron fields respectively. The rho fields $\rho_\mu^a(x)$ also form an isovector, and we use the notation $\underline{\rho}_\mu(x) \equiv \frac{1}{2}\boldsymbol{\tau} \cdot \boldsymbol{\rho}_\mu(x)$.

A nonlinear realization of the chiral group $SU(2)_\mathrm{L} \times SU(2)_\mathrm{R}$ is now defined such that for arbitrary *global* matrices $\underline{L} \in SU(2)_\mathrm{L}$ and $\underline{R} \in SU(2)_\mathrm{R}$, there is a mapping

$$L \otimes R : \quad (\underline{\xi}, \underline{\rho}_\mu, \underline{N}) \longrightarrow (\underline{\xi}', \underline{\rho}'_\mu, \underline{N}') . \tag{5.24}$$

Because of the parity operation $\mathcal{P}$, which produces the transformation

$$\mathcal{P} : \quad L \longleftrightarrow R , \quad \pi^a(t, \mathbf{x}) \longrightarrow -\pi^a(t, -\mathbf{x}) , \quad \xi(t, \mathbf{x}) \longrightarrow \xi^\dagger(t, -\mathbf{x}) , \tag{5.25}$$

the chiral mapping (5.21) can be written as [compare Eqs. (5.14) and (5.15)]

$$\underline{\xi}'(x) = \underline{L}\underline{\xi}(x)\underline{h}^\dagger(x) = \underline{h}(x)\underline{\xi}(x)\underline{R}^\dagger , \tag{5.26}$$

$$\underline{\rho}'_\mu(x) = \underline{h}(x)\underline{\rho}_\mu(x)\underline{h}^\dagger(x) , \tag{5.27}$$

$$\underline{N}'(x) = \underline{h}(x)\underline{N}(x) . \tag{5.28}$$

The second equality in Eq. (5.26) defines $\underline{h}(x)$ as a function of $\underline{L}$, $\underline{R}$, and the local pion fields: $\underline{h}(x) = \underline{h}(\underline{L}, \underline{R}, \boldsymbol{\pi}(x))$. It follows from Eq. (5.26) that $\underline{h}(x)$ is invariant under the parity operation (5.25), that is,

$$\underline{h}(x) \in SU(2)_V , \tag{5.29}$$

with $SU(2)_V$ the unbroken vector subgroup of $SU(2)_L \times SU(2)_R$.[23] Equations (5.27) and (5.28) ensure that the rho and nucleon fields transform linearly under $SU(2)_V$ in accordance with their isospins. Note that the matrix $\underline{h}(x)$ becomes constant only when $\underline{L} = \underline{R}$, in which case $g \in H = SU(2)_V$ and $\underline{h} = \underline{L} = \underline{R}$. The isoscalar fields $V^\mu(x)$ and $\phi(x)$ are chiral scalars and are unaffected by both chiral and isospin transformations.

For discussing purely pionic interactions, it is convenient to define the matrix [compare Eqs. (5.5) and (5.11)]

$$\underline{U}(x) \equiv \underline{\xi}^2(x) = \exp(2i\,\underline{\pi}(x)/f_\pi) , \tag{5.30}$$

since the transformation law (5.26) then implies

$$\underline{U}(x) \longrightarrow \underline{U}'(x) = \underline{L}\underline{U}(x)\underline{R}^\dagger , \tag{5.31}$$

so that $\underline{U}(x)$ *always transforms globally* [see Eq. (5.17)]. Thus derivatives of $\underline{U}(x)$ transform the same way as $\underline{U}(x)$, and chirally invariant interactions involving pions alone can be constructed from products of $\underline{U}(x)$, $\underline{U}^\dagger(x)$, and their derivatives. As is well known, these terms can be organized according to the number of derivatives, with the lowest-order term [Wa95]

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[23]We can express $\underline{h}$ in terms of the matrices $\underline{L}$, $\underline{R}$, and $\underline{U}$ [see Eqs. (5.30) and (5.31)] as $\underline{h}(x) = \sqrt{\underline{U}'^\dagger(x)}\,\underline{L}\sqrt{\underline{U}(x)} = \sqrt{\underline{R}\underline{U}^\dagger(x)\underline{L}^\dagger}\,\underline{L}\sqrt{\underline{U}(x)}$. Given the decomposition [Co69] $\underline{L} = \exp(i\boldsymbol{\alpha} \cdot \boldsymbol{\tau})\exp(i\boldsymbol{\beta} \cdot \boldsymbol{\tau})$, $\underline{R} = \exp(-i\boldsymbol{\alpha} \cdot \boldsymbol{\tau})\exp(i\boldsymbol{\beta} \cdot \boldsymbol{\tau})$, and $\underline{h} = \exp(i\boldsymbol{\gamma} \cdot \boldsymbol{\tau})$, with $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and $\boldsymbol{\gamma}$ real, the infinitesimal expansion of $\underline{h}$ is determined by $\boldsymbol{\gamma} = \boldsymbol{\beta} - (\boldsymbol{\alpha} \times \boldsymbol{\pi})/2f_\pi + O(\boldsymbol{\alpha}^2, \boldsymbol{\beta}^2, \boldsymbol{\pi}^2)$ [Me93a,Be95].

$$\mathcal{L}_2 = \frac{1}{4} f_\pi^2 \, \mathrm{tr}\, (\partial_\mu \underline{U} \partial^\mu \underline{U}^\dagger) = \mathrm{tr}\, (\partial_\mu \underline{\pi} \partial^\mu \underline{\pi}) + \frac{1}{3f_\pi^2} \, \mathrm{tr}\, ([\,\underline{\pi}, \partial_\mu \underline{\pi}\,]^2) + \cdots \ . \tag{5.32}$$

$\mathcal{L}_2$ determines all multipion scattering amplitudes to second order in external momenta in terms of the single constant f_π. Terms with more derivatives $(\mathcal{L}_4, \mathcal{L}_6, \ldots)$ can be used to describe pion dynamics within the framework of ChPT [Ga84]. We will not need more than $\mathcal{L}_2$ in the model studied here.

For describing the interactions of pions with other particles, $\underline{U}(x)$ is not convenient, because other fields transform with the local $\underline{h}(x)$ of the unbroken isovector subgroup $SU(2)_\mathrm{V}$. It follows from the transformation laws given earlier, that interaction terms that are invariant under *local* isospin rotations will be invariant under *global* transformations of the full group $SU(2)_\mathrm{L} \times SU(2)_\mathrm{R}$. Thus, to form chirally invariant interactions involving pions and other fields, we need functions of the pion field that transform with $\underline{h}(x)$ only.

The desired functions involving one derivative of the pion field can be written as [compare Eq. (5.13)]

$$\underline{a}_\mu \equiv -\frac{i}{2}(\underline{\xi}^\dagger \partial_\mu \underline{\xi} - \underline{\xi} \partial_\mu \underline{\xi}^\dagger) = \underline{a}_\mu^\dagger \ , \tag{5.33}$$

$$\underline{v}_\mu \equiv -\frac{i}{2}(\underline{\xi}^\dagger \partial_\mu \underline{\xi} + \underline{\xi} \partial_\mu \underline{\xi}^\dagger) = \underline{v}_\mu^\dagger \ , \tag{5.34}$$

where the hermiticity follows from $\partial_\mu(\underline{\xi}^\dagger \underline{\xi}) = 0 = \partial_\mu(\underline{\xi}\underline{\xi}^\dagger)$. Under parity transformations, we have

$$\mathcal{P}: \qquad \underline{a}_\mu(t, \mathbf{x}) \longrightarrow -\underline{a}^\mu(t, -\mathbf{x}) \ , \quad \underline{v}_\mu(t, \mathbf{x}) \longrightarrow \underline{v}^\mu(t, -\mathbf{x}) \ , \tag{5.35}$$

so that $\underline{a}_\mu$ is an axial vector and $\underline{v}_\mu$ is a polar vector. To leading order in derivatives, one finds

$$\underline{a}_\mu = \frac{1}{f_\pi} \partial_\mu \underline{\pi} + \cdots \ , \tag{5.36}$$

$$\underline{v}_\mu = -\frac{i}{2f_\pi^2}[\underline{\pi}, \partial_\mu \underline{\pi}] + \cdots \ . \tag{5.37}$$

Moreover, under a chiral transformation, Eq. (5.26) implies

$$\underline{a}_\mu \rightarrow \underline{a}_\mu' = \underline{h}\underline{a}_\mu \underline{h}^\dagger \ , \tag{5.38}$$

$$\underline{v}_\mu \rightarrow \underline{v}_\mu' = \underline{h}\underline{v}_\mu \underline{h}^\dagger - i\underline{h}\partial_\mu \underline{h}^\dagger = \underline{h}\underline{v}_\mu \underline{h}^\dagger + i(\partial_\mu \underline{h})\underline{h}^\dagger \ . \tag{5.39}$$

Thus $\underline{a}_\mu$ transforms *homogeneously* under the local $SU(2)_\mathrm{V}$ group and can be interpreted as a covariant derivative of the pion-field matrix $\underline{\xi}(x)$. In contrast, the *inhomogeneous* transformation law for $\underline{v}_\mu$ resembles that of a gauge field, so that $\underline{v}_\mu$ allows us to construct chirally covariant derivatives of the other fields. For example, it is straightforward to verify that the covariant derivatives

$$D_\mu \underline{N} \equiv (\partial_\mu + i\underline{v}_\mu)\underline{N} \ , \quad D_\mu \underline{\rho}_\nu \equiv \partial_\mu \underline{\rho}_\nu + i[\underline{v}_\mu, \underline{\rho}_\nu] \tag{5.40}$$

transform homogeneously with $\underline{h}(x)$ under the full group:

$$(D_\mu \underline{N})' = \underline{h}(D_\mu \underline{N}) \ , \quad (D_\mu \underline{\rho}_\nu)' = \underline{h}(D_\mu \underline{\rho}_\nu)\underline{h}^\dagger \ . \tag{5.41}$$

The covariant derivative of $\underline{\rho}_\mu$ can be used to construct the covariant field tensor

$$\underline{\rho}_{\mu\nu} = D_\mu \underline{\rho}_\nu - D_\nu \underline{\rho}_\mu + ig_\rho[\underline{\rho}_\mu, \underline{\rho}_\nu] \ . \tag{5.42}$$

The antisymmetric combination of derivatives implies that the timelike components ρ_0^a of the rho field have no conjugate momenta and are thus determined by equations of constraint, as appropriate for a massive vector field with three dynamical degrees of freedom. The final term in Eq. (5.42) has the usual form for a non-abelian vector field and enables the $\boldsymbol{\rho}$ meson to couple to a conserved isovector current [Se86]. We can also construct a covariant tensor for the pion field as

$$\underline{v}_{\mu\nu} = \partial_\mu \underline{v}_\nu - \partial_\nu \underline{v}_\mu + i[\underline{v}_\mu, \underline{v}_\nu] = -i[\underline{a}_\mu, \underline{a}_\nu] \ , \tag{5.43}$$

which transforms homogeneously with $\underline{h}$, and which will allow us to produce an invariant $\rho\pi\pi$ coupling through an interaction of the form $\text{tr}\,(\underline{\rho}_{\mu\nu}\underline{v}^{\mu\nu})$.

Before exhibiting the lagrangian for the model, we consider electromagnetic interactions. As discussed in [Fu96a], these can be included straightforwardly by defining appropriate charge operators for the particles and by modifying the preceding covariant derivatives so that they remain covariant under the local U(1) transformations of electromagnetism. We will not present the intermediate steps here and simply show the contributions to the model lagrangian below. The electromagnetic interactions induce small violations of both the isospin and chiral symmetries.

To write a general effective lagrangian, we need an organizational scheme for the interaction terms. We organize the lagrangian in increasing powers of the fields and their derivatives, as motivated by the principles discussed earlier. We assign to each interaction term an index

$$\nu = d + \frac{n}{2} + b \ , \tag{5.44}$$

where d is the number of derivatives, n is the number of nucleon fields, and b is the number of non-Goldstone boson fields in the interaction term. The first two terms in Eq. (5.44) are suggested by Weinberg's work [We90a]. Derivatives on the nucleon fields are *not counted in* d because they will generally introduce powers of the nucleon mass M, which will not lead to small expansion parameters. The last term is a generalization that arises because a non-Goldstone boson couples to *two* nucleon fields. Equation (5.44) is also consistent with finite-density applications when the density is not too much higher than the nuclear-matter equilibrium density, as we will see below.

The effective lagrangian for the full model can be written as

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_M + \mathcal{L}_{EM} \ , \tag{5.45}$$

where each term will be truncated by considering the various values of ν, as defined above. Only non-redundant terms will be exhibited, and we will return to consider the redundancy problem after describing the lagrangian.

The part of the effective lagrangian involving nucleons can be written through order $\nu = 3$ as

$$\mathcal{L}_{\mathrm{N}}(x) = \overline{N}\Big[i\gamma^\mu(\partial_\mu + i\underline{v}_\mu + ig_\rho\,\underline{\rho}_\mu + ig_{\mathrm{v}}V_\mu) + g_{\mathrm{A}}\gamma^\mu\gamma_5\underline{a}_\mu - M + g_{\mathrm{s}}\phi\Big]N$$

$$-\frac{f_\rho g_\rho}{4M}\,\overline{N}\,\underline{\rho}_{\mu\nu}\sigma^{\mu\nu}N - \frac{f_{\mathrm{v}}g_{\mathrm{v}}}{4M}\,\overline{N}V_{\mu\nu}\sigma^{\mu\nu}N + \cdots , \qquad (5.46)$$

where $V^{\mu\nu} \equiv \partial^\mu V^\nu - \partial^\nu V^\mu$. Note that both vector and tensor couplings are included for the ρ and ω mesons, together with a Yukawa coupling for the effective scalar field ϕ; we explain the motivation for a simple Yukawa coupling below. The ellipsis represents terms involving πN interactions that are not needed in the following discussion, as well as terms with derivatives on the nucleon field, which will be considered below. Four-nucleon contact terms are not included, since they can be represented by appropriate powers of other meson fields [Fu96a], as discussed shortly. Additional terms with $\nu = 4$ are either redundant or tiny.

We emphasize that because the nucleon field obeys the transformation law (5.28), the mass term in Eq. (5.46) is chirally invariant. Thus the nature of the spontaneous chiral symmetry breaking is *outside* the realm of our effective theory, unlike a linear realization, where the symmetry breaking is implemented by the fields in the model [see Eq. (3.8)]. In a linear model, the mechanism that generates the free-space mass M is also responsible for shifting $M \to M^*$ at finite density, which leads to the problems discussed in Section 3. A nonlinear realization of the symmetry allows these two aspects of the nucleon mass to be treated independently.

The mesonic part of the lagrangian is also organized in powers of ν. Keeping terms up to order $\nu = 4$, we find[24]

$$\mathcal{L}_{\mathrm{M}}(x) = \frac{1}{4}\,f_\pi^2\,\mathrm{tr}\,(\partial_\mu\underline{U}\partial^\mu\underline{U}^\dagger) + \frac{1}{4}\,m_\pi^2 f_\pi^2\,\mathrm{tr}\,(\underline{U} + \underline{U}^\dagger - 2) + \frac{1}{2}\Big(1 + \alpha_1\frac{g_{\mathrm{s}}\phi}{M}\Big)\partial_\mu\phi\partial^\mu\phi$$

$$-\frac{1}{2}\,\mathrm{tr}\,(\underline{\rho}_{\mu\nu}\underline{\rho}^{\mu\nu}) - \frac{1}{4}\Big(1 + \alpha_2\frac{g_{\mathrm{s}}\phi}{M}\Big)V_{\mu\nu}V^{\mu\nu} - g_{\rho\pi\pi}\frac{2f_\pi^2}{m_\rho^2}\,\mathrm{tr}\,(\underline{\rho}_{\mu\nu}\underline{v}^{\mu\nu})$$

$$+\frac{1}{2}\Big(1 + \eta_1\frac{g_{\mathrm{s}}\phi}{M} + \frac{\eta_2}{2}\frac{g_{\mathrm{s}}^2\phi^2}{M^2}\Big)m_{\mathrm{v}}^2 V_\mu V^\mu + \frac{1}{4!}\zeta_0 g_{\mathrm{v}}^2(V_\mu V^\mu)^2$$

$$+\Big(1 + \eta_\rho\frac{g_{\mathrm{s}}\phi}{M}\Big)m_\rho^2\,\mathrm{tr}\,(\underline{\rho}_\mu\underline{\rho}^\mu) - m_{\mathrm{s}}^2\phi^2\Big(\frac{1}{2} + \frac{\kappa_3}{3!}\frac{g_{\mathrm{s}}\phi}{M} + \frac{\kappa_4}{4!}\frac{g_{\mathrm{s}}^2\phi^2}{M^2}\Big) , \qquad (5.47)$$

where we have included a small chiral-symmetry-violating term involving m_π^2. Apart from conventional definitions of some couplings (g_{s} and g_{v}) and the masses, the parameters are defined so that they are of order unity according to the naive dimensional analysis discussed earlier. This hypothesis will be tested by fitting the parameters to nuclear properties. Moreover, since the expectation value of the ρ field is typically an order of magnitude smaller

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[24]Note that the leading-order coupling of the scalar-isoscalar field to pions has the form $\phi\,\mathrm{tr}\,(\partial_\mu\underline{U}\partial^\mu\underline{U}^\dagger)$.

than that of the ω field, we have retained nonlinear ρ couplings only through order $\nu = 3$. Note that the α_1 and α_2 terms involve derivatives of the meson fields and have $\nu = 5$, but these give contributions to the nuclear surface energy that are numerically of the same magnitude as the quartic scalar term, so we have retained them. Thus numerical factors such as $1/n!$, which are cancelled in scattering amplitudes, are relevant in deciding the importance of contributions to the energy.

Higher-order self-couplings and derivatives involving meson fields alone [$e.g.$, ϕ^5, or $(V_\mu V^\mu)^3$, or $(\partial_\mu \phi \partial^\mu \phi)^2$] should be numerically small unless their coefficients are "unnaturally" large. We show below that the parameters that have been retained indeed exhibit naturalness, so that the omission of these terms is justified at the level of accuracy we can expect in comparisons with observables in finite nuclei.

Finally, the electromagnetic interactions are described by

$$
\begin{aligned}
\mathcal{L}_{\mathrm{EM}}(x) = {} & -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - e\, \overline{N} \gamma^\mu \frac{1}{2}(1 + \tau_3)\, \underline{N} A_\mu \\
& -\frac{e}{4M} F_{\mu\nu}\, \overline{N} \underline{\lambda}' \sigma^{\mu\nu} \underline{N} - \frac{e}{2M^2}\, \overline{N} \gamma_\mu (\beta^{(0)} + \beta^{(1)} \tau_3) \underline{N} \partial_\nu F^{\mu\nu} \\
& -2e f_\pi^2 A^\mu \operatorname{tr}(\underline{v}_\mu \tau_3) - \frac{e}{2g_\gamma} F_{\mu\nu} \Big[\operatorname{tr}(\tau_3 \underline{\rho}^{\mu\nu}) + \frac{1}{3} V^{\mu\nu} \Big] + \cdots ,
\end{aligned} \tag{5.48}
$$

where A^μ is the electromagnetic potential and $F^{\mu\nu}$ is now the usual field tensor. The lagrangian $\mathcal{L}_{\mathrm{EM}}$ is invariant under the $U(1)$ group of electromagnetism, and the resulting current [see Eq. (6.3), below] is conserved, at least to $O(e)$.[25] The composite structure of the nucleon is included here through an anomalous moment $\underline{\lambda}'$ [see Eq. (6.1), below] and through terms that will generate a q^2 dependence in both the isoscalar ($\beta^{(0)}$) and isovector ($\beta^{(1)}$) electromagnetic form factors. Moreover, the coupling between the massive vector mesons and the photon generate contributions to the nucleon form factor in accord with vector-meson dominance. The end result is momentum dependence that resembles the empirical "dipole" form. Similar contributions will arise in the pion form factor due to the $\rho\pi\pi$ coupling in Eq. (5.47). We will return to the electromagnetic structure of the nucleon in this model in Section 6.

Turning now to "redundant" terms that have been omitted in the preceding equations, we emphasize that there is considerable freedom in the choice of generalized coordinates (fields) for the lagrangian. It is known that a wide class of point transformations of the fields do not change the on-shell scattering amplitudes [Co69].[26] Thus the relevant question is which choice of coordinates leads to the most practical and accurate truncation scheme, and this is currently under active investigation.

Some redundant terms that we have omitted through our choice of field variables are:

1. Contact terms involving nucleon fields beyond bilinear order [$e.g.$, $(\overline{N} \underline{N})(\overline{N} \underline{N})$ or $(\overline{N} \gamma^\mu \underline{N})(\overline{N} \gamma_\mu \underline{N})$]. We observe that if one constructs the hamiltonian from the la-

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[25]The fully U(1)-invariant lagrangian is discussed in [Fu96a].

[26]We assume that this is also true for finite-density observables, but we know of no proof.

grangian given above and then eliminates the meson fields using their equations of motion, contact terms involving products of nucleon bilinears will arise. Conversely, contact terms included originally in the hamiltonian can be eliminated in favor of the scalar and vector fields, if we allow products of fields to all orders. The issue then becomes one of efficiency, since one will always have to truncate in practice. Although both ways of representing these nonlinear interactions may turn out to be practical [Bo77,Bo91,Ni92,Fu96], the fits to nuclei presented below show that nonlinear meson interactions lead to an efficient and natural truncation.

2. More complicated meson–nucleon couplings [$e.g.$, $g_{\rm s}(\phi)\,\overline{N}\underline{N}\phi$]; these are often motivated by the claim that they are necessary to incorporate the compositeness of the nucleon. Here we rely on the freedom to redefine the fields to rewrite complicated couplings in simple Yukawa form: rather than work with $g_{\rm s}(\phi)\,\overline{N}\underline{N}\phi$, where $g_{\rm s}(\phi) = g_{\rm s}(1 + c_1\phi + c_2\phi^2 + \cdots)$, we could define a new scalar field $\widetilde{\phi}$ by $\widetilde{g}_{\rm s}\widetilde{\phi} \equiv g_{\rm s}(\phi)\phi$. Inversion of this relation and substitution into the lagrangian would result in additional nonlinear interactions in powers of $\widetilde{\phi}$ that have the same form as those that have already been included. Note that this procedure can actually be used on all possible scalar-isoscalar field combinations, so the following couplings are all redundant:

$$\overline{N}\underline{N}\phi^2\,,\quad \overline{N}\underline{N}\phi^3\,,\quad \overline{N}\underline{N}\partial^2\phi\,,\quad \overline{N}\underline{N}V_\mu V^\mu\,.$$

A similar observation holds for the vector-isoscalar and vector-isovector couplings. This discussion illustrates the important point that nucleon compositeness can be incorporated through nonlinear meson interactions.

3. Couplings involving higher derivatives of the nucleon field [$e.g.$, $(\overline{N}\partial^2\underline{N})\phi$, $\overline{N}iV^\mu\partial_\mu N$, or $(\overline{N}\partial_\mu\underline{N})(\overline{N}\partial^\mu\underline{N})$].[27] These are the most problematic, since derivatives acting on the nucleon field produce factors of the nucleon energies E_i, and since $E_i/M \approx 1$, this would spoil the expansion and truncation procedure outlined above. Fortunately, through partial integration, redefinition of the baryon field, and the use of the equations of motion in the construction of the hamiltonian, these terms can be recast in the form of the terms we have retained or can be shown to produce terms that should give only small contributions. Further discussion is contained in [Ba88,Ge91,Fu96a] and in §7.7 of [We95].

The mean-field equations and energy density resulting from the lagrangian (5.45) can be derived straightforwardly. For symmetric nuclear matter ($\gamma = 4$), one finds through order $\nu = 4$

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[27]Higher-derivative terms should of course be written in terms of *covariant* derivatives [see Eq. (5.40)]. The contributions from the meson fields to these terms have already been classified; here we focus on the gradient pieces.

$$\mathcal{E}[\Phi, W; \rho_{\mathrm{B}}] = W\rho_{\mathrm{B}} + \frac{4}{(2\pi)^3}\int_0^{k_{\mathrm{F}}} \mathrm{d}^3 k\,\sqrt{\mathbf{k}^2 + M^{*2}} + \frac{1}{g_{\mathrm{s}}^2}\Big(\frac{1}{2} + \frac{\kappa_3}{3!}\frac{\Phi}{M} + \frac{\kappa_4}{4!}\frac{\Phi^2}{M^2}\Big)m_{\mathrm{s}}^2\Phi^2$$

$$-\frac{1}{2g_{\mathrm{v}}^2}\Big(1 + \eta_1\frac{\Phi}{M} + \frac{\eta_2}{2}\frac{\Phi^2}{M^2}\Big)m_{\mathrm{v}}^2 W^2 - \frac{1}{4!g_{\mathrm{v}}^2}\zeta_0 W^4 \; , \tag{5.49}$$

where $\Phi = g_{\mathrm{s}}\phi_0$ and $W = g_{\mathrm{v}}V_0$ are the scaled fields defined earlier. One can also compute the bulk symmetry-energy coefficient [Se86]

$$a_4 = \frac{g_\rho^2}{12\pi^2 m_\rho^{*2}}\,k_{\mathrm{F}}^3 + \frac{1}{6}\frac{k_{\mathrm{F}}^2}{\sqrt{k_{\mathrm{F}}^2 + M^{*2}}} \; , \tag{5.50}$$

where the effective rho mass m_ρ^* is defined by

$$m_\rho^{*2} \equiv m_\rho^2(1 + \eta_\rho \Phi/M) \; . \tag{5.51}$$

A comparison with Eq. (2.17) shows that the nuclear matter energy has been generalized to include additional nonlinearities that are not allowed in the renormalizable model QHD–I. The fields Φ and W are again determined by extremization.

The Dirac–Hartree equations for finite nuclei can also be derived using the procedures in Section 2.B. The resulting equations are lengthy and will not be reproduced here; the interested reader is referred to [Fu96a] for details. One important result is that due to the additional nonrenormalizable interactions between the nucleon and the electromagnetic field, and also due to vector-meson dominance, the computed nuclear charge density automatically contains the effects of nucleon structure, and it is unnecessary to introduce an *ad hoc* form factor.

It is clear from the discussion in Section 2 that the present model has more than enough parameters to give an accurate reproduction of nuclear properties. The more important question is whether the parameters fitted to nuclei are natural. In [Fu96a], the parameters were determined by calculating a set of observables $\{X_{\mathrm{th}}^{(i)}\}$ for each nucleus and by adjusting the parameters to minimize the generalized χ^2 defined by [Ni92]

$$\chi^2 = \sum_i \sum_X \Big[\frac{X_{\mathrm{exp}}^{(i)} - X_{\mathrm{th}}^{(i)}}{W_X^{(i)} X_{\mathrm{exp}}^{(i)}}\Big]^2 \; , \tag{5.52}$$

where i runs over the set of nuclei, X runs over the set of observables, the subscript "exp" indicates the experimental value of the observable, and $W_X^{(i)}$ are the relative weights. The weights are chosen to be the relative accuracy expected for the given observable in a good fit.[28] The nuclei chosen were ^{16}O, ^{40}Ca, ^{48}Ca, ^{88}Sr, and ^{208}Pb.

A total of 29 observables and their relative weights were taken as follows:

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[28]In practice, a reasonable range of weights was tested, and the qualitative conclusions discussed below were always reproduced. Some of the considerations relevant in choosing the weights are discussed in Section 6.

- The binding energies per nucleon ϵ/B, with a relative weight of 0.15%

- The rms charge radii $\langle r^2 \rangle_{\text{chg}}^{\frac{1}{2}}$, with a relative weight of 0.2%

- The d.m.s. radii R_{dms}, with a relative weight of 0.15%

- The spin-orbit splittings ΔE_{SO} of the least-bound protons and neutrons, with a relative weight of 5% for ^{16}O, 15% for ^{208}Pb, 25% for ^{40}Ca and ^{48}Ca, and 50% for ^{88}Sr

- The proton energy $E_{\text{p}}(1h_{9/2})$ and the proton level splitting $E_{\text{p}}(2d_{3/2}) - E_{\text{p}}(1h_{11/2})$ in ^{208}Pb, with relative weights of 5% and 25%, respectively

- The surface-energy and symmetry-energy deviation coefficients δa_2 and δa_4, each with a weight of 0.08.

The so-called diffraction-minimum-sharp (d.m.s.) radius of a nucleus is defined to be [Fr82]

$$R_{\text{dms}} \equiv 4.493/Q_0^{(1)} \, , \tag{5.53}$$

where $Q_0^{(1)}$ is the three-momentum transfer at the first zero of the nuclear charge form factor $F(Q) \equiv F_{\text{chg}}(\mathbf{q})$ with $Q = |\mathbf{q}|$. The surface-energy and symmetry-energy deviation coefficients δa_2 and δa_4 are *defined* by fitting the difference between experimental and calculated binding energies $\delta\epsilon_i \equiv (\epsilon_i)_{\text{exp}} - (\epsilon_i)_{\text{th}}$ according to[29]

$$\delta\epsilon_i = \delta a_1 \, A_i - \delta a_2 \, A_i^{2/3} - \delta a_4 \, (N_i - Z_i)^2/A_i \, . \tag{5.54}$$

Here N_i and Z_i are the number of neutrons and protons in the ith nucleus and $A_i = N_i + Z_i$. (An exact fit to the energies would have $\delta a_1 = \delta a_2 = \delta a_4 = 0$.) The deviations δa_2 and δa_4 are included as separate terms in Eq. (5.52) in the form $[\delta a_i/W_{\delta a_i}]^2$, with $W_{\delta a_i} = 0.08$. The motivation for this choice of observables is discussed more fully in [Fu96a].

The nucleon, ω, and ρ masses are taken to have their experimental values: $M = 939 \, \text{MeV}$, $m_{\text{v}} = 782 \, \text{MeV}$, and $m_\rho = 770 \, \text{MeV}$. (Including the heavy meson masses as free parameters produces only minor changes in the fits.) The anomalous magnetic moments of the nucleon are fixed at $\lambda'_{\text{p}} = 1.793$ and $\lambda'_{\text{n}} = -1.913$, and $g_\gamma = 5.01$ is chosen to reproduce the experimental partial width $\Gamma(\rho^0 \to e^+ e^-) = 6.8 \, \text{keV}$. The empirical free-space charge radii of the nucleon are used to fix β_{s}, β_{v}, and f_ρ by solving Eqs. (6.8) and (6.9), below. The remaining thirteen parameters g_{s}, g_{v}, g_ρ, η_1, η_2, η_ρ, κ_3, κ_4, ζ_0, m_{s}, f_{v}, α_1, and α_2 for the $\nu = 4$ parametrization are then obtained by optimization of the generalized χ^2.

In Table II, we show two parameter sets (G1 and G2) obtained from fits with roughly equal accuracy when all terms through order $\nu = 4$ are retained. The parameters have been displayed in such a way that they should all be of order unity according to NDA and the

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[29]Note that phenomenological surface-energy and symmetry-energy coefficients are *not* used, so there is no direct input from nuclear matter to the fitting procedure.

TABLE II. Parameter sets from fits to finite nuclei, as described in the text. Note that sets W1 and Q1 include the same interaction terms as sets L2 and NLC in Table I.

	ν	$W1$	$C1$	$Q1$	$Q2$	$G1$	$G2$
$m_{\rm s}/M$	2	0.60305	0.53874	0.53735	0.54268	0.53963	0.55410
$g_{\rm s}/4\pi$	2	0.93797	0.77756	0.81024	0.78661	0.78532	0.83522
$g_{\rm v}/4\pi$	2	1.13652	0.98486	1.02125	0.97202	0.96512	1.01560
$g_{\rho}/4\pi$	2	0.77787	0.65053	0.70261	0.68096	0.69844	0.75467
η_1	3		0.29577			0.07060	0.64992
η_2	4					-0.96161	0.10975
κ_3	3		1.6698	1.6582	1.7424	2.2067	3.2467
κ_4	4			-6.6045	-8.4836	-10.090	0.63152
ζ_0	4				-1.7750	3.5249	2.6416
η_ρ	4					-0.2722	0.3901
α_1	5					1.8549	1.7234
α_2	5					1.7880	-1.5798
$f_{\rm v}/4$	3					0.1079	0.1734
$f_{\rho}/4$	3	0.9332	1.1159	1.0332	1.0660	1.0393	0.9619
$\beta_{\rm s}$	4	-0.38482	-0.01915	-0.10689	0.01181	0.02844	-0.09328
$\beta_{\rm v}$	4	-0.54618	-0.07120	-0.26545	-0.18470	-0.24992	-0.45964

naturalness assumption. This is seen to be the case.[30] Most importantly, it is found that the accuracy of the fit and the contributions to the nuclear matter energy/particle are not driven by the last terms retained, as illustrated in Fig. 8. This result was further checked [Fu96a] by including the $\nu = 5$ interactions

$$\mathcal{L}_5 = -\frac{1}{5!}\kappa_5\frac{g_{\rm s}^3\phi^3}{M^3}m_{\rm s}^2\phi^2 + \frac{1}{3!}\eta_3\frac{g_{\rm s}^3\phi^3}{M^3}\cdot\frac{1}{2}m_{\rm v}^2V_\mu V^\mu + \frac{1}{4!}\zeta_1\frac{g_{\rm s}\phi}{M}g_{\rm v}^2(V_\mu V^\mu)^2 ,\qquad(5.55)$$

which do not improve the fits to the data; these contributions are essentially negligible unless the coefficients are unnaturally large, and no indication for such large parameters was found. *Thus we conclude that NDA and the naturalness assumption are valid when applied to finite nuclei, and that the truncation procedure defined above is practical, at least for moderate densities.* Moreover, although the parameters were obtained from a fit to a specific set of nuclei, one can now extrapolate to study other features of nuclear structure, such as nuclear deformations, isotope shifts in charge radii, etc.

Also shown in Table II are the results of fits with fewer parameters, which were obtained to verify that the best possible accuracy for this set of input data has been achieved by keeping terms through order $\nu = 4$. Roughly speaking, set W1 had $\chi^2 \approx 1700$, set C1 had $\chi^2 \approx 400$, sets Q1 and Q2 had $\chi^2 \approx 100$, while sets G1 and G2 achieved $\chi^2 \approx 50$. Including

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[30]Natural parameters have also been obtained in a "point-coupling" model that describes the NN interaction through contact terms [Ni92,Fr96].
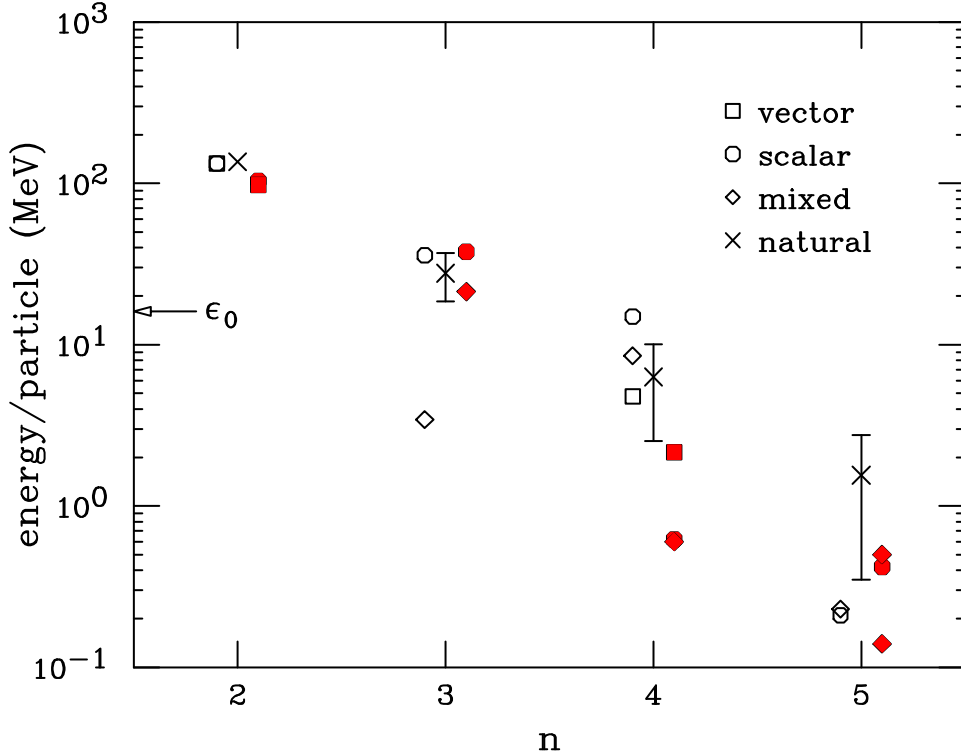
FIG. 8. Contributions to the energy per particle in nuclear matter for parameter sets G1 and G2 from the nth-order terms of the form $\Phi^\ell W^m$, where $n = \ell + m$. The boxes are terms with $\ell = 0$, the circles are terms with $m = 0$, and absolute values are shown. Results from set G1 are open and those from G2 are filled. The crosses are estimates based on Eq. (5.2). The arrow indicates the total binding energy $\epsilon_0 = 16.1$ MeV.

the $\nu = 5$ parameters in Eq. (5.55) improved χ^2 only slightly (< 2 units). Thus keeping terms through order $\nu = 4$ is essentially the best one can do, and in fact, the parameters are already underdetermined at this level, as is evident by the differences between sets G1 and G2.

Nuclear matter properties *predicted* by these parameter sets are given in Table III. Observe that sets G1 and G2 yield similar results in spite of the differences in the parameters, implying that the nuclear matter properties are better determined than the parameters themselves. Note also that all sets including parameters through orders $\nu = 3$ or $\nu = 4$ predict $M^* \approx 0.6\,M$ at equilibrium, in agreement with our discussion in Section 2. Although this result was obtained when spin-orbit information was included as input, fits *without* such information leads to similar values for M^* [Fu96a], as was first shown in [Re86].

E. The Quantum Vacuum in QHD-I

In any consistent relativistic field theory, one must ultimately consider loop diagrams. These contributions are an integral part of a fully relativistic description of nuclear structure, and as described in Section 2, it is impossible to construct a meaningful nuclear response or consistent nuclear currents without including the negative-energy baryon states. Although

TABLE III. Nuclear matter equilibrium properties for sets from Table II and for the point-coupling model of Ref. [Ni92] (set PC). Values are given for the binding energy per nucleon (in MeV), the Fermi momentum k_F (in fm^{-1}), the compression modulus K (in MeV), the bulk symmetry energy coefficient a_4 (in MeV), M^*/M, and $g_v V_0$ (in MeV) at equilibrium.

Set	$E/B - M$	k_F	K	a_4	$M^* M$	$g_v V_0$
W1	−16.46	1.279	569	40.9	0.532	363
C1	−16.19	1.293	304	32.0	0.657	255
Q1	−16.10	1.299	242	36.4	0.597	306
Q2	−16.13	1.303	279	35.2	0.614	292
G1	−16.14	1.314	215	38.5	0.634	274
G2	−16.07	1.315	215	36.4	0.664	248
PC	−16.13	1.299	264	37.0	0.575	322

the MFT ground state is causal and consistent with Lorentz covariance and thermodynamics by itself, it is natural to ask about the role of contributions from the filled Dirac sea. This is one of the motivations for constructing QHD–I as a renormalizable theory [Wa74,Se86]. Our goal in this subsection is to determine if the simplest evaluation of these effects in QHD–I produces results that are consistent with NDA and naturalness.

In Section 2, we studied the consequences of the mean-field hamiltonian of Eq. (2.11) and its generalization to finite nuclei. Let us now return to infinite nuclear matter and include the contribution from δH in Eq. (2.12). The inclusion of this term defines the so-called relativistic Hartree approximation or RHA. (This is also often called the one-baryon-loop approximation.)

An inspection of δH reveals that, even with the indicated vacuum subtraction, the sum still diverges. Since QHD–I is a renormalizable model, however, the sum can be rendered finite by including counterterms in the lagrangian (2.7). These counterterms also appear in the hamiltonian, and they can be grouped with δH, resulting in a correction to the energy density of the form

$$\Delta \mathcal{E}(M^*) = -\frac{1}{V} \sum_{k\lambda} \left[(\mathbf{k}^2 + M^{*2})^{1/2} - (\mathbf{k}^2 + M^2)^{1/2} \right] - \sum_{n=1}^{4} \frac{\alpha_n}{n!} \phi_0^n \ . \tag{5.56}$$

The counterterms enter as a quartic polynomial in ϕ_0, and the (infinite) coefficients α_n are determined by specifying appropriate renormalization conditions on the energy. Following [Ch77] and [Se86,Wa95], we will choose the counterterms to cancel the first four powers of ϕ_0 appearing in the expansion of the infinite sum. This is equivalent to defining the renormalized parameters κ and λ to be zero. Although this procedure is not unique (and is also unnatural), it minimizes the contributions from this vacuum correction, and it is easy to verify that only the first four terms in this expansion produce divergent results. The divergences can be defined by converting the sum to an integral and then by regularizing dimensionally [Ch77,Se86,Wa95].

After removing the divergences with the counterterms, the remaining terms are finite, and one finds (for spin-isospin degeneracy $\gamma = 4$)

$$\Delta \mathcal{E}(M^*) = -\frac{1}{4\pi^2}\Big\{M^{*4}\ln(M^*/M) + M^3(M-M^*) - \frac{7}{2}M^2(M-M^*)^2$$

$$+ \frac{13}{3}M(M-M^*)^3 - \frac{25}{12}(M-M^*)^4\Big\} \tag{5.57}$$

$$= \frac{M^4}{4\pi^2}\Big\{\frac{\Phi^5}{5M^5} + \frac{\Phi^6}{30M^6} + \frac{\Phi^7}{105M^7} + \cdots + \frac{4!(n-5)!}{n!}\frac{\Phi^n}{M^n} + \cdots\Big\}\,, \tag{5.58}$$

where $M^* \equiv M - g_{\mathrm{s}}\phi_0 \equiv M - \Phi$. $\Delta\mathcal{E}$ is the finite shift in the baryon zero-point energy that occurs at finite density and is analogous to the "Casimir energy" that arises in quantum electrodynamics. Just as in the MFT, M^* is determined at each ρ_{B} by minimization, which produces the one-loop (RHA) self-consistency condition [compare Eq. (2.19)]

$$M^* = M - \frac{g_{\mathrm{s}}^2}{m_{\mathrm{s}}^2}\rho_{\mathrm{s}} + \frac{g_{\mathrm{s}}^2}{m_{\mathrm{s}}^2}\frac{1}{\pi^2}\Big\{M^{*3}\ln(M^*/M) - M^2(M^*-M)$$

$$- \frac{5}{2}M(M^*-M)^2 - \frac{11}{6}(M^*-M)^3\Big\}\,. \tag{5.59}$$

Note that the solution to this equation contains all orders in the coupling g_{s}.

To discuss the size of the one-loop vacuum correction, we apply the NDA. Based on the scaling rules discussed above, a term of $O(\phi_0^5)$ should be scaled as

$$\frac{M^2}{5!f_\pi^3}\phi_0^5\,, \tag{5.60}$$

and if this contribution is natural, any residual overall constant should be of order unity. However, if we perform a similar scaling on the leading term in Eq. (5.58), we find

$$\frac{M^4}{4\pi^2}\frac{g_{\mathrm{s}}^5\phi_0^5}{5M^5} \longrightarrow \frac{4}{5}\frac{M^2}{f_\pi^3}\phi_0^5 = 96\Big(\frac{M^2}{5!f_\pi^3}\phi_0^5\Big)\,, \tag{5.61}$$

where we used $4\pi f_\pi \approx M$ and $g_{\mathrm{s}} \approx M/f_\pi$. Thus the one-baryon-loop contribution to the vacuum energy in QHD–I is roughly two orders of magnitude *larger* than naturalness requires. It is not hard to show from Eq. (5.58) that all higher powers of Φ contain essentially the same large overall factor.

Similar behavior occurs in the linear sigma model. If we rewrite the coefficients in Eq. (3.10) in terms of the nonlinear parameters in Eq. (5.3), we find (in the chiral limit) $\kappa_3 = -\kappa_4 = -3$, so that the nonlinear parameters are natural at the mean-field level. However, if one includes the one-baryon-loop vacuum corrections, renormalized in a fashion that preserves the chiral symmetry [Ma82,Fu93], one finds unnatural corrections to the cubic and quartic couplings: $\Delta\kappa_3 = 2M^2/\pi^2 f_\pi^2 \approx 20$, $\Delta\kappa_4 = -8M^2/\pi^2 f_\pi^2 \approx -80$. The quintic and higher corrections are exactly the same as in Eq. (5.58). Thus the one-baryon-loop vacuum contributions again produce unnatural coefficients.

These unnatural coefficients generate correspondingly large corrections to the MFT. If we consider QHD–I and adjust the model parameters from their MFT values to reproduce the desired nuclear matter properties in the RHA (equilibrium at $k_{\mathrm{F}}^0 = 1.30\,\mathrm{fm}^{-1}$ with a binding energy of 15.75 MeV), the baryon effective mass at equilibrium becomes $M^*/M \approx 0.73$. This
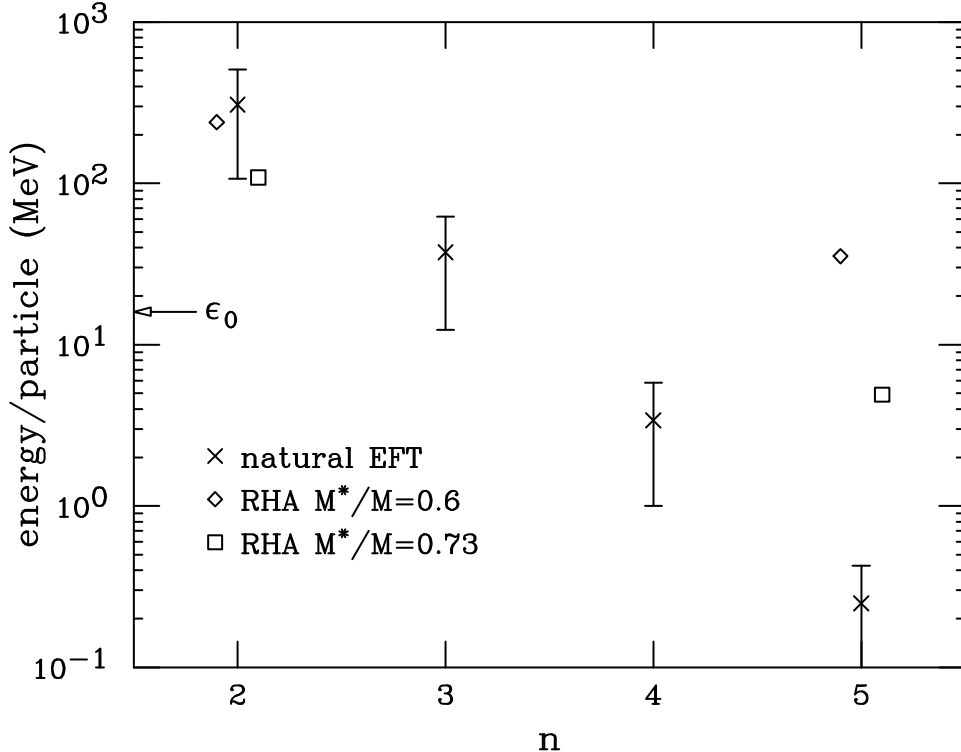
FIG. 9. Contributions to the scalar potential per particle in nuclear matter from the nth-order terms of the form Φ^n for the RHA model. The crosses are estimates based on Eq. (5.2). The arrow indicates the total binding energy $\epsilon_0 = 15.75$ MeV.

translates into a change in the scalar potential Φ from 430 MeV in the MFT to 250 MeV in the RHA, which is a large effect. In fact, the $O(\Phi^5)$ term in the energy density forces Φ to significantly lower values, and the contributions to the energy/nucleon from $\Delta\mathcal{E}$ are much larger than what one would expect from this term in a natural model, like the one discussed in the preceding subsection. This is illustrated in Fig. 9, where the RHA $O(\Phi^5)$ contribution for $M^*/M = 0.6$ is as large as a typical $O(\Phi^3)$ contribution in a natural model.

If one accepts the assumption of naturalness, the conclusion is that the treatment of the quantum vacuum at the one-baryon-loop level in the renormalizable model QHD–I is, at best, inadequate.[31] Although higher-order corrections (some of which are discussed in Section 8) might reduce the size of the one-loop terms and ultimately yield a natural size for the vacuum contributions, this can only occur through sensitive cancellations against the one-loop terms.

In contrast, in an effective field theory, the presence of nonrenormalizable couplings will generally cause the sum in Eq. (5.56) to have divergences *at all orders in* ϕ_0, and

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[31]We emphasize, however, that naturalness is a strong assumption, just as the assumption of renormalizability is. In the latter, one sets all but a few of the model parameters to zero, while in the former, one maintains that all parameters are roughly of the same magnitude.

an infinite number of counterterms must be added to produce finite results. (This will happen, for example, if $M^* = M - \Phi$ becomes a more complicated function of Φ.) An explicit calculation of the counterterms is unnecessary, however, since the end result is simply an infinite polynomial in the scalar field, with finite, unknown, and presumably natural coefficients arising from the underlying dynamics of the QCD vacuum. To have any predictive power, one must rely either on the truncation scheme discussed above, so that only a small, finite number of unknown coefficients are relevant, or on some other dynamics to constrain the form of the renormalized scalar potential.[32] There is, of course, no guarantee that an effective hadronic theory *should* be predictive, and until one can derive the hadronic theory from the underlying QCD, the range of predictive power must be determined by comparison with experiment. Moreover, since the non-Goldstone bosons are always off mass shell in nuclear structure calculations, it is likely that the ability to make realistic predictions depends on the choice of generalized coordinates (fields).

In summary, even in an effective hadronic field theory, one must include loop contributions that contain negative-energy baryon wave functions, since it is essential to maintain the completeness of the Dirac basis, which plays a crucial role in the field theory [We95]. The new ingredient, compared to renormalizable models, is that there is now an *infinite* number of counterterms. Since the divergent contributions must also respect the symmetries in the lagrangian, the appropriate counterterms to remove the divergences will always exist, but final, finite results will generally depend on an infinite number of unknown parameters. Thus one must rely on truncation or on additional dynamical input to limit the parameters to a manageable number. We will discuss these observations further in Section 8.

6. RELATIVISTIC MEAN-FIELD THEORY

The background for relativistic mean-field theory (RMFT) and its application to finite nuclei[33] is developed in detail in [Se86,Se92,Wa95]; we have summarized this material in Section 2. Important background references also include [Bo77,Ho81,Re86, Fu87,Ru88,Fu89c,Re89,Ga90,Fu91a]. The main conclusion from this material is that by using local relativistic Hartree equations for the baryon and meson fields (ψ, σ, ω), with linear couplings and a minimal set of parameters fitted to the properties of nuclear matter, *one derives the nuclear shell model.*

Moreover, by including phenomenological nonlinear meson couplings of the form $(\kappa/3!)\phi^3 + (\lambda/4!)\phi^4$, one can extend this model, fit the nuclear compression modulus and baryon effective mass [Bo91,Fu96], and obtain an accurate description of nuclear deformations in light nuclei [Bo77,Bo84,Re86,Fu87,Ru88,Fi89,Se92,Va92].

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[32]For example, a simple model is used in [Fu95] to show how the broken scale invariance of QCD leads to dynamical constraints on the scalar potential, and fits to the properties of finite nuclei also generate coefficients that are natural.

[33]We sometimes use the terminology "Dirac–Hartree theory" for the RMFT of finite nuclei.

A. Density Functional Theory

To understand the success of the RMFT and to put the calculations discussed in this section in context, it us useful to consider some elements of density functional theory (DFT). A discussion of DFT applied to nonrelativistic systems can be found, for example, in [Dr90]. Speicher, Dreizler, and Engel [Sp92] extend the framework to study the relativistic many-body problem. These authors outline the density functional approach to the strong-interaction model of QHD and apply the Hohenberg–Kohn theorem, widely used in *ab initio* calculations of the structure of solids, to this situation. The nonrelativistic version of this theorem can be stated as follows: The ground-state expectation value of any observable is a *unique* functional of the exact ground-state density; moreover, if the expectation value of the hamiltonian is considered as a *functional* of the density, the exact ground-state density can be determined by minimizing the energy functional.

Speicher, Dreizler, and Engel derive an approximate energy functional by using a gradient expansion of the noninteracting kinetic energy to order $\hbar^2$. The energy functional includes the effects of four-vector meson exchange and of vacuum contributions, and the variational equations of the corresponding extended Thomas–Fermi model are discussed. A great deal of work has been done on the extended Thomas–Fermi model, where it is relatively straightforward to generalize the results of infinite nuclear matter calculations to finite systems. The extended Thomas-Fermi approach is discussed in detail in [Ce92b,Mu92,Vo92b,Ce93a,Ce93b,Sp93,Ce94,Ha94,Vo94a,Vo94b,Sc95,Sc95a].

In more general terms, the central object in a DFT formulation of the relativistic nuclear many-body problem is an energy functional of scalar and vector densities (or more precisely, vector four-currents). Minimization of the functional gives rise to variational equations that determine the ground-state densities. By introducing a complete set of Dirac wave functions, one can recast these variational equations as Dirac equations for occupied orbitals; the single-particle hamiltonian contains *local* scalar and vector potentials, not only in the Hartree approximation, but in the general case as well.[34] Rather than work solely with the Dirac wave functions and the resulting densities (as in [Ni92,Sc95,Fr96]), one can introduce auxiliary fields corresponding to the local potentials, so that the energy functional depends also on classical meson fields. The resulting DFT formulation produces field equations that resemble those in a Dirac–Hartree calculation, but correlation effects can be included, *if* the proper functional can be found.

The procedure described above is analogous to the well-known Kohn–Sham [Ko65] approach in DFT, which is based on the following theorem [Dr90] (generalized here to relativistic systems):

> The exact ground-state scalar and vector densities, energy, and chemical potential for the fully interacting many-fermion system can be reproduced by a collection of (quasi)fermions moving in appropriately defined local, classical fields.

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[34]Note that the Dirac eigenvalues do not correspond precisely to physical energy levels in the general case [Dr90,Sp92].

In the QHD case, the local scalar and vector fields play the role of (relativistic) Kohn–Sham potentials, and by introducing nonlinear couplings between these fields, one can implicitly include additional density dependence in the single-particle potentials, as well as the composite nature of the nucleon [Fu96a]. Thus, even though the Dirac nucleons in an RMFT calculation move in local, classical potentials, this does not preclude an *exact* description of the observables mentioned in the theorem.

The exact energy functional has kinetic-energy and Hartree parts (which are combined in the relativistic formulation) plus an "exchange-correlation" functional, which is a nonlocal, nonanalytic functional of the densities that contains all the other many-body and relativistic effects [Mu96]. Rather than try to *construct* the latter functional from the lagrangian using explicit many-body techniques [Se86,Se92,Sc95], the basic idea behind the RMFT approach is to *approximate* the functional using an expansion in classical meson fields and their derivatives, based on the observation that the ratios of these quantities to the nucleon mass are small, at least up to moderate density.[35] The parameters introduced in the expansion can be fitted to experiment, and if we have a systematic way to truncate the expansion, the framework is predictive.

Thus a conventional RMFT energy functional fitted directly to nuclear properties, if allowed to be sufficiently general, will automatically incorporate effects beyond the Hartree approximation, such as those due to short-range correlations. Future work can then be focused on the explicit inclusion of higher-order many-body effects (as discussed later in this work), to examine the accuracy and limitations of the relativistic Kohn–Sham approach.

Why should we expect an approximate, mean-field functional to work well? We observe that while the mean scalar and vector potentials Φ and W are small compared to the nucleon mass, they are large on nuclear energy scales [Bo91,Fu95]. Moreover, as is illustrated in Dirac–Brueckner–Hartree–Fock (DBHF) calculations [Ho87,Te87,Ma89], the scalar and vector potentials (or self-energies) are nearly state independent and are nearly equal to those obtained in the Hartree approximation. Thus the Hartree contributions to the energy functional should dominate, and an expansion of the exchange-correlation functional in terms of mean fields should be reasonable. This "Hartree dominance" also implies that it should be a good approximation to associate the single-particle Dirac eigenvalues with the empirical nuclear energy levels, at least for states near the Fermi surface [Dr90].

We also observe that the nuclear properties of interest include: 1) nuclear shape properties, such as charge radii and charge densities, 2) nuclear binding-energy systematics, and 3) single-particle properties such as level spacings and orderings, which reflect spin-orbit splittings and shell structure. Since the Kohn–Sham approach is formulated to reproduce exactly the ground-state energy and density, and the Hartree contributions are expected to dominate the Dirac single-particle potentials, these observables are indeed the ones for

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[35]Since the meson fields are roughly proportional to the nuclear density, and since the spatial variations in nuclei are determined by the momentum distributions of the valence-nucleon wave functions, this organizational scheme is essentially an expansion in k_F/M, for k_F corresponding to ordinary nuclear densities. Here the nucleon mass M is the generic large mass scale characterizing physics beyond the Goldstone bosons.

which meaningful comparisons with experiment should be possible. Nevertheless, because the Dirac eigenvalues do not correspond precisely to observed single-particle energies (except exactly at the Fermi surface), we should not expect to reproduce spin-orbit splittings at the same level of accuracy as rms charge radii and total binding energies.[36]

As discussed in Section 5.D, an RMFT energy functional of the form in Eq. (2.33), extended to include meson self-interactions as in Eq. (5.49), successfully reproduces these nuclear observables with parameters of natural size [Fu96a]. This justifies a truncation of the energy functional at the first few powers of the fields and their derivatives, as is evident from Fig. 8. Moreover, the full complement of parameters is underdetermined, so keeping only a subset does not preclude the possibility of a realistic fit to nuclei. Both the early RMFT calculations mentioned above and the newer calculations discussed below should be interpreted within the context of this Kohn–Sham approach to DFT, since they typically involve truncation at low powers of the fields and include only a subset of the possible parameters. We also emphasize that the Dirac–Hartree approach to nuclei is not really a Hartree approximation in a "strict" sense, in which one would determine the parameters in the lagrangian from other sources (NN scattering, for example) and then solve the mean-field equations for nuclei with the *same* parameters. The DFT interpretation implies that the model parameters fitted to nuclei implicitly contain effects of both short-distance physics and many-body corrections.

B. Nuclear Structure

Recent applications of these concepts exist for a wide variety of nuclei. Properties of light nuclei with neutron halos are examined in [Ta92,Zh94]. The single-particle structure of odd-A nuclei is discussed in [Fu89c,Pe94,Wa94,Ne95]. "Islands of inversion" in neutron-rich Ne, Na, and Mg nuclei are studied in [Pa91]. Exotic nuclei near $Z = 34$ and the proton drip line, which play a role in the nuclear r-process and in astrophysics, are studied in [Sh93b,Ga94a]. The shapes of nuclei with $N = Z$ and $20 \leq A \leq 48$ are discussed in [Pa93b,Ma96a], those of superdeformed Hg isotopes, in [Pa94a], and superdeformation for $140 \leq A \leq 150$, in [Af96]. The shapes of neutron-deficient Pt isotopes are discussed in [Sh92a]; similar discussions exist for Pt, Hg, and Pb in [Yo94], for Sr and Zr in [Ma92a], for Sr and Zn isotopes near the proton drip line in [Ma92b], for Sn in [En93a,Ho94a], for Ho in [Pa93], and for rare-earth nuclei in [La96]. Hexadecapole moments of Yb isotopes are examined in [Pa95]. Exotic Ba isotopes are studied in [Sh93c], and superheavy nuclei are studied in [Bo93a,La96a]. Fission barriers in heavy nuclei are discussed in [Ru95].

Light nuclei are also examined in [Pa93a], and the effects of pairing are included in [Pa93c]. Kinks in the isotope shifts of charge radii near $Z = 40$ are examined in [La95] and for the Pb isotopes, in [Sh93a]. They appear to come out quite naturally in the RMFT.

The general conclusion of this body of work is that the relativistic mean-field theory provides an economical means of describing much of the structure of observed nuclei and a

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[36]These arguments are relevant to the choice of weights for the fitting procedure described in Section 5.D.

relatively reliable way to extrapolate to new regions of nuclear structure.

The charged mesons $(\boldsymbol{\pi}, \boldsymbol{\rho})$ first enter these calculations at the relativistic Hartree–Fock (HF) level, where the exchange interaction is included. Systematic studies at this level are contained in [Vo92a,Sh93,Zh93,Su93,Bo94,Ha94,Sc95a]. In nonrelativistic Hartree–Fock calculations with Skyrme interactions, one makes a phenomenological fit to nuclei using a contact NN interaction containing various powers of the density. (A computer code to carry out such calculations now exists in the literature [Re91].) Comparisons between the RMFT and Skyrme calculations of various nuclear properties are presented in [Ma92b,Sh92,Su93,Pe94,Sh94].

The effects of retardation and of medium modifications to the Dirac wave functions are examined in more detail in [Mi92,Zh91,Za92,Zh92]. The relationship to Landau Fermi-liquid theory is studied in [Ue92,Ta93]. Extensions to include other types of phenomenological nonlinear self-couplings of the meson and baryon fields are discussed in [Gr92,Ni92].

As noted above, an important goal is to relate calculations of nuclear properties more directly to parameters determined from NN observables by explicitly computing many-body contributions. Because the relevant equations must be solved self-consistently, this is a very difficult problem for finite systems, and an important advance was made by Gmuca [Gm92,Gm92a]. Here the DBHF calculations of nuclear matter [Br90a,De91] are *parametrized* by fitting the RMFT with scalar and vector nonlinear self-interactions to the DBHF results for the energy/nucleon *and* the self-energies Φ and W over a range of densities. (The nonlinear parameters used correspond to κ_3, κ_4, and ζ_0 in Eq. (5.49). The important advance is that it is more efficient to fit the nucleon self-energies than the effective NN interaction (or G matrix), as in [Br92,Bo94,Ha94].) The effective interactions thus obtained are used in RMFT studies of the structure of ^{16}O and ^{40}Ca nuclei *without the introduction of additional free parameters.* The calculated binding energies, single-particle spectra, and charge radii agree reasonably well (although not completely satisfactorily) with experimental data and present an improvement over the nonrelativistic Brueckner–Hartree–Fock approximation. This approach provides a framework for relating the RMFT to the DBHF calculations of nuclear matter in a quantitative manner, and thus, ultimately, to the free NN interaction. The relationship between the RMFT calculations and DBHF studies of nuclear matter is also explored in [Zh92,Za92,Su94].

Lenske, Fuchs, and Wolter have also made an important contribution [Le95,Fu95b]. Here a fully covariant, density-dependent hadronic field theory is obtained in which nonlinear effects are described through a functional dependence of meson–nucleon vertices on the baryon field operators. Rearrangement self-energies arise in the baryon field equations from the variational derivatives of the vertices. Solutions are studied in the Hartree limit and compared to the local-density approximation to DBHF theory. Parametrizations of nonlinear effects in terms of the scalar density or baryon (vector) density are discussed. Hartree calculations for nuclei between ^{16}O and ^{208}Pb show that rearrangement corrections simultaneously improve the description of binding energies, root-mean-square radii, and density distributions. This approach provides an alternative to the method used by Gmuca for connecting nuclear observables to the NN interaction.

In Section 3.A, we noted that a linear realization of chiral symmetry, with the usual form of symmetry breaking, cannot produce successful nuclear phenomenology at the RMFT level. This was the primary motivation for constructing a model with a nonlinear realization of

the symmetry in Section 5.D, which includes a light scalar to simulate the exchange of two correlated pions between nucleons. The failure of the linear models, however, lies more with the form of the scalar potential responsible for spontaneous symmetry breaking than with the linear realization of the symmetry.

In [He94a,Ca96], nuclear matter and finite nuclei are studied in the RMFT with a chiral lagrangian that generalizes the linear σ model and also accounts for the QCD trace anomaly. A logarithmic meson potential that involves the σ and $\boldsymbol{\pi}$ fields and also a heavy glueball field φ is used to spontaneously break both scale invariance and chiral symmetry. The scale-invariant term that leads to an ω meson mass after spontaneous symmetry breaking is strongly favored to be of the form $\omega_\mu \omega^\mu \varphi^2$ by the bulk properties of nuclei; they also rather strongly constrain the other parameters. A reasonable description of the closed-shell nuclei oxygen, calcium, and lead can be achieved, and the results are improved by including a quartic omega self-interaction term in the lagrangian. These results are consistent with the discussion at the beginning of this section, since this linear chiral model contains meson self-interactions corresponding to the κ_3, κ_4, and ζ_0 terms in Eq. (5.49). (See also parameter set Q2 in Table II.) The important point is that the use of a more general (*i.e.*, nonrenormalizable) interaction potential provides more freedom to adjust the self-interactions, and the number of free parameters is sufficient to describe nuclei, in contrast to the usual (renormalizable) sigma model, as discussed after Eq. (3.9).

In [Pr94], a single, adjustable scalar-nonlinearity parameter is included in the energy, together with the zero-point energy $\Delta\mathcal{E}$ discussed in Section 5.E. Although observed neutron star masses do not constrain this parameter, it can be chosen to provide a reasonable description of bulk nuclear properties, when the scalar mass is roughly $600\,\text{MeV}$. In this case, however, the coefficients of the nonlinear Φ^3 and Φ^4 terms are unnatural; in particular, the quartic coefficient is unnaturally large and negative. This is necessary to cancel the effects of the unnaturally large, positive coefficients introduced by $\Delta\mathcal{E}$ [see Eqs. (5.58) and (5.61)].

C. Electroweak Interactions in Nuclei

1. Electromagnetic currents in QHD

In QHD–I there are no charged mesons. One can, however, introduce an effective electromagnetic current, to be used in lowest order, that incorporates some of the nucleon's internal structure [Se86,Se92,Wa95]:

$$J_\mu = \overline{\psi}\gamma_\mu \underline{Q}\psi + \frac{1}{2M}\,\partial^\nu\big(\overline{\psi}\underline{\lambda}'\sigma_{\mu\nu}\psi\big)\ ,$$

$$\underline{Q} = \frac{1}{2}(1+\tau_3)\ , \qquad \underline{\lambda}' = \lambda'_{\text{p}}\frac{1}{2}(1+\tau_3) + \lambda'_{\text{n}}\frac{1}{2}(1-\tau_3)\ . \tag{6.1}$$

Here $\lambda'_{\text{p}} = 1.7928$ and $\lambda'_{\text{n}} = -1.9131$. This current is covariant and local, and it is conserved by virtue of the QHD–I field equations. It also contains the correct anomalous magnetic moment. To include the spatial extent of the nucleon, one can introduce a single *overall* form factor

$$f_{\text{s.n.}}(q^2) = \left[\frac{1}{1-q^2/(855\,\text{MeV})^2}\right]^2\ , \tag{6.2}$$

where q^μ is the four-momentum transfer, to be used in all matrix elements of the current (6.1). This is equivalent to replacing the photon propagator $1/q^2$ with the effective Møller potential $f_{\text{s.n.}}(q^2)/q^2$. This current can be used consistently with the RMFT solutions in QHD–I, such as the Dirac–Hartree wave functions. Many applications exist [Se86,Se92,Wa95].

The use of a single function $f_{\text{s.n.}}(q^2)$ assumes that the momentum dependence of the charge and anomalous form factors is the same. This assumption breaks down at large q^2, since it is actually the Sachs form factors that scale similarly. This observation can be implemented easily with a simple change to $f_{\text{s.n.}}(q^2)$ and J_μ [Wa95].

Alternatively, one can attempt to calculate the single-nucleon structure by starting with the electromagnetic current in the renormalizable model QHD–II, which contains charged ρ and π mesons, and then by evaluating quantum loop diagrams. This is discussed in [Se86]. The structure of the electromagnetic current in QHD–III [Se92b], a chirally invariant, renormalizable extension of QHD–II, is currently under investigation [Pr96].

In the context of effective field theory, the composite structure of the particles is described with increasing detail by including more and more *nonrenormalizable* interactions in a derivative expansion. For example, the electromagnetic current obtained from Eq. (5.48) by taking $\delta\mathcal{L}/\delta(eA_\mu)$ is, after some partial integration,

$$J^\mu = \frac{1}{2}\overline{N}(1+\tau_3)\gamma^\mu N + \frac{1}{2M}\partial_\nu(\overline{N}\lambda'\sigma^{\mu\nu}N) - \frac{1}{2M^2}\partial^2[\overline{N}(\beta^{(0)}+\beta^{(1)}\tau_3)\gamma^\mu N]$$

$$+\frac{1}{2M^2}\partial^\mu\partial^\nu[\overline{N}\beta^{(1)}\tau_3\gamma_\nu N] + \frac{1}{g_\gamma}(\partial_\nu\rho_3^{\mu\nu} + \frac{1}{3}\partial_\nu V^{\mu\nu}) + 2f_\pi^2\operatorname{tr}(\underline{v}^\mu\tau_3) . \tag{6.3}$$

Note that the photon can couple to the nucleon either directly or through the exchange of neutral vector mesons (rho or omega).

We can determine the tree-level electromagnetic form factors of the nucleon from the current (6.3) and the lagrangian (5.46). For spacelike momentum transfers $Q^2 = -q^2$, the isoscalar and isovector charge form factors are

$$F_1^{(0)}(Q^2) = \frac{1}{2} - \frac{\beta^{(0)}}{2}\frac{Q^2}{M^2} - \frac{g_\text{v}}{3g_\gamma}\frac{Q^2}{Q^2+m_\text{v}^2} + \cdots , \tag{6.4}$$

$$F_1^{(1)}(Q^2) = \frac{1}{2} - \frac{\beta^{(1)}}{2}\frac{Q^2}{M^2} - \frac{g_\rho}{2g_\gamma}\frac{Q^2}{Q^2+m_\rho^2} + \cdots , \tag{6.5}$$

and the anomalous form factors are

$$F_2^{(0)}(Q^2) = \frac{\lambda'_p + \lambda'_n}{2} - \frac{f_\text{v}g_\text{v}}{3g_\gamma}\frac{Q^2}{Q^2+m_\text{v}^2} + \cdots , \tag{6.6}$$

$$F_2^{(1)}(Q^2) = \frac{\lambda'_p - \lambda'_n}{2} - \frac{f_\rho g_\rho}{2g_\gamma}\frac{Q^2}{Q^2+m_\rho^2} + \cdots . \tag{6.7}$$

The corresponding mean-square charge radii are

$$\langle r^2\rangle_1^{(0)} = 6\left(\frac{\beta^{(0)}}{M^2} + \frac{2g_\text{v}}{3g_\gamma m_\text{v}^2}\right) , \qquad \langle r^2\rangle_1^{(1)} = 6\left(\frac{\beta^{(1)}}{M^2} + \frac{g_\rho}{g_\gamma m_\rho^2}\right) , \tag{6.8}$$

$$\langle r^2\rangle_2^{(0)} = \frac{4}{\lambda'_p + \lambda'_n}\frac{f_\text{v}g_\text{v}}{g_\gamma m_\text{v}^2} , \qquad \langle r^2\rangle_2^{(1)} = \frac{6}{\lambda'_p - \lambda'_n}\frac{f_\rho g_\rho}{g_\gamma m_\rho^2} . \tag{6.9}$$

The form factors have a contribution from vector dominance and a correction from the intrinsic structure of order Q^2, that is, to second order in a derivative expansion. This correction is adequate for most applications to nuclear structure; thus, when the model of Section 5.D is applied to finite nuclei, an *ad hoc* form factor need not be introduced.[37] In practice, as the values of $g_{\rm v}$ and g_ρ are varied to fit nuclear properties, $\beta^{(0)}$, $\beta^{(1)}$, and f_ρ are chosen to reproduce the empirical isoscalar and isovector charge radii and isovector anomalous radius of the nucleon. (The parameter $f_{\rm v}$ is determined from nuclear properties, since the nucleon's isoscalar anomalous radius is poorly known.)

We note, however, that although the derivative expansion for the current is adequate for most RMFT calculations, applications involving large energy-momentum transfers will require additional terms of higher order in Q^2. The utility of this expansion for $Q^2 \gtrsim M^2$ is an open question that is currently under active investigation.

2. Weak currents in QHD

For the effective weak currents, one can proceed analogously. The effective weak current to be used in lowest order in QHD–I is given as the sum of a polar-vector and an axial-vector part by [Se86,Wa95]

$$\mathcal{J}_\mu^\pm = J_\mu^\pm + J_{\mu5}^\pm \; , \tag{6.10}$$

where the charge-changing, polar-vector current is defined by

$$J_\mu^\pm = \overline{\psi}\gamma_\mu\tau_\pm\psi + \frac{(\lambda_p' - \lambda_n')}{2M}\,\partial^\nu\!\left(\overline{\psi}\tau_\pm\sigma_{\mu\nu}\psi\right) \; , \tag{6.11}$$

with $\tau_\pm = \frac{1}{2}(\tau_1 \pm i\tau_2)$. Similarly, the charge-changing, axial-vector current is given by

$$J_{\mu5}^\pm = F_A(0)\left(g_\mu{}^\nu - \frac{1}{m_\pi^2 + \partial^2}\partial_\mu\partial^\nu\right)\overline{\psi}\gamma_\nu\gamma_5\tau_\pm\psi \; . \tag{6.12}$$

The weak current so defined is covariant, it satisfies PCAC (partial conservation of the axial-vector current), it contains a nonlocality implied by pion-pole dominance, and it gives the correct result for semileptonic weak interactions on a free nucleon.

The effective weak *neutral* current for QHD–I is defined to have the symmetry properties of the standard model [Se86,Wa95]

$$\mathcal{J}_\mu^0 = J_\mu^0 + J_{\mu5}^0 - 2\sin^2\theta_{\rm W}J_\mu \; , \tag{6.13}$$

where J_μ^0 and $J_{\mu5}^0$ are obtained from Eqs. (6.11) and (6.12) by the replacement $\tau_\pm \to \frac{1}{2}\tau_3$, and J_μ in the final term is the electromagnetic current. Several applications of these currents exist [Se86,Wa95]. The weak current in the effective theory of Section 5.D remains to be studied.

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[37] A form factor for the pion also arises directly from vector-meson dominance [Fu96a].

3. Electroweak exchange currents

Within the framework of a consistent hadronic field theory, one can also calculate two-body exchange currents. Early work on electromagnetic exchange currents is summarized and referenced in [Se86], where explicit expressions are given for the long-range pion and pair currents. By the conserved-vector-current theory (CVC), these also give the exchange-current contributions to the weak vector current. Exchange currents play a crucial role in the accurate reproduction of the high-momentum-transfer behavior of electromagnetic observables for light nuclei, as discussed in Section 4.A.

One can also compute the exchange-current contributions to the weak axial-vector current. Instead of strict current conservation, one must now respect chiral symmetry and PCAC [Wa95]. While the forms of the relativistic one-body and two-body axial-vector currents are model dependent, a simple and useful version arises from the chirally transformed (nonlinear) sigma model discussed in Section 3.B, where a projection operator of the form in Eq. (6.12) appears directly in the current [An96]. The long-range pion-exchange-current contribution to the isovector axial charge operator in this approach is given by

$$
\mathbf{J}_{05}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}) = -\frac{F_A f^2}{4\pi} [\boldsymbol{\tau}(1) \times \boldsymbol{\tau}(2)][\delta(\mathbf{x}_1 - \mathbf{x})\boldsymbol{\sigma}_2 + \delta(\mathbf{x}_2 - \mathbf{x})\boldsymbol{\sigma}_1] \cdot \hat{r} \, (1 + x_\pi) \frac{\exp(-x_\pi)}{x_\pi^2} \,,
$$

$$
(6.14)
$$

where $x_\pi = m_\pi|\mathbf{r}|$, with $\mathbf{r} \equiv \mathbf{x}_1 - \mathbf{x}_2$, and $f/m_\pi = g_\pi/2M$. The Goldberger–Treiman relation, which follows from PCAC, states that $MF_A(0) = -g_\pi f_\pi$, where $f_\pi \approx 93\,\mathrm{MeV}$ is the pion decay constant [Wa95]. Many applications of the above expression, including additional, shorter-range, hadronic-exchange contributions, also appear in the literature [To95].[38]

4. Recent developments in electromagnetic interactions

The simplest and most informative electromagnetic process to study is electron scattering (e, e'). Relativistic analyses of scattering in the region of the quasielastic peak, corresponding to single-nucleon knockout, are found in [De92,Ji92,Ei94,Ri94,Fr94,Pi95]. The systematics of the location and shape of the peak, from low energies to the high-energy data from SLAC on ^{56}Fe, is examined in [De92,Fr94]. In the former work, the roles of M^* and exchange currents at high q^2 are emphasized. In the latter, it is shown that one must include momentum dependence in the relativistic self-energy $\Sigma(k)$ to understand the data; the RMFT gives a constant Σ.

The failure of the (e, e') data to satisfy the Coulomb sum rule throughout the periodic table has long been one of the significant puzzles in nuclear physics [Wa95]. The correct form of the relativistic sum rule, the effects of nuclear binding, and the use of the off-shell current are analyzed in [Fe94,Ko95]. An important recent contribution [Jo95] indicates that the solution to this problem may now be in hand. In this work, the world data on inclusive

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[38]Equation (6.14), based on pion-pole dominance of the relevant amplitudes, contains an extra factor of F_A^2 relative to that used in [To95].
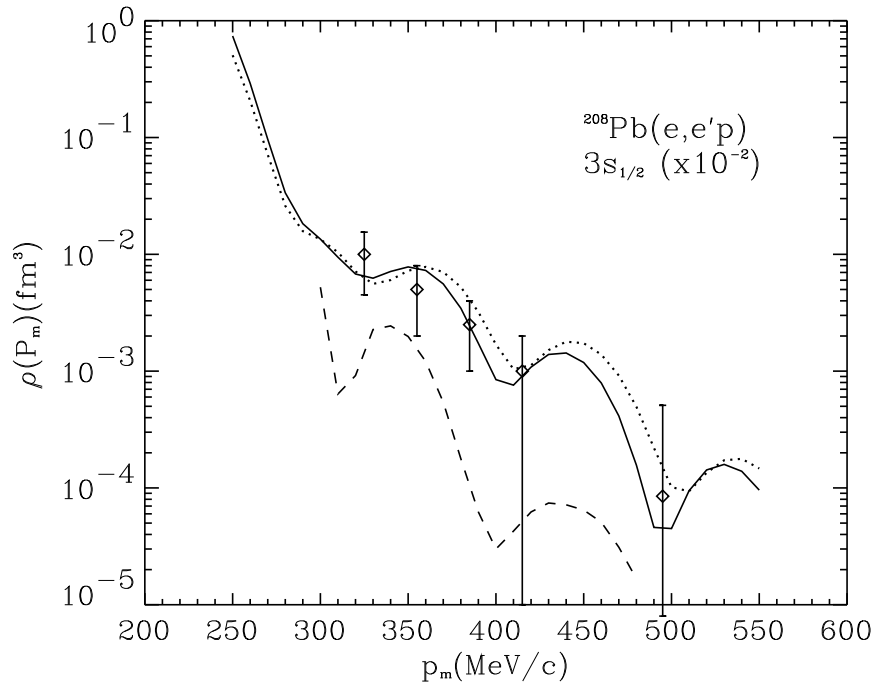
FIG. 10. The $(e, e'p[3s_{1/2}])$ reduced cross section of ^{208}Pb for high missing momentum with incoming kinetic energy $E_i = 487$ MeV and the binding energy $E_b = 10$ MeV. The solid and dotted lines are the results of Kim and Wright [Ki96], who use an RMFT model with a spectroscopic factor fitted to low-missing-momentum results. The solid line includes Coulomb distortion in an approximate way, while the dotted line is the plane-wave result. The dashed line is a nonrelativistic result by the Belgium group [Va96], which includes only the one-body current. The data are from NIKHEF [Bo94b].

quasielastic electron scattering have been used to separate the longitudinal and transverse response functions of ^{56}Fe and ^{12}C. The resulting longitudinal response functions lack the "quenching" that has been such a problem. There are still some inconsistencies in the total collection of world data, however, and this is a subject that cries for a complete experiment with all the kinematic flexibility of CEBAF for definitive resolution.

With coincidence capabilities at extreme kinematics now available at Bates, Mainz, NIKHEF, and CEBAF, one can use the reaction $(e, e'X)$ to probe, among other things, the high-momentum tail of the momentum distribution in the nucleus. There are many effects that will contribute at high momentum transfer and at high missing momentum, in $(e, e'p)$ from a filled orbital. For example, in ^{208}Pb$(e, e'p)$ from the $3s_{1/2}$ level, one must consider long- and short-range correlations, meson-exchange currents, isobar currents, the spin-orbit interaction, other relativistic effects, etc. One of the prime motivations for such experiments is to disentangle and study these effects. The RMFT analysis of such a process, where the initial wave function is the solution to the Dirac–Hartree equations and the final wave function is generated in the RIA optical potential, has the distinct advantage that much of this physics, as in nucleon–nucleus scattering (Section 2.C), is incorporated in the starting approximation. Figure 10, for example, shows recent calculations of Kim and Wright [Ki96] for this transition. This calculation, with no free parameters, accurately

reproduces the existing data. It is only by pushing to more extreme kinematics, with precision experiments, and with many nuclear transitions, that one will be able to unambiguously disentangle the role of the various additional contributions.

The subject of $(e, e'X)$ is also discussed within the framework of QHD in [Pi92,Ji93,Ga94]. It is emphasized in the final paper that one must understand relativistic hadron dynamics before drawing conclusions about new phenomena, such as color transparency.

Relativistic analyses of the photonuclear reactions (γ, p) and $(\gamma, \pi^- p)$ are discussed in [Lo92,He94,Jo94]. Other phenomena such as $(\sigma\omega\gamma)$ mixing, $(\rho\omega)$ mixing, charge-symmetry breaking, and the production of (e^+e^-) pairs to probe the quark-gluon plasma are discussed in [Kr92,Li95a,Li95b].

In a recent review article on *Electromagnetic Response Functions in Quantum Hadrodynamics* [We93], elastic, inelastic, and quasielastic electron scattering over a wide range of energy and momentum transfers are discussed as a probe of the electromagnetic response of nuclei. Electromagnetic response functions obtained with different QHD models of nuclear structure and at different levels of approximation are compared with data. It is shown that RPA correlations are important and have different effects in different kinematical regimes.

A second recent review of *Nuclear Response in Electromagnetic Interactions with Complex Nuclei* is given in [Bo93]. Here the response of nucleons and complex nuclei to an external electromagnetic probe at intermediate and high energies is illustrated by considering both inclusive and semi-inclusive electron scattering. Form factors for elastic scattering and structure functions for inelastic scattering are derived, and several examples are discussed, including polarization observables.

5. Recent developments in electroweak interactions

One can obtain information about the single-nucleon matrix elements of the weak current by studying quasielastic neutrino scattering (ν, ν') from nuclei. The importance of the single-nucleon self-energies and of a realistic (RPA) description of the nuclear excitation spectrum in the extraction of the axial-vector form factor of the nucleon $F_A(q^2)$ is discussed in [En93,Ki95].

The single-nucleon matrix element of the weak axial-vector current contains a one-pion-exchange contribution leading to the induced pseudoscalar coupling $F_P(q^2)$, and the weak axial-vector exchange current also has a one-pion-exchange contribution (6.14); one might expect this longest-range contribution to be modified in the nuclear medium. A relativistic hadronic field theory of the nucleus (QHD) allows one to estimate these corrections. A great deal of recent work has progressed in this direction [To92,Ga92,Ba93,Iz94,Pa94,Gi95], much of it carried out within the RMFT of nuclear structure. The basic conclusions are that weak axial-vector exchange currents can play an important role if one looks in the right place, for example, in first-forbidden β-decay, and that there can be significant nuclear-structure effects in the weak axial-vector coupling to the nucleus.

In [To92], the "enhancement factor", defined as the ratio of the axial-charge matrix element in first-forbidden β-decay to its one-body value, is calculated in a meson-exchange model for both light and heavy nuclei. Pion-exchange processes are computed for a chirally symmetric, phenomenological lagrangian and compared with results obtained in the soft-pion approximation [Eq. (6.14)]. Heavy-meson pair graphs are included, with coupling constants

determined from the Bonn NN potential [Ma89]. Nonrelativistic reductions are performed to both leading order and next-to-leading order. The results are sensitive to the choice of the short-range correlation function used in conjunction with harmonic oscillator wave functions. A ratio of transition matrix elements, however, is less sensitive to this choice. The main result in this paper is the comparison of a transition in a heavy nucleus with a transition in a light nucleus: $r = \delta_{\mathrm{mec}}(A = 208; 1g_{9/2} \to 0h_{9/2})/\delta_{\mathrm{mec}}(A = 16; 1s_{1/2} \to 0p_{1/2}) = 1.38$. This result is smaller than the value of 1.58 ± 0.09 deduced by Warburton [Wa94a] in a fit between one-body shell-model matrix elements and experiment.

In [Pa94], shell-model matrix elements of the axial-charge exchange-current operator are calculated through next-to-leading order in heavy-fermion chiral perturbation theory. It is found that the loop corrections to the one-soft-pion-exchange contribution in Eq. (6.14) are small (roughly 10%) and have no significant dependence on the nuclear mass number or on the valence-nucleon orbits.

Radiative μ-capture is studied in the RMFT in [Fe92].

6. Parity violation in $(\vec{e}, e')$

One of the most important areas of future research at CEBAF will be the study of parity violation in electron scattering. This arises from the interference between the amplitudes for photon exchange and for Z^0 exchange, which involve couplings to the electromagnetic and weak neutral currents, respectively [Wa95]. Feasibility has been demonstrated in pioneering experiments at Bates and Mainz.

Two particularly interesting contributions are [Al93,Ba94]. In [Al93], the impact of pionic correlations and meson-exchange currents in determining the (vector) response functions for electroweak, quasielastic lepton scattering from nuclei is discussed. The Fermi-gas model is used to maintain consistency in treating forces and currents (gauge invariance) and to provide a Lorentz-covariant framework. Results obtained in first-order perturbation theory are compared with HF and RPA calculations and are found to provide quite successful approximations for the quasielastic response functions. The role of pionic correlations is investigated in some detail, and meson-exchange currents are shown to provide a small, but non-negligible contribution to the vector response.

In the second paper, parity-violating quasielastic electron scattering is studied within the context of the relativistic Fermi-gas model and its extension to include pionic correlations and meson-exchange currents. The work builds on previous studies using the same model; here, the part of the parity-violating asymmetry that contains axial-vector hadronic currents is developed in detail, and a link is provided to the transverse vector-isovector response. Various integrated observables are constructed from the differential asymmetry, and the most favorable observables for studying pionic correlations and the strangeness form factors of the nucleon are determined. Comparisons are also made with recent predictions based on the RMFT.

Nuclear-structure effects pertinent to the extraction of the weak, neutral, axial-vector form factor of the nucleon, a quantity of particular interest because of the potential role played by $s\bar{s}$ quark-antiquark pairs, are emphasized in [Ho93,Ho93a].

Parity-violation in the structure of heavy nuclei is examined in the relativistic HF approximation in [Ho94]. The role of nuclear structure in atomic parity-violating experiments

is discussed in the definitive work [Po92a].

D. Strangeness in Nuclei

The addition of strangeness adds another dimension to nuclear structure. Properties of hypernuclei with $S = -1$ are investigated in the RMFT in [Co93,Gl93,Do95,Lo95]. The two primary systematic features of Λ hypernuclei are the relatively small depth of the central potential and the small spin-orbit splittings. The scalar and vector couplings in a (σ, ω) model can be adjusted phenomenologically to describe the central potential, but the spin-orbit splittings have been a more vexing problem.

It was observed in [Je90,Co91] that a tensor coupling of the ω to the hyperon, with the appropriate sign, can indeed produce a small spin-orbit splitting. The systematics of the interaction of hyperons with this additional tensor coupling are investigated in [Co92,Co94,Co95,Ma94,Ma95]. In particular, it is shown in [Co94,Co95] that a relativistic optical potential with a tensor coupling can describe the data.

If one views QHD as an effective hadronic field theory, such a coupling will exist in the lagrangian. Even within a renormalizable framework, one has an induced coupling of this form. While motivation for a tensor coupling appears to exist within a quark-model framework, the theoretical challenge is to quantitatively explain the difference in size between spin-orbit effects in the hyperon and nucleon sectors.

One interesting goal of the study of strangeness in nuclei is to extrapolate to large $|S|$; the RMFT provides a convenient basis for this extrapolation, and this problem is examined in [Sc92,Do93,Ma93a,Sc93,Ga95]. A large class of bound, multi-strange objects is one distinct possibility raised by these studies.

The K^+ has a relatively weak coupling to nucleons, and thus it provides a useful hadronic probe for viewing the nuclear interior. K^+ scattering and production from nuclei are studied within the framework of the RMFT in [Ca92,La92,Ru92,Ko95a]. In [Ko95b], it is shown that K^- quasielastic scattering from nuclei can be explained within the framework of the local-density RPA and isoscalar correlations contained in QHD–I. Subthreshold kaon production is discussed in [Li95], and strong-interaction effects in Σ^- atoms, in [Ma95a]. Other properties of hypernuclei are examined in [Lo94].

On a more basic level, the properties of a hyperon in nuclear matter are examined within the framework of QCD sum rules in [Ji94,Ji95].

7. MATTER UNDER EXTREME CONDITIONS

A. Relativistic Transport Theory

One of the principal thrusts of nuclear physics has been, and will continue to be, the use of relativistic heavy-ion reactions to study the properties of nuclear matter under extreme conditions of density, temperature, and flow. A great deal of work has been done within the framework of QHD using relativistic transport theory to describe heavy-ion collisions. For example, the foundations of relativistic transport theory are discussed in [Ko87,Bl88a,Ko88,Li89a,La90,Ma92,Ma93,Mo94,Sc94] and further developed in

[De91,De92a,Me93,Ni93,Mr94,Fu95a,Po95]. In this section, we start the discussion with a simple introduction to transport theory.

The Vlasov–Uehling–Uhlenbeck (VUU) model describes the transport of the distribution function in phase space. It is a one-body transport model that includes the effects of a long-range mean field on the one-body dynamics, together with a short-range, two-body collision term. The mean field is calculated from the distribution function, and when parametrized as a function of the density, it probes the equation of state of the medium. The dynamical description is classical, except that collisions are treated stochastically, and a Pauli-blocking factor is included for the final states in the collision term. The approach can be generalized to include inelastic channels. The basic ideas behind various transport models are described in the review article by Greiner and Stöcker [St86], and a computer code to carry out such calculations is now available [Ha93a]. We review some of the basic concepts.

Consider the microcanonical ensemble that consists of a collection of identical, randomly prepared, microscopic systems (particles). The ensemble can be characterized by the distribution in phase space:

$$\mathrm{d}N = \text{Number of members of the ensemble in } \frac{\mathrm{d}^3 p\, \mathrm{d}^3 q}{(2\pi)^3}$$

$$\equiv f(\mathbf{p}, \mathbf{q}, t)\, \frac{\mathrm{d}^3 p\, \mathrm{d}^3 q}{(2\pi)^3} \ . \tag{7.1}$$

The *probability* to find a particle in this region of the ensemble is the probability of picking such a member of the ensemble at random; this is equal to $\mathrm{d}N/N$, where N is the total number of members in the ensemble, obtained by integrating Eq. (7.1) over all phase space. This probability can be used to compute expectation values over the ensemble.

As a function of time, a particle moves from $(\mathbf{p}_0, \mathbf{q}_0)$ to $(\mathbf{p}, \mathbf{q})$ in phase space. Liouville's theorem (see, for example, [Wa89]) states that with hamiltonian dynamics, the volume in phase space is unchanged with time. Since the number of particles $\mathrm{d}N$ in this volume is also conserved, one concludes that the distribution function is unchanged along a phase trajectory:

$$f[\mathbf{p}(t), \mathbf{q}(t), t] = f[\mathbf{p}_0, \mathbf{q}_0, t_0] = \text{constant.} \tag{7.2}$$

Differentiation with respect to time yields

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial \mathbf{p}} \cdot \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} + \frac{\partial f}{\partial \mathbf{q}} \cdot \frac{\mathrm{d}\mathbf{q}}{\mathrm{d}t} = 0 \ . \tag{7.3}$$

Now Hamilton's equations state

$$\frac{\mathrm{d}\mathbf{q}}{\mathrm{d}t} = \mathbf{v} = \frac{\partial H}{\partial \mathbf{p}} \ , \qquad \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = \mathbf{F} = -\frac{\partial H}{\partial \mathbf{q}} \ , \tag{7.4}$$

and insertion into Eq. (7.3) gives

$$\frac{\partial f}{\partial t} = \frac{\partial H}{\partial \mathbf{q}} \cdot \frac{\partial f}{\partial \mathbf{p}} - \frac{\partial H}{\partial \mathbf{p}} \cdot \frac{\partial f}{\partial \mathbf{q}} \equiv \{H, f\}_{\mathrm{PB}} \ . \tag{7.5}$$

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The last equality identifies the Poisson bracket. In *equilibrium*, there is no time dependence to the distribution function, and this expression must vanish. The solution to this condition is that $f = f(H)$, and f is a constant of the motion.

A short-range, two-body Boltzmann collision term is now included. Go to the canonical ensemble, which consists of identical, randomly prepared assemblies (collections of systems). Particles are now scattered into and out of the region in phase space in Eq. (7.1). Assume that

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \left(\frac{\partial f}{\partial t}\right)_{\text{collisions}} . \tag{7.6}$$

Momentum is conserved in the collisions, so that $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}'_1 + \mathbf{p}'_2$. Detailed balance then states that $\text{Rate}(i \to f) = \text{Rate}(f \to i)$, or $v_{12}\sigma = v_{1'2'}\sigma'$, where v_{ij} is the relative velocity.[39] The number of transitions per unit time in the direction $i \to f$ in the assembly is given by

$$\left[\frac{\text{number of transitions}}{\text{time}}\right]_{i \to f} = \left[f(\mathbf{p}_1, \mathbf{q}, t)\frac{\mathrm{d}^3 p_1}{(2\pi)^3}v_{12}\right]\sigma\left[f(\mathbf{p}_2, \mathbf{q}, t)\frac{\mathrm{d}^3 p_2\,\mathrm{d}^3 q}{(2\pi)^3}\right] . \tag{7.7}$$

The first factor is the incident flux, and the last factor is the number of target particles, both in the appropriate momentum interval. The Boltzmann collision term at the point $\mathbf{q}$ can thus be written as [St86]

$$\left(\frac{\partial f}{\partial t}\right)_{\text{collision}} = \int \mathrm{d}^3 p_2\,\mathrm{d}^3 p'_1\,\mathrm{d}^3 p'_2\,\frac{1}{(2\pi)^6}\,\delta^{(3)}(\mathbf{p} + \mathbf{p}_2 - \mathbf{p}'_1 - \mathbf{p}'_2)\sigma v_{12}$$
$$\times [f(\mathbf{p}'_1, \mathbf{q}, t)f(\mathbf{p}'_2, \mathbf{q}, t) - f(\mathbf{p}, \mathbf{q}, t)f(\mathbf{p}_2, \mathbf{q}, t)] . \tag{7.8}$$

The last term counts particles scattered *out* of this region in phase space, and the first term counts those particles scattered *in*. This relation will be abbreviated as

$$\left(\frac{\partial f}{\partial t}\right)_{\text{coll}} = \int \mathrm{d}^3 p_2\,\mathrm{d}^3 p'_1\,\mathrm{d}^3 p'_2\,\frac{1}{(2\pi)^6}\,[f'_1 f'_2 - f f_2]\sigma v_{12}\delta^{(3)}(\mathbf{p} + \mathbf{p}_2 - \mathbf{p}'_1 - \mathbf{p}'_2) . \tag{7.9}$$

This is the result of classical transport theory; with zero-range collisions, the angular distributions will be isotropic.

If the one-body hamiltonian has the form

$$H = \frac{\mathbf{p}^2}{2m} + U(\mathbf{q}) , \tag{7.10}$$

where U is a mean field, then the transport equations take the form

$$\left(\frac{\partial f}{\partial t}\right) + \mathbf{v}\cdot\nabla_{\mathbf{q}}f - \nabla_{\mathbf{q}}U\cdot\nabla_{\mathbf{p}}f = \left(\frac{\partial f}{\partial t}\right)_{\text{coll}} . \tag{7.11}$$

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[39]Note that the number of final states in momentum space $\mathrm{d}^3 n_f = \mathrm{d}^3 p_f/(2\pi)^3$ has now been explicitly removed from the definition of σ.

These are nonlinear, integro-differential equations for the distribution function f. In equilibrium, with no time dependence, the collision term must vanish, which implies $f_1' f_2' = f f_2$, or $f(E_1')f(E_2') = f(E)f(E_2)$. Since energy is conserved in the two-body collision cross section, one has $E + E_2 = E_1' + E_2'$. If these relations are to hold for all E and E_2, then $f(E)f(E_2) = g(E + E_2)$. Differentiation with respect to E and E_2 in turn leads to $f'(E)/f(E) = \text{constant} \equiv -1/k_\mathrm{B}T$, which yields the Boltzmann distribution $f(H) = \exp{(-H/k_\mathrm{B}T)}$ as the equilibrium solution to these kinetic equations.

In *molecular dynamics* calculations [with no $U(\mathbf{q})$], one simply follows the classical equations of motion of all the members of the canonical ensemble numerically. In contrast, in the *Boltzmann–Uehling–Uhlenbeck* approach [still with no $U(\mathbf{q})$],[40] one includes a Pauli-blocking term for identical fermions that prevents them from scattering into occupied states. An occupied state is assumed to consist of one particle in the unit cell $\mathrm{d}^3p\,\mathrm{d}^3q/(2\pi)^3$ in phase space. Thus one makes the replacement

$$[f_1' f_2' - f f_2] \rightarrow [f_1' f_2'(1 - f)(1 - f_2) - f f_2(1 - f_1')(1 - f_2')] . \tag{7.12}$$

The equilibrium solutions to these kinetic equations are the familiar Fermi distribution functions.

In the *VUU* approach, one also includes the additional long-range mean field $U(\mathbf{q})$ produced by the local particle density ρ, which is obtained in turn by integrating the distribution function over $\mathrm{d}^3p/(2\pi)^3$. Parametrization of the ρ dependence of U then probes various equations of state.

What advantages does QHD have for studying these transport equations?

- It provides a covariant description of the nuclear many-body system.

- It thus provides a basis for a relativistic treatment of the transport equations.

- It allows for a consistent treatment of all hadronic channels.

- In the RMFT, it provides an excellent first approximation to the nuclear mean field, including an energy dependence in the equivalent nonrelativistic optical potential [Se86].

A great deal of work on relativistic transport theory, mostly in connection with relativistic heavy-ion reactions, has been done in the past few years. A covariant Boltzmann–Uehling–Uhlenbeck (BUU) approach, the basic ideas of which have been presented above, is developed in [Bl88a,La90,Ma92,Ma94a] and applied in [Bl89,Bl91,Ko90,Ko91]. Relativistic transport coefficients are discussed in [Ha93b,Ay94,Mo94].

The connection of the scalar and vector mean fields to the underlying relativistic two-body theory is explored in [El92,Fu92,We92]. Effective cross sections in the medium are studied in [Li93,Ma94b]. Additional momentum-dependent scalar and vector potentials, which provide a more accurate description of the optical potential, are introduced in [We92,We93a]. Shock waves are discussed in [Mo95]. The role of the Dirac sea in such collisions is discussed

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[40]This is sometimes called the Nordheim–Uehling–Uhlenbeck approach.

in [Ju92]. The production of kaons is examined in [Fa93,Fa93a,Fa94] and of antinucleons, in [Te93,Te94,Li94].

Interesting extensions include studies of a classical version of QHD [Bu93,Bu93a,Bu95] and of a transport theory for quarks and mesons [Zh92a]. Experimental implications of a relativistic, mean-field, two-fluid model are explored in [Iv94,Ru94].

B. Extrapolation and Connections to QCD

The original motivation for the development of QHD was to find a theoretical framework that allows extrapolation of the properties of observed nuclear matter to extreme conditions of density and temperature, while retaining general principles of physics, such as quantum mechanics, Lorentz invariance, and microscopic causality. The only consistent theoretical framework we have for describing such a strongly interacting quantum many-body system is relativistic quantum field theory based on a local lagrangian density. The first model attempt using hadronic degrees of freedom, QHD–I, consisted of a renormalizable theory with baryons and neutral scalar and vector mesons. When solved at the level of mean-field theory (RMFT), one finds an equation of state with a minimal number of parameters that can be fitted to the equilibrium point of nuclear matter, which then provides a simple equation of state at all densities $\rho_{\rm B}$ and temperatures T. These results have been discussed in Section 2.

As noted earlier, when viewed as an effective hadronic field theory describing the underlying QCD, all possible couplings consistent with the relevant symmetries should be included in the lagrangian. Without some organizing principle, one soon loses predictive power. The organizing principle presented here is that while the mean fields are large on the scale of the nuclear binding energy, the dimensionless ratios Φ/M and W/M are still relatively small, and one can sensibly expand in these. This observation allows one to understand the success of the RMFT treatment of nuclear structure. While providing a systematic basis for extrapolating away from the properties of observed nuclear matter, this approach clearly limits the extent over which one can reliably carry out this extrapolation.

A major goal of the extrapolation to high baryon density $\rho_{\rm B}$ is to describe the properties of neutron stars. Extrapolations within the framework of RMFT with effective couplings are studied in [Ro92,Gl92,Ba94a,Mo94a,Pr94,Su95,Mu96]. In [Mo94a], cold nuclear matter is investigated in nonlinear, mean-field, scalar–vector models that include density-dependent meson parameters. This dependence can be both explicit and implicit through the nucleon effective mass. Interactions between the scalar and vector fields are included, and the properties of neutron stars are investigated using the resulting equation of state. The implications of the model of Zimanyi and Moszkowski [Zi90] with derivative scalar couplings are examined in [Gl92a,Sa94a,De95]. The connection between the RMFT results and Dirac–Brueckner theory is studied in [Li92].

A study of the uncertainties in extrapolation is contained in [Mu96], where the properties of high-density nuclear and neutron matter are computed using a relativistic mean-field approximation to the energy functional. Various types of nonlinearities involving scalar-isoscalar (σ), vector-isoscalar (ω), and vector-isovector ($\boldsymbol{\rho}$) fields are introduced to parametrize the density dependence of the energy functional. The model parameters are

calibrated at equilibrium nuclear matter density, and it is possible to build different models that reproduce *exactly* the same nuclear matter properties, but which yield maximum neutron star masses that differ by as much as one solar mass, even when the nonlinear parameters are restricted to be of natural size. Moreover, with enough nonlinear couplings, one can reduce the predicted maximum neutron star mass to $M_{\max}/M_{\odot} \approx 1.6$, which is only 10% larger than that of the most massive observed neutron stars. Implications for the existence of kaon condensates or quark cores in neutron stars are discussed.

Properties of neutron stars are examined within a similar RMFT framework in [Ma92c] and within DBHF theory in [Hu94]. The interesting possibility of the transition of nuclear matter into a "Peierls' type" of periodic structure is investigated in the RMFT in [Lo92a,Lo94a].

Another goal of QHD is to provide a reliable description of the hadronic phase of matter as one approaches the transition to the quark-gluon plasma (QGP) [Se92,Wa95], assuming that such a transition exists. The role of the QGP in the neutron matter equation of state is examined in [Ro92] and more generally in [Ma93b,Dr95,Ka95,Ri95].

The less spectacular, but more easily accessed liquid-gas phase transition is studied within the framework of RMFT in [So93,Ha94a,So94,Mu95]. In [Mu95], a RMFT model of nuclear matter with arbitrary proton fraction is studied at finite temperature. An analysis is performed of the liquid-gas phase transition in a system with two conserved charges (baryon and isospin) using the stability conditions on the free energy, the conservation laws, and Gibbs' criteria for phase equilibrium. For a binary system with two phases, the coexistence surface (binodal) is two-dimensional, and thus the liquid-gas phase transition is continuous (second order by Ehrenfest's definition) rather than discontinuous (first order), as in familiar one-component systems. Using a mean-field equation of state calibrated to the properties of nuclear matter and finite nuclei, various phase-separation scenarios are considered, and the model is applied to the liquid-gas phase transition that may occur in the warm, dilute matter produced in energetic heavy-ion collisions. In asymmetric matter, instabilities that produce a liquid-gas phase separation arise from fluctuations in the proton concentration, rather than from fluctuations in the baryon density.

The description of nuclear matter at higher temperature is also of great interest; for example, very hot nuclear matter existed in the early universe. A similar substance will be produced at the Relativistic Heavy-Ion Collider (RHIC), and supernovae also involve a large range of $\rho_{\rm B}$ and T. An analysis of hot nuclear matter within the framework of QHD is developed in [Fu90]. General principles of covariant thermodynamics and thermodynamic consistency are discussed, and these principles are illustrated by computing nuclear matter properties in an arbitrary reference frame, using the RMFT of QHD–I. The results are shown to be Lorentz covariant, and thermodynamic consistency is demonstrated by proving the equality of the "thermodynamic" and "hydrostatic" pressures. The mean-field results are used in a simple hydrodynamic picture to discuss the phenomenology of heavy-ion collisions and astrophysical systems, with an emphasis on new features that arise in a covariant approach. Covariant Feynman rules for going beyond the RMFT are discussed in [Fu91,Fu91b].

The nucleon mean free path is considered in [Ha92,Ha94b], thermal fluctuations and quantum corrections are examined in [Ku91,Su92a,Qi93,Ag94,Su95a], and screening is discussed in [Ga94c]. Supernovae and neutron stars are studied together in [Su92]. Statistical

properties are examined in [Vo92,Ra93], and collective modes at finite T in [Ni93]. Various aspects of relativistic heavy-ion physics are discussed in [Du95].

A novel $1/N$ expansion of the theory is pursued in [Ta92a,He93,Ta93,He95], where, for isospin, one has $SU(N)$ with $N = 2$. The thermodynamic potential with exchange and RPA corrections to the RMFT is calculated within this framework.

A key element of the RMFT of nuclear matter in QHD–I is that the baryon acquires a density-dependent effective mass M^* due to the classical scalar field, which in turn arises from the baryons themselves. M^* must be determined self-consistently at all densities and temperatures, and in this model, M^*/M goes to zero at high ρ_{B} and high T. The successful description of the location and shape of the quasielastic peak in (e, e') at the RMFT level gives additional evidence for the model values of M^*/M in the nuclear medium [Se86,Wa95]. There is no shift in the meson masses at this level of approximation, although meson masses are modified when one includes polarization insertions [Se86,Wa95]. The modification of meson masses is crucial, for example, for deciding whether pion or kaon condensation takes place in the nuclear medium, and at what density. The modification of the masses of vector mesons can be studied experimentally by looking at in-medium lepton-pair formation; there are plans in place to do so at CEBAF.

A great deal of work in the past few years has gone into the investigation of the modification of hadron masses in the nuclear medium [Ga94b,We94,Ga95a,Ko95c,Li95c,Sa95]. The effect on the nuclear force is examined in [Ga95a] and on relativistic transport in [Ko95c,Li95c]. In [We94], aspects of chiral symmetry and their implications for changes in hadron structure in the nuclear medium are summarized and discussed. This includes issues such as the density dependence of the chiral (quark) condensate, the related appearance of strong scalar mean fields in nuclei, the stability of the pion mass against compression in dense matter, and recent explorations of s-wave kaon–nuclear interactions. Kaon condensation is also discussed in [Ma94d,Sc94a,El95]. Fortunately, there is an extensive survey article available on this subject in [Ad93]. Here various scenarios for chiral symmetry restoration and deconfinement at finite temperature or density are studied assuming universal scaling relations for some hadron masses.

An interesting extension of QHD focuses on the incorporation of the broken scale invariance of QCD. Reference [Ja93a] examines various chiral lagrangians in which the QCD scale anomaly is implemented by introducing a dilaton field representing the gluon condensate. The lagrangians are used to study the chiral phase transition and the propagators of both scalar and vector mesons in dense baryonic matter. A hybrid model is proposed that allows for an unambiguous definition of meson masses and coupling constants in dense matter.

At the level of QCD, there is a quark condensate, that is, a vacuum expectation value $\langle 0|\overline{q}q|0\rangle$, whose presence signals the spontaneous breakdown of chiral symmetry and leads to a baryon mass. QCD sum rules combine hadronic amplitudes, quark and gluon condensates, and asymptotic freedom to obtain constraints on the properties of hadrons. This quark condensate will be modified in the presence of other baryons; using sum rules, one can relate the change in baryon (or more generally, hadron) masses in nuclear matter to the underlying QCD. Important papers in this regard include [Co91a,El91,Ce92a,Co92a,Fu92a,Ce93,Ji93a].

In [El91], chiral and scale symmetries of QCD are used to describe the interaction between these condensates and hadrons. The resulting equations are solved self-consistently in the RMFT approximation. For these QCD condensates to be driven towards zero at high density,

their coupling to scalar and vector mesons must be such that the masses of these mesons do not decrease with density. In this case, a physically sensible phase transition to quark matter ensues.

In [Co91a,Co92a,Fu92a], the self-energies for quasinucleon states in nuclear matter are studied using QCD sum rules. A correlator of nucleon interpolating fields, evaluated in the finite-density ground state, is calculated using both an operator product expansion and a dispersion relation with a spectral *Ansatz*. This approach relates the nucleon spectral properties (such as the quasinucleon self-energies) to matrix elements of QCD composite operators (condensates). With increasing nucleon density, large changes in Lorentz scalar and vector self-energies arise naturally; the self-energies are found to be comparable to those arising in RMFT phenomenology. The most important phenomenological inputs are the baryon density and the value of the nucleon "sigma term" (σ_{N}) divided by the average current mass of the light quarks. The successful comparison to RMFT phenomenology is, however, sensitive to assumptions about the density dependence of certain four-quark condensates [Ji93a,Ji94a].

One extension of QHD involves developing model field theories where the mesons couple directly to quarks. Three important papers in this regard are [Ja92,Sa94,Sa94b]. In [Ja92], chiral symmetry restoration at finite baryon density is studied in a quark model involving both scalar and vector interactions. The presence of vector interactions makes chiral symmetry restoration more difficult. On-shell masses and coupling constants are calculated for the $\boldsymbol{\pi}, \omega, \boldsymbol{\rho}$, and $\mathbf{a}_1$ mesons. An attempt is made to relate the quark–meson interactions to the observed nucleon–meson coupling constants.

In *summary*, relativistic quantum field theories of the many-hadron system, with local lagrangian densities, appear to provide the appropriate framework for extrapolating the properties of ordinary hadronic nuclear matter to all temperatures and densities up to the transition to the quark-gluon phase. We can understand this based on our earlier discussion. Since the truncation of an effective lagrangian involves an expansion in powers of k_{F}/M, then even at 10 times equilibrium density, this ratio is only 0.6. Moreover, the relevant temperatures are $T \lesssim 200\,\mathrm{MeV}$, which are small compared to the baryon and heavy meson masses. Nevertheless, interaction terms that are small at normal densities and temperatures, and thus difficult to calibrate, can become important at densities and temperatures relevant to the phase transition, thus producing significant uncertainties in the extrapolation [Mu96,Mu96a]. Finding appropriate ways to calibrate these interactions is a major challenge for future investigations.

8. LOOPS IN QHD

We have seen that the mean-field approximation to QHD gives a concise and highly successful nuclear phenomenology. We have also noted, based on ideas from density functional theory, that a mean-field energy functional fitted to nuclear properties implicitly includes some effects of higher-order, many-body corrections. Nevertheless, mean fields are insufficient for a detailed understanding of nuclear structure and its relation to the underlying NN, NNN, ... interactions. Moreover, various experimental observables probe aspects of nuclear dynamics that go beyond a single-particle description, which can tell us about the

modification of the NN interaction inside a nucleus. We must therefore develop reliable, relativistic techniques to extend the RMFT calculations discussed above. Useful tools in this endeavor are Feynman diagrams, dispersion relations, and path integrals, as discussed in [Se86], where a historical development is presented, and in [Se92], which updates the 1986 volume. References to the original literature are given in these review articles. Here we briefly summarize some of those results and discuss some of the work that has been done since that time. A more detailed discussion of the theoretical background is contained in the recent text [Wa95].

Corrections to the RMFT will generically involve "loop diagrams", which are important for including several different aspects of the quantum nature of the system. For example, loop diagrams must be included to ensure the unitarity of scattering amplitudes. Baryon loops are necessary for incorporating familiar many-body effects, like the exchange of identical nucleons or the summation of ladder and ring diagrams [Fe71]. Moreover, loops introduce effects arising from the modification of the quantum vacuum in the presence of valence nucleons, and meson loops in particular generate contributions to the extended structure of the nucleon. The relevant question is how to treat these loops most efficiently, consistent with the notion that QHD is intended to be a large-distance, hadronic model of the underlying QCD. We begin the discussion by considering pion loops and then turn to the more difficult question of heavy-meson and baryon loops.

A. Pion Loops

Because the pion has a small mass, pion loops make significant contributions to both scattering and nuclear-structure observables. Moreover, since the energies and momenta of interest are typically of the order of the pion mass (or even larger), pion-loop contributions generally involve nonanalytic functions of the external four-momenta. Thus these contributions cannot be absorbed in a local effective lagrangian, which, by definition, contains only finite powers of derivatives [Ge93,Ba94b]. They must be computed explicitly, whether one is using a renormalizable model or an effective, nonrenormalizable lagrangian.

A highly developed framework for systematically including pion loops in scattering processes is chiral perturbation theory (ChPT) [Ga84,Me93a]. Chiral symmetry implies that pion interactions in the lagrangian can be grouped order-by-order in the number of derivatives and powers of m_π^2 (as in the nonlinear models discussed earlier), so that there is a systematic expansion for scattering amplitudes at low energies. Moreover, an expansion in the number of loops also proceeds in powers of momenta [We67,We90a], so one can systematically include loop corrections. ChPT has been successful in describing scattering in both the $B = 0$ and $B = 1$ sectors of low-energy QCD [Ga84,Me88,Me93a]. Studies of two- and many-nucleon systems have been initiated and are currently under active investigation [Or92,Va93,Or96].

Properties of the baryon also arise through pion-loop integrals in QHD. For example, the vertex diagram consisting of the emission of a pion, its interaction with the virtual electromagnetic field, and its reabsorption by the baryon contributes to the baryon's anomalous magnetic moment. The two-pion contribution gives the low-mass, or long-distance, part of the spectral weight function $\rho_2(\sigma^2)$ for the anomalous magnetic form factor $F_2(q^2)$:

$$F_2(q^2) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{\rho_2(\sigma^2)}{\sigma^2 - q^2} \, \mathrm{d}\sigma^2 \ . \tag{8.1}$$

Assume that the two-pion contribution arising from this vertex diagram dominates the spectral weight function everywhere. This yields a qualitative description of the isovector anomalous magnetic moment and its mean-square radius [Se86,Wa95].[41]

Pion loops also give hadronic contributions to vacuum polarization. For example, a spectral analysis of the strong-interaction contribution to electromagnetic vacuum polarization shows that the spectral weight function starts at $4m_\pi^2$:

$$\Pi_{\mu\nu}^{\mathrm{str}}(q) = (q_\mu q_\nu - q^2 g_{\mu\nu})\Pi(q^2) \ , \tag{8.2}$$

$$\Pi(q^2) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{\rho(\sigma^2)}{\sigma^2 - q^2} \, \mathrm{d}\sigma^2 \ . \tag{8.3}$$

In the complex q^2 plane, $\Pi(q^2)$ is an analytic function with a branch cut running along the real axis from $4m_\pi^2$ to infinity. The discontinuity across that cut for $4m_\pi^2 \leq q^2 \leq 9m_\pi^2$ comes from the electroproduction of two real pions. Thus the low-mass singularities of propagator and vertex functions are most efficiently expressed in terms of hadronic variables. The higher-mass singularities are more complicated, and hadrons are less efficient. Nevertheless, if one is interested in studying the low-momentum behavior of the vacuum polarization, one can emphasize the low-mass part of the spectral integral by making several subtractions from the integral and by determining the unknown coefficients empirically [Do96].

The exchange of two correlated pions between nucleons also involves multiple pion loops, which can be treated with dispersion relations [Ja75,Du77,Du80,Li89,Li90,Ki94]. The result is a strong NN attraction that can be simulated by introducing an effective scalar-isoscalar field coupled directly to the nucleon, as discussed in Sections 3.B and 5.D. More detailed studies of the scalar-isoscalar NN interaction can be performed by returning to the description with explicit pion loops [Du93,Ao95]. Similar observations can be made in exchange channels with vector meson quantum numbers. In particular, a tree-level effective lagrangian with vector mesons is found to be essentially equivalent to ChPT in the pion sector at one-pion-loop order [Me88,Do89,Ec89,Ec89a]. Thus these non-Goldstone (heavy) bosons can be introduced into an effective lagrangian containing baryons to efficiently account for the intermediate-range NN interactions and to conveniently describe nonvanishing expectation values of nuclear bilinears (*e.g.*, $\overline{N}N$ and $\overline{N}\gamma_\mu N$), as in Section 5.D.

Moreover, as noted in Section 3.C, the sum of πN ladder diagrams with nucleon exchange can be investigated with partial-wave dispersion relations, leading to a resonance in the Δ channel. Thus the $\Delta(1232)$ can be included in an effective hadronic lagrangian to incorporate this important physics [De92a,We93,Ta96]. Alternatively, the dynamical model of the resonance provides a means to investigate many interesting questions concerning the behavior of the Δ in the nuclear many-body system, such as its binding energy in nuclear matter, its optical potential, and the modification of its electroweak properties.

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[41]The *isoscalar* anomalous moment vanishes in this approximation; experimentally, it is indeed very small.

Virtual pion effects have been studied extensively. Pion loops and pion dressing are examined in [Pa92] using a coherent-state approach. Reference [Fr93] studies in depth the contribution of the virtual pion cloud to the electromagnetic properties of the nucleon.

B. Loops in Renormalizable QHD Theories

By starting with the Feynman rules for the Green's functions in a renormalizable QHD theory, one can *in principle* go beyond the RMFT, compute observables in terms of a finite number of couplings and masses, and then compare with experiment. *In practice*, this program is extremely difficult, since QHD is a strong-coupling relativistic quantum field theory. Moreover, renormalizable theories explicitly include contributions from all length scales, and the QHD couplings get stronger at short distances, since these theories are not asymptotically free.

Nevertheless, just as in nonrelativistic many-body theory, one can use intuition to sum selected infinite sets of diagrams, determine the renormalized coupling constants by refitting nuclear matter properties, and then see whether the RMFT results are *stable* under the inclusion of these additional contributions, while investigating new physical phenomena. All of the extensions we discuss involve loop corrections to the RMFT of one sort or another. These corrections include familiar many-body effects, where the loop momenta are typically of the size of k_F (or at most, several times k_F), and also corrections from the dynamical quantum vacuum, which involve shorter distance scales.

In the RMFT, the baryon Green's function can be written as [Se86]

$$G(k) = (\gamma_\mu k^{*\mu} + M^*)\Big\{ \frac{1}{k^{*2} - M^{*2} + i\epsilon} + \frac{i\pi}{E^*(k)}\, \delta[k_0^* - E^*(k)]\theta(k_\mathrm{F} - |\mathbf{k}|) \Big\}$$
$$\equiv G_F(k) + G_D(k) \ . \tag{8.4}$$

in the rest frame of the nuclear matter. Here $k^{*\mu} \equiv (k_0 - g_\mathrm{v}V_0, \mathbf{k})$ is the kinetic four-momentum[42] and $E^*(k) \equiv \sqrt{\mathbf{k}^2 + M^{*2}}$, with $M^* \equiv M - g_\mathrm{s}\phi_0$. The first term $G_F(k)$ is the Feynman propagator for a baryon of mass M^*, and the second term is the contribution arising from baryons already present at finite density; this latter contribution reproduces the RMFT results. In discussing the following extensions, we shall frequently distinguish results obtained with the full baryon propagator $G(k)$ from those obtained with just the second, or "density-dependent", contribution $G_D(k)$. Since the three- and four-momenta are constrained in $G_D(k)$, loop integrals over this second term give well-defined, finite results that are direct analogues of the terms arising in nonrelativistic many-body theory.

For example, relativistic Hartree theory is obtained by self-consistently summing the tadpole graphs in the baryon self-energy [Se86]. Retention of G_D in the tadpoles gives rise to the MFT, while the full G (together with appropriate counterterms $\delta\mathcal{L}$) produces the

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[42]Note that in closed-loop integrals, such as those involved in computing the ground-state energy, a simple shift of integration variables allows one to eliminate the dependence on $g_\mathrm{v}V_0$ [Se86,Fu89].

RHA, as discussed previously.[43] A characteristic result of QHD–I is that the Lorentz scalar and vector self-energies are large on the scale of nuclear energies; these contributions *cancel* in the binding energy but *add* in the spin-orbit interaction.

Hartree–Fock (HF) theory is obtained by including the meson emission and reabsorption ("exchange") graphs in the baryon proper self-energy [Se86,Se92]. Retention of G_D only in this HF theory leads to exchange terms that are the direct relativistic generalization of those arising when Slater determinants are used to determine the best single-particle wave functions in nonrelativistic many-body theory. The inclusion of the exchange terms does not qualitatively alter the size of the large scalar and vector self-energies found in the RMFT, and thus the RMFT is *stable* with respect to these corrections. In fact, after renormalization to equilibrium nuclear matter properties, the binding energy curves in the RMFT and HF approximations are almost indistinguishable [Se86]. Moreover, the equation of state approaches that of the RMFT at high baryon density. Further discussion is contained in [Se92].

Relativistic HF calculations that include charged mesons and that examine the relation of the scalar and vector self-energies to DBHF calculations in nuclear matter are carried out in [Bo87,Ce92,Be93,Fr93a,Bo94a,Fr94a,Ma94c,Zh94a,Be96]. The existence of large Lorentz scalar and vector self-energies appears to be a firm conclusion. (Recall our discussion of DBHF results in Section 4.B.) The connection to semiclassical approximations is given in [Vo94]. The general structure of the self-energy in matter is analyzed in [He91,Ru95a].

The fully self-consistent HF theory retaining the complete $G(k)$ and meson retardation is complicated [Se92]; it has not yet been successfully solved. The summation of exchange diagrams only (*i.e.*, HF theory at zero baryon density) is discussed in [Bi83,Bi84,Kr93].

The sum of fermion ring diagrams involving $G(k)$ is equivalent to the random-phase approximation. We shall refer to the calculation of the rings that keeps only terms with at least one factor of G_D as the RPA. This includes loops that are at least linear in the density (particle-hole parts and admixtures between filled valence states and those in the Dirac sea) and is the direct relativistic extension of the RPA in nonrelativistic many-body theory. The calculation that also includes the modification of the strong vacuum polarization in the nuclear medium due to the shift $M \rightarrow M^*$ will be called RRPA.

The RPA as applied to finite nuclei was discussed in Section 2.D. We emphasized the need to include contributions from negative-energy states in order to maintain the conservation laws in the theory. Some important results from RPA studies of nuclear matter are as follows. The scalar and vector propagators mix in nuclear matter, and at high density, vector meson exchange dominates in QHD–I. The excitation spectrum of nuclear matter in the RPA is that of zero sound, where the sound velocity c_0 approaches the speed of light from below as the baryon density gets large ($c_0 \rightarrow 1^-$ as $k_F \rightarrow \infty$). This implies that signals in the medium cannot propagate faster than the speed of light, in accord with special relativity. There are other branches in the excitation spectrum corresponding to meson propagation.

We turn now to calculations involving strong vacuum polarization (RRPA). This polar-

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[43]One can also solve 1+1 dimensional QHD exactly for *finite* systems in the one-loop approximation; this is carried out in [Fe93].

ization does not have any explicit density dependence, but depends on it implicitly through the baryon mass M^*. These vacuum loops correspond to the one-baryon loops contained in the RHA (Section 5.E), and just as we found unnaturally large contributions in the RHA, analogous large effects appear in the RRPA. In consequence, in nuclear matter, Landau "ghost" poles appear in the meson propagators at zero frequency $q_0 = 0$ and finite wave number $|\mathbf{q}| \neq 0$; the value of this wave number is a few times the nucleon mass in QHD–I [Co87,Pe87,Fu88,We90,Li90a]. Such poles imply an instability of the system against density fluctuations of the corresponding wavelength and are a manifestation of the lack of asymptotic freedom in the theory. Apparently, a description of the quantum vacuum by summing simple baryon loops is inadequate.

Similar problems occur when one extends the RHA to include two-loop contributions [Fu89], which consist of a closed baryon loop with an internal meson line. When the full propagator G is used in this calculation, one incorporates the many-body modifications to both the strong vacuum polarization and the baryon self-energy that arise from the Pauli exclusion principle and from the shifted baryon mass M^*. One again finds unnaturally large contributions to the nuclear matter energy.

The problem in all of these calculations of vacuum effects is that an expansion in powers of loops is basically an expansion in the dimensionless coupling constants, which are large in QHD. The quantum corrections are correspondingly large, the series is not converging, and the RMFT *is not stable against this perturbative loop expansion.* Clearly, an alternative procedure must be found to systematically and reliably calculate the vacuum-fluctuation corrections to the RMFT in renormalizable QHD, if that is indeed possible.

One idea is to include corrections that sum diagrams to *all orders* in loops, such as the "ladder" summation discussed in Section 4.B. Alternatively, one observes that in a theory with a vector boson coupled to a fermion, the vertex form factor is highly damped at large spacelike momentum transfers $q^2 < 0$; this is analogous to the result first derived by Sudakov in quantum electrodynamics [Mi91]. Such a form factor would decrease the sensitivity to high-momentum (or short-distance) contributions to loop integrals and would provide a favorable situation for QHD. Moreover, it is essential to include vertex corrections, as these reflect the internal hadron structure present in renormalizable QHD. Remember that baryons are complicated objects in the full field theory; they are surrounded by a cloud of virtual mesons and baryon-antibaryon pairs. It was an initial hope that the theory might contain vertex functions that would damp the contributions from loop integrals before one reaches distance scales where the lack of asymptotic freedom becomes manifest.[44]

In [Al92], vacuum polarization is studied in QHD–I (without the scalar meson). The lowest-order (one-baryon-loop) polarization produces a "ghost" pole when summed to all orders in the RRPA, as discussed above. It is first verified that the infrared structure of the meson–baryon vertex in this model produces an on-shell proper vertex function that is strongly damped at large spacelike momentum transfer. When the model vertex function is approximated by its on-shell form and combined with the lowest-order polarization, the

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[44]Note that in a theory with a vector field coupled to a conserved fermion current, Ward's identity implies that the structure of the vertex function is unrelated to the running of the coupling constant.

vacuum contributions are significantly reduced. The resulting RRPA meson propagator has no "ghost" poles and is finite at large spacelike momenta.

In [Se95], it is shown that a similarly damped form factor arises for the scalar–nucleon vertex in QHD–I due to the vector-meson dressing. The on-shell approximations to the vertex functions are used to investigate the two-loop contributions to the properties of nuclear matter, and it is found that they are greatly reduced by the inclusion of vertex functions. Similar results are found using *ad hoc* form factors in [Pr92,Fr92].

RRPA calculations are performed with *ad hoc* vertex cutoffs in [Pr92a] and with the on-shell model described above in [Ta93a]. A two-loop calculation that also consistently treats the electromagnetic interaction to the same level of approximation is developed in [Be92]. Extensions of RPA and RRPA to include the self-consistent sum of ring diagrams with additional Δ-hole propagation are examined in [Be93a]. The relation of the density dependence of zero sound to the renormalization prescription is discussed in [Ca95].

The fully off-shell vertex function is complicated in any field theory. An attempt to include the off-shell vertex in the RRPA calculation for QHD–I has been initiated in [Al95]; no concrete results exist yet for this very difficult calculation. A simplified approach to the baryon self-energy, using an *ad hoc* off-shell form factor and dispersion relations, is contained in [Kr93].

Nuclear Schwinger–Dyson equations, which provide a basis for an analysis of the relativistic field theory content of the nuclear many-body problem in terms of propagators and vertices, are developed in [Na91,Ko93,Na94,Na94a]. Truncated applications usually use some sort of spectral representation or dispersion relation [Ta91] to eliminate the Landau "ghost" poles in the meson propagators discussed earlier.

Pion propagation in the nuclear medium, which involves baryon-antibaryon loops, is studied in [He92a,Ka92]; again, the "ghost" poles are generally removed through one of the mechanisms described above. Vector meson propagation is examined in [Je94] and vector meson mixing through baryon loops is examined in [Pi93,Pi93a]. (Note that here the strength of the mixing is *finite* without any renormalization.) Meson modes in nuclear matter are also examined in [Ja93,Ja93a].

A chirally symmetric, renormalizable, anomaly-free theory that contains baryons and $(\boldsymbol{\pi}, \sigma, \omega, \boldsymbol{\rho}, \mathbf{a}_1)$ mesons is developed in [Se92b]; we refer to this theory as QHD–III. This model provides a hadronic description of strongly interacting matter that includes isovector, pseudoscalar and vector fields in addition to the isoscalar, scalar and vector fields of QHD–I. Although this theory provides a consistent, self-contained description of nuclear physics, including loop processes, its phenomenology remains to be investigated [Wa95].

To summarize, the systematic calculation of loop corrections to the RMFT is an important goal in renormalizable QHD. At present, vacuum contributions evaluated at various orders in loops are all unnaturally large, signaling the inadequacy of these approximations for representing the vacuum dynamics. Although these large effects can be reduced by introducing vertex form factors *external* to the renormalizable theory, the question of whether such form factors can be generated *internally*, and indeed, whether calculations with off-shell vertices are even feasible, is still open. At best, even if renormalizable QHD provides a realistic description of vacuum dynamics, the results will involve sensitive cancellations between unnaturally large contributions. We are thus motivated to consider other options.

C. Loops in Effective QHD Theories

The results discussed above present significant evidence that the requirement of renormalizability is too restrictive, and that it is more appropriate to consider QHD as an effective, nonrenormalizable hadronic field theory. In this approach, short-distance effects and vacuum modifications are parametrized through nonrenormalizable interaction terms in the lagrangian, with couplings that are determined by fitting to data.[45] Nucleon and non-Goldstone boson fields are still needed in the lagrangian to account for the valence nucleons and to treat the large mean fields conveniently. Loop diagrams involving these degrees of freedom must also be included to ensure the unitarity of scattering amplitudes and to incorporate many-body, density-dependent effects. The relevant question is how to separate the short-range contributions, which are to be absorbed in the model parameters, from the long-range, many-body effects.

The baryon propagator in Eq. (8.4) separates into an explicitly density-dependent part G_D and a part G_F that has no explicit density dependence. The familiar many-body contributions arise from loop integrals involving G_D alone (as in the RMFT or HF approximations) or combinations of G_D and G_F (as in the DBHF approximation or the RPA); these are to be computed explicitly. One must also retain various density-dependent contributions that contain negative-energy states, but that are finite, such as those needed to preserve the conservation laws in the RPA. The remaining contributions involve divergent loop integrals that contain vacuum dynamics and short-distance behavior; these are to be absorbed in the parameters of the lagrangian that are fitted to empirical data. Some of these contributions will have explicit density dependence (like the strong Lamb shift), while some will not (vacuum polarization). We emphasize that this is a more difficult problem than the inclusion of loops in ChPT, since the loop corrections do not produce an expansion in powers of external momenta, and thus corrections at a given order in loops renormalize the parameters at *all lower orders* in loops.

In a renormalizable theory, the divergences can be removed by cancelling them against a finite number of counterterms. In a nonrenormalizable theory, however, the number of divergences is, in principle, infinite. Nevertheless, these divergences can always be formally cancelled by the appropriate counterterms, *since all possible interaction terms consistent with the symmetries are already included in the effective lagrangian.* For example, in infinite nuclear matter, any vacuum contribution to the energy arising from loops containing only factors of G_F must appear in the form of a polynomial in the scalar field ϕ_0.[46] As another example, contributions from the nucleon's Lamb shift can be (formally) cancelled by counterterms of the form $\overline{N}N\phi^n$ or $\overline{N}(i\gamma_\mu\partial^\mu - M)^m N\phi^n$. The important issue thus becomes how to limit the required counterterms to a finite and manageable number. Moreover, due

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[45]These couplings also implicitly include the effects of heavier degrees of freedom not included in the model lagrangian.

[46]Although nonanalytic functions of ϕ_0 may arise in principle, we assume that any function involving heavy mesons can be expanded in a Taylor series in the relevant density regime.

to the freedom one has to redefine the field variables, the most efficient way to write the counterterms is still an unsolved problem.

One way to constrain the parameters is to rely on the broken scale invariance of QCD, which restricts the interactions in the scalar-isoscalar channel. In [Fu95], an effective relativistic hadronic model for nuclear matter that incorporates nonlinear chiral symmetry and broken scale invariance is applied at the one-baryon-loop level. The model contains an effective light scalar field that is responsible for the mid-range NN attraction and that has anomalous scaling behavior. One-loop vacuum contributions in this background scalar field at finite density are constrained by low-energy theorems that reflect the broken scale invariance of QCD, so that the scalar effective potential contains only three free parameters. The resulting mean-field energy functional for nuclear matter and nuclei has only a finite number of parameters and yields good fits to the bulk and single-particle properties of nuclei.

These results are consistent with the discussions in Sections 5 and 6, where it was observed that only the first few terms in an expansion in powers of the fields (and their derivatives) are relevant up to moderate densities; in particular, only three parameters are needed in the scalar potential [m_s^2, κ_3, and κ_4 in Eq. (5.47)]. This result obtains because the empirically fitted parameters are of natural size. Thus one may expect that the assumption of naturalness is sufficient to limit the unknown parameters in the lagrangian to a manageable number. This expectation remains to be confirmed by calculations beyond one-baryon-loop order, which are currently in progress.

To *summarize* this section on loop processes in QHD: although there are many qualitative and even semi-quantitative insights, and applications of the ideas of nonrenormalizable effective field theory look promising, *there is still no consistent, reliable, practical approach to the relativistic nuclear many-body problem that includes all loop terms.*

9. SUMMARY

Our goals in this paper are to describe consistent microscopic treatments of the relativistic nuclear many-body problem based on hadrons (quantum hadrodynamics) and to summarize work in this field from early 1992 through 1995. Although QCD is the underlying theory of the strong interaction, the QCD couplings are strong at large distances, and hadrons are more efficient degrees of freedom. But since it is still impossible to derive the low-energy hadronic lagrangian directly from QCD, we must rely on more indirect information, such as the symmetries of the QCD lagrangian, and on well-known nuclear phenomenology to guide us in the construction of the low-energy theory. The framework is based on local, Lorentz-invariant lagrangian densities, as this is the most general way to parametrize observables consistent with the desired constraints of quantum mechanics, special relativity, unitarity, causality, cluster decomposition, and the intrinsic QCD symmetries. Historically, the hadronic lagrangian was required to be renormalizable, so that one could calibrate the theory and then extrapolate without the appearance of new, unknown parameters. There are now strong indications, however, that the constraint of renormalizability is too restrictive, so it is important to generalize our viewpoint to include nonrenormalizable, effective field theories, which can still provide a consistent treatment of the relativistic nuclear many-body problem.

The simplest model that we study (QHD–I) contains protons, neutrons, and neutral scalar and vector mesons, and is renormalizable. At large enough densities, the meson field operators can be replaced by their expectation values, and the result is a relativistic mean-field theory (RMFT), which must be solved self-consistently. In the original version of the model, which omits scalar self-couplings, one obtains a simple, two-parameter description of the equilibrium properties of nuclear matter that can be extrapolated to arbitrary density, temperature, and proton fraction. The classical meson fields are large (several hundred MeV), and nuclear saturation occurs because the nucleon effective mass M^* decreases as the density increases, so that the attractive forces saturate, and the binding curve develops a minimum. If this RMFT is applied to finite nuclei (with the addition of classical fields for the neutral rho meson and Coulomb potential), then from this minimal set of parameters fitted to the properties of nuclear matter, one derives the nuclear shell model. Moreover, when cubic and quartic scalar self-interactions are included, one can tune the equilibrium nuclear matter properties to provide a realistic description of nuclear charge densities, charge radii, binding energies, single-particle spectra, and quadrupole deformations throughout the periodic table.

To explicitly include pions, we need a lagrangian consistent with global chiral $SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}}$ symmetry, which is a symmetry of QCD in the limit of massless u and d quarks. It is simplest to use a linear representation of the symmetry, in which the fields enter as chiral multiplets. If one also demands renormalizability, one is led to the linear sigma model, with an additional neutral vector field. The neutral scalar field plays a dual role as the chiral partner of the pion and the mediator of the attractive NN force. One finds, however, that the mean-field approximation to this model is unable to provide a realistic description of nuclei, primarily due to the strong constraints on the scalar self-interactions arising from the scalar potential used to induce spontaneous symmetry breaking. This is evidence, even at the level of the RMFT, that the simultaneous constraints of linear chiral symmetry and renormalizability are too restrictive.

We observe, however, that it is possible to make a chiral, point transformation of the fields and to recast the lagrangian so that the symmetry is realized *nonlinearly.* The new scalar field is a chiral singlet that can be decoupled from the theory by taking its mass to be very large, thus removing the unwanted nonlinear interactions. (In the limit of infinite scalar mass, the model becomes nonrenormalizable.) Although the mid-range NN attraction is apparently destroyed by this procedure, it can be restored by including correlated two-pion exchange in the scalar-isoscalar channel. Moreover, this two-pion exchange can be efficiently and adequately simulated by introducing a *new* scalar field into the theory with a mass of roughly $500\,\mathrm{MeV}$.[47] The new field is also a chiral scalar, so the lagrangian remains chirally invariant, and the RMFT of this chiral model is identical to that of QHD–I. Thus we draw the important conclusion that the scalar field in QHD–I is to be interpreted as an *effective* field that incorporates the pion-exchange contributions that are the most important

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[47]We observe with some amusement that the possibility of introducing such an additional scalar into a chiral theory was already noted in the early work of Coleman, Wess, and Zumino on nonlinear realizations of chiral symmetry. See footnote 9 in [Co69].

for describing the bulk properties of nuclear matter.

To get a deeper understanding of the large mean fields and to connect the RMFT description to the underlying NN interaction, we consider corrections from nucleon exchange and short-range correlations in nuclear matter. This can be done in the Dirac–Brueckner–Hartree–Fock (DBHF) framework. Here an NN quasipotential fitted to two-nucleon data is used to determine a self-consistent NN interaction in the medium, which includes the effects of the Pauli exclusion principle and the modifications to the single-nucleon Dirac wave functions at finite density. Three important conclusions from this work are: the nucleon scalar and vector self-energies (which are analogous to the scalar and vector fields in the RMFT) are essentially the same size as the scalar and vector mean fields studied earlier, and the state dependence of the self-energies is small; the depletion of the Fermi sea due to correlations is considerably smaller in the DBHF theory than in nonrelativistic Brueckner–Goldstone theory; the DBHF effective NN interaction contains density dependence that goes beyond what is included in nonrelativistic Brueckner–Goldstone theory, and it is therefore possible to simultaneously fit both the NN phase shifts and the nuclear matter equilibrium point at the two-hole-line level. This last result can be understood by considering an expansion of the RMFT energy/particle in powers of the Fermi momentum; one finds that even two-body interactions involving a Lorentz scalar field lead to terms that would be interpreted as many-body forces in a nonrelativistic framework.

Motivated by the appearance of an effective scalar field in our discussion of chiral symmetry, we generalize the QHD framework to embrace the ideas of nonrenormalizable, effective field theory. We still retain a local, Lorentz-invariant lagrangian, but now the lagrangian must contain all possible (non-redundant) interaction terms consistent with the symmetries of the underlying QCD. The coefficients of these terms parametrize the short-distance dynamics that will be modelled by the effective hadronic lagrangian. Since we cannot calculate these coefficients directly from QCD, they must be determined by fitting to data, and we have the important freedom to choose our generalized coordinates (fields) to make this fitting as efficient as possible. Moreover, since the effective lagrangian has, in principle, an infinite number of terms, we must have an organizing principle to retain some predictive power, and this is also influenced by the choice of dynamical variables.

Based on the successes of the hadronic description of both the NN interaction and the nuclear-structure observables, we choose as degrees of freedom the nucleon, pion, and low-mass scalar and vector fields. By retaining the "heavy" non-Goldstone bosons, we can describe the NN interaction without the explicit calculation of multi-pion loops, and we can efficiently describe the expectation values of nucleon bilinears using mean fields. We then construct a lagrangian consistent with the underlying symmetries of QCD, in which the chiral symmetry is realized nonlinearly and electromagnetic interactions are introduced both through minimal-coupling terms and through derivative couplings of the nucleon to the photon, which allows us to incorporate the nucleon electromagnetic structure directly in the theory.

To organize the lagrangian, we rely on naive dimensional analysis, which allows us to extract the dimensional scales of any term, on the assumption of naturalness, which says that the remaining dimensionless coefficient for each term should be of order unity, and on the observation that the ratios of mean meson fields $g_s\phi_0$ and $g_v V_0$ (and their derivatives) to the nucleon mass M (which generically represents the "heavy" mass scale) are good expansion

parameters. Since the meson fields are roughly proportional to the nuclear density, and since the spatial variations in nuclei are determined by the momentum distributions of the valence-nucleon wave functions, this organizational scheme is essentially an expansion in k_F/M, where k_F is a Fermi wavenumber corresponding to ordinary nuclear densities. If naive dimensional analysis and the naturalness assumption are valid, we can expand the lagrangian in powers of the fields and their derivatives, truncate at some finite order, and thus have some predictive power for the properties of nuclei. The model parameters fitted to bulk and single-particle nuclear properties show that this is indeed the case.

We also observe, based on concepts from density functional theory, that RMFT parameters fitted to nuclear properties implicitly include not only the short-distance effects parametrized in the lagrangian, but also long-range, many-body effects arising from corrections to the RMFT. For evidence that mean fields should provide a reasonably accurate way to parametrize these many-body effects, we rely on the DBHF results, which show that exchange and short-range-correlation corrections do not significantly modify the mean fields obtained at the RMFT level. Thus we conclude that densities "large enough" to justify the use of mean-field theory are present in the interiors of medium and heavy nuclei, since the Hartree contributions dominate the nucleon self-energies already at these densities. The numerous successful RMFT calculations of nuclei throughout the periodic table should be interpreted within this density functional context. By explicitly calculating higher-order corrections, one can improve the description of the density functional, and by using the properties of nuclei to determine the parameters, one will ultimately obtain the values that should appear in the effective lagrangian.

This observation leads to another important conclusion. No matter how well one can compute the many-body contributions, there will *always* be additional terms in the energy functional that depend (for example) on powers of the scalar field (or scalar density). The coefficients of these terms, which parametrize the short-distance behavior of QCD, are unknown and are presently impossible to calculate. Moreover, these contributions will be significant on the scale of the nuclear binding energy when their coefficients are of natural size. Unless one has a concrete argument for why these coefficients should be *anomalously small*, they will always be present to adjust the energy functional in a way that allows one to fit nuclear matter saturation or nuclear properties. The goal of predicting the nuclear matter equilibrium point from an NN interaction fitted to two-body data therefore loses some of its importance. In the conventional nonrelativistic language, these additional terms correspond to the short-range parts of the three-body (and many-body) forces that cannot be calculated and that must be fitted to the desired observables.[48]

The problems of extrapolation to extreme conditions of density and temperature, or to large energy-momentum transfers, and the systematic computation of loop corrections still pose challenges for QHD. Although one can argue that the effective lagrangian can be truncated at a few terms for studies of neutron stars or the transition to the quark-gluon plasma, interaction terms that are small at normal densities and temperatures, and thus

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[48]The situation has been succinctly described by Jackson [Ja92a]: "The problem is that there is too little dirt and too much rug under which to sweep it."

difficult to calibrate, can become important under extreme conditions. This incomplete calibration leads to significant uncertainties in the extrapolation, even for parameters of natural size. As for loop calculations, there is now significant evidence that it is insufficient to represent vacuum modifications by simple loop integrals in renormalizable QHD theories. Although it may be possible to reduce the size of these corrections by incorporating vertex modifications, calculations involving off-shell vertex functions computed entirely within the context of the theory remain to be done.

Nonrenormalizable effective theories appear more promising, since short-range and vacuum effects are absorbed into the parameters of the lagrangian, and long-range, many-body effects are (typically) straightforward relativistic generalizations of their nonrelativistic counterparts.[49] Moreover, calculations of pion loops based on chiral perturbation theory or dispersion relations can be performed systematically. Nevertheless, a fully consistent, practical, relativistic many-body description of nuclei based on effective hadronic field theory remains to be formulated.

In *summary*, nuclear physics is the study of the structure of strongly interacting baryonic matter ($B \geq 1$), and the only consistent theoretical framework we have for describing such a relativistic, interacting, quantum-mechanical, many-body system is relativistic quantum field theory based on a local lagrangian density. Although QCD of quarks and gluons provides the basic underlying theory, lagrangians based on hadronic degrees of freedom (QHD), which are the particles observed in the laboratory, provide the most efficient description of the physics in the strong-coupling, nuclear domain. While it is not surprising that initial QHD attempts to model the system within the subset of renormalizable lagrangians appears to be too restrictive, some results of the initial simple models remain robust. In particular, the notion of strong isoscalar, Lorentz scalar and vector mean fields remains valid in a density-functional framework where correlations and other higher-order effects are incorporated in the functional. In the modern effective field theory approach to QCD, one incorporates only the underlying symmetries of QCD in the hadronic lagrangian. With the crucial observation that while the scalar and vector fields in nuclei are comparable to the rest mass of the nucleon, the ratios $g_s\phi_0(x)/M$ and $g_v V_0(x)/M$ still provide expansion parameters less than unity, one is able to understand the multitude of successful applications of the relativistic mean-field treatment of nuclei (RMFT). Applications of QHD to nuclear structure, electroweak interactions with nuclei, the hadronic region of the nuclear phase diagram, relativistic heavy-ion reactions, and to many other phenomena, now abound. The challenge for the future is to further understand the successful applications, the failures where they occur, the full role of hadronic loops, and the deeper connection to QCD.

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[49]There are, however, some nontrivial aspects to these generalizations from a nonrelativistic to a relativistic description. For example, the dynamical nature of the mesons introduces retardation effects that have not yet been adequately studied.

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REFERENCES

Due to the large number of references, we feel that it is most useful to divide the reference list into topics. To use this list most efficiently, the reader will probably find it advantageous to first become familiar with the topic headings in the text. Our convention is to place each reference under the topic heading where the first citation to that reference occurs. The initial references in the list provide background material.

[Se86]   B. D. Serot and J. D. Walecka, *Adv. Nucl. Phys.* **16**, 1 (1986).

[Se92]   B. D. Serot, *Rep. Prog. Phys.* **55**, 1855 (1992).

[Se92a]   B. D. Serot and J. D. Walecka, in *Recent Progress in Many-Body Theories*, vol. 3, ed. T. L. Ainsworth, C. E. Campbell, B. E. Clements, and E. Krotscheck (Plenum, 1992), p. 49.

[Se92b]   B. D. Serot and J. D. Walecka, *Acta Phys. Pol.* **B23**, 655 (1992).

[Wa95]   J. D. Walecka, *Theoretical Nuclear and Subnuclear Physics* (Oxford Univ. Press, 1995).

1. Introduction

[Ba61]   G. Baym and L. P. Kadanoff, *Phys. Rev.* **124**, 287 (1961).

[Ba62]   G. Baym, *Phys. Rev.* **127**, 1391 (1962).

[Bj64]   J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw–Hill, 1964).

[Bj65]   J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw–Hill, 1965).

[Br76]   G. E. Brown and A. D. Jackson, *The Nucleon–Nucleon Interaction* (North-Holland, 1976).

[Ca82]   W. E. Caswell and A. D. Kennedy, *Phys. Rev.* **D25**, 392 (1982).

[Cl83]   B. C. Clark, S. Hama, R. L. Mercer, L. Ray, and B. D. Serot, *Phys. Rev. Lett.* **50**, 1644 (1983).

[Co84]   J. C. Collins, *Renormalization* (Cambridge Univ. Press, 1984).

[Da83]   B. D. Day, *Comments Nucl. Part. Phys.* **11**, 115 (1983).

[Da85]   B. D. Day and R. B. Wiringa, *Phys. Rev.* **C32**, 1057 (1985).

[Da86]   E. Dagotto, A. Moreo, and U. Wolff, *Phys. Rev. Lett.* **57**, 1292 (1986).

[Dr90]   R. M. Dreizler and E. K. U. Gross, *Density Functional Theory* (Springer, 1990).

[Ei87]   J. M. Eisenberg and W. Greiner, *Nuclear Theory*, vols. I–III (North-Holland, 1987).

[Fe71]   A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw–Hill, 1971).

[Fr95]   J. L. Friar, in *Proc.* XIVth *Int. Conf. on Few-Body Problems in Physics*, ed. F. Gross, AIP Conference Proceedings **334**, 323 (1995).

[Fu89]   R. J. Furnstahl, R. J. Perry, and B. D. Serot, *Phys. Rev.* **C40**, 321 (1989); **C41**, 404 (E) (1990).

[Ga84]   J. Gasser and H. Leutwyler, *Ann. Phys. (N.Y.)* **158**, 142 (1984); *Nucl. Phys.* **B250**, 465, 517, 539 (1985).

[Ge60]   M. Gell-Mann and M. Lévy, *Nuovo Cim.* **16**, 705 (1960).

[Ge93]   H. Georgi, *Annu. Rev. Nucl. Part. Sci.* **43**, 209 (1993).

[Go79]   D. Gogny, in *Nuclear Physics with Electromagnetic Interactions*, Lecture Notes in Physics **108**, ed. H. Arenhövel and D. Drechsel (Springer, 1979), p. 88.

[Gr93]   F. Gross, *Relativistic Quantum Mechanics and Field Theory* (Wiley-Interscience, 1993).

[It80]   C. Itzykson and J.-B. Zuber, *Quantum Field Theory* (McGraw–Hill, 1980).

[Ja92a] A. D. Jackson *Recent Progress in Many-Body Theories*, vol. 3, ed. T. L. Ainsworth, C. E. Campbell, B. E. Clements, and E. Krotscheck (Plenum, 1992), p. 1.

[Ka87]   F. Karsch, *Nucl. Phys.* **A461**, 305c (1987).

[Ka89]   J. I. Kapusta, *Finite-Temperature Field Theory* (Cambridge Univ. Press, 1989).

[Ka96a] K. Kanaya, *Nucl. Phys. B (Proc. Suppl.)* **47**, 144 (1996), sec. 5.

[Ko95d] J. B. Kogut, M.-P. Lombardo, and D. K. Sinclair, *Nucl. Phys. B (Proc. Suppl.)* **42**, 514 (1995).

[Le72]   B. W. Lee, *Chiral Dynamics* (Gordon and Breach, 1972).

[Ma82]   T. Matsui and B. D. Serot, *Ann. Phys. (N.Y.)* **144**, 107 (1982).

[Ma84]   F. Mandl and G. Shaw, *Quantum Field Theory* (Wiley, 1984).

[Ma87]   R. Machleidt, K. Holinde, and Ch. Elster, *Phys. Rep.* **149**, 1 (1987).

[Ma89]   R. Machleidt, *Adv. Nucl. Phys.* **19**, 189 (1989).

[Ma96]   A. V. Manohar, Lectures at Schladming Winter School, March, 1996, e-print hep-ph/9606222.

[Mc83]   J. A. McNeil, J. R. Shepard, and S. J. Wallace, *Phys. Rev. Lett.* **50**, 1439 (1983).

[Me93a]   Ulf-G. Meissner, *Rep. Prog. Phys.* **56**, 903 (1993).

[Na73]   M. M. Nagels, T. A. Rijken, and J. J. de Swart, *Ann. Phys. (NY)* **79**, 338 (1973); *Phys. Rev.* **D17**, 768 (1978); **D20**, 1633 (1979).

[Na78]   C. Nash, *Relativistic Quantum Fields* (Academic, 1978).

[Pa79]   R. Vinh Mau, in *Mesons in Nuclei*, vol. I, ed. M. Rho and D. H. Wilkinson (North-Holland, 1979), p. 151.

[Pl94]   D. Plümper, J. Flender, and M. Gari, *Phys. Rev.* **C49**, 2370 (1994).

[Pu95]   B. S. Pudliner, V. R. Pandharipande, J. Carlson, and R. B. Wiringa, *Phys. Rev. Lett.* **74**, 4396 (1995).

[Ra81]   P. Ramond *Field Theory: A Modern Primer*, 2nd ed. (Addison–Wesley, 1990).

[Ry85]   L. H. Ryder, *Quantum Field Theory* (Cambridge Univ. Press, 1985).

[Sc57]   J. Schwinger, *Ann. Phys. (N.Y.)* **2**, 407 (1957).

[Se79]   B. D. Serot, *Phys. Lett.* **86B**, 146 (1979); **87B**, 403 (E) (1979).

[Sh83]   J. R. Shepard, J. A. McNeil, and S. J. Wallace, *Phys. Rev. Lett.* **50**, 1443 (1983).

[St93]   G. Sterman, *An Introduction to Quantum Field Theory* (Cambridge Univ. Press, 1993).

[St94]   V. G. J. Stoks, R. A. M. Klomp, C. P. F. Terheggen, and J. J. de Swart, *Phys. Rev.* **C49**, 2950 (1994).

[Te87]   B. ter Haar and R. Malfliet, *Phys. Rep.* **149**, 207 (1987).

[Um93]   H. Umezawa, *Advanced Field Theory: Micro, Macro, and Thermal Physics* (American Inst. of Physics, 1993).

[Wa74]   J. D. Walecka, *Ann. Phys. (N.Y.)* **83**, 491 (1974).

[Wa87]   S. J. Wallace, *Annu. Rev. Nucl. Part. Sci.* **37**, 267 (1987).

[We90a]   S. Weinberg, *Phys. Lett.* **B251**, 288 (1990).

[We91]   S. Weinberg, *Nucl. Phys.* **B363**, 3 (1991).

[We92a]   S. Weinberg, *Phys. Lett.* **B295**, 114 (1992).

[We95]   S. Weinberg, *The Quantum Theory of Fields, vol. I: Foundations* (Cambridge Univ. Press, 1995).

[We96]   S. Weinberg, *The Quantum Theory of Fields, vol. II: Modern Applications* (Cambridge Univ. Press, 1996).

[Wi92]   R. B. Wiringa, *Recent Progress in Many-Body Theories*, vol. 3, ed. T. L. Ainsworth, C. E. Campbell, B. E. Clements, and E. Krotscheck (Plenum, 1992), p. 39.

[Zu81]   M. J. Zuilhof and J. A. Tjon, *Phys. Rev.* **C24**, 736 (1981); **C26**, 1277 (1982).

2. Quantum Hadrodynamics (QHD)

[Ab73]   E. S. Abers and B. W. Lee, *Phys. Rep.* **9C**, 1 (1973).

[Bl88]   P. G. Blunden and P. McCorquodale, *Phys. Rev.* **C38**, 1861 (1988).

[Bo69]   A. Bohr and B. Mottelson, *Nuclear Structure*, vol. I (Benjamin, 1969).

[Bo70]   D. G. Boulware, *Ann. Phys. (N.Y.)* **56**, 140 (1970).

[Bo77]   J. Boguta and A. R. Bodmer, *Nucl. Phys.* **A292**, 413 (1977).

[Bo89]   A. R. Bodmer and C. E. Price, *Nucl. Phys.* **A505**, 123 (1989).

[Bo91]   A. R. Bodmer, *Nucl. Phys.* **A526**, 703 (1991).

[Ch77]   S. A. Chin, *Ann. Phys. (N.Y.)* **108**, 301 (1977).

[Co90]   W. R. Coker and L. Ray, *Phys. Rev.* **C42**, 659 (1990).

[Da90]   J. F. Dawson and R. J. Furnstahl, *Phys. Rev.* **C42**, 2009 (1990).

[De87]   H. de Vries, C. W. de Jager, and C. de Vries, *At. Data Nucl. Data Tables* **36**, 495 (1987).

[Du56]   H. P. Duerr, *Phys. Rev.* **103**, 469 (1956).

[Ed57]   A. R. Edmonds, *Angular Momentum in Quantum Mechanics*, 2nd ed. (Princeton Univ. Press, 1960).

[Fr78]   R. A. Freedman, Ph.D. Thesis, Stanford Univ., 1978.

[Fu36]   W. H. Furry, *Phys. Rev.* **50**, 784 (1936).

[Fu85]   R. J. Furnstahl, Ph.D. Thesis, Stanford Univ., 1985.

[Fu85a]   R. J. Furnstahl, *Phys. Lett.* **152B**, 313 (1985).

[Fu87]   R. J. Furnstahl, C. E. Price, and G. E. Walker, *Phys. Rev.* **C36**, 2590 (1987).

[Fu88a]   R. J. Furnstahl, *Phys. Rev.* **C38**, 370 (1988).

[Fu89b]   R. J. Furnstahl and C. E. Price, *Phys. Rev.* **C40**, 1398 (1989).

[Fu90]   R. J. Furnstahl and B. D. Serot, *Phys. Rev.* **C41**, 262 (1990).

[Fu91]   R. J. Furnstahl and B. D. Serot, *Phys. Rev.* **C43**, 105 (1991).

[Fu96]   R. J. Furnstahl, B. D. Serot, and H.-B. Tang, *Nucl. Phys.* **A598**, 539 (1996).

[Ho81]   C. J. Horowitz and B. D. Serot, *Nucl. Phys.* **A368**, 503 (1981).

[Ho90]   C. J. Horowitz and J. Piekarewicz, *Nucl. Phys.* **A511**, 461 (1990).

[Ho91]   C. J. Horowitz, D. P. Murdock, and B. D. Serot, in *Computational Nuclear Physics I: Nuclear Structure*, ed. K. Langanke, J. A. Maruhn, and S. E. Koonin (Springer-Verlag, 1991), p. 129.

[Hy85]   M. V. Hynes, A. Picklesimer, P. C. Tandy, and R. M. Thaler, *Phys. Rev.* **C31**, 1438 (1985).

[Jo55]   M. H. Johnson and E. Teller, *Phys. Rev.* **98**, 783 (1955).

[Kr73]   V. A. Krutov and L. N. Savushkin, *J. Phys.* **A6**, 93 (1973).

[Kr74]   V. A. Krutov, V. N. Fomenko, and L. N. Savushkin, *J. Phys.* **A7**, 372, A2093 (1974).

[Le75]   G. Leander and S. E. Larsson, *Nucl. Phys.* **A239**, 93 (1975).

[Le86]   S.-J. Lee, J. Fink, A. B. Balantekin, M. R. Strayer, A. S. Umar, P.-G. Reinhard, J. A. Maruhn, and W. Greiner, *Phys. Rev. Lett.* **57**, 2916 (1986); **59**, 1171 (E) (1987); **60**, 163 (1988).

[Lu87]   J. D. Lumpe and L. Ray, *Phys. Rev.* **C35**, 1040 (1987).

[Mc83a]   J. A. McNeil, L. Ray, and S. J. Wallace, *Phys. Rev.* **C27**, 2123 (1983).

[Mi72]   L. D. Miller and A. E. S. Green, *Phys. Rev.* **C5**, 241 (1972).

[Mi74]   L. D. Miller, *Phys. Rev.* **C9**, 537 (1974).

[Mi75]   L. D. Miller, *Phys. Rev.* **C12**, 710 (1975).

[Ni86]   S. Nishizaki, H. Kurasawa, and T. Suzuki, *Phys. Lett.* **B171**, 1 (1986).

[Pa87]   W. Pannert, P. Ring, and J. Boguta, *Phys. Rev. Lett.* **59**, 2420 (1987).

[Pi84]   A. Picklesimer, P. C. Tandy, R. M. Thaler, and D. H. Wolfe, *Phys. Rev.* **C29**, 1582 (1984); **C30**, 1861 (1984).

[Pi90]   J. Piekarewicz, *Nucl. Phys.* **A511**, 487 (1990).

[Pr87]   C. E. Price and G. E. Walker, *Phys. Rev.* **C36**, 354 (1987).

[Pr92a]   C. E. Price, E. Rost, J. R. Shepard, and J. A. McNeil, *Phys. Rev.* **C45**, 1089 (1992).

[Ra79]   L. Ray and P. E. Hodgson, *Phys. Rev.* **C20**, 2403 (1979).

[Ra90]   L. Ray, *Phys. Rev.* **C41**, 2816 (1990).

[Sc51]   L. I. Schiff, *Phys. Rev.* **84**, 1, 10 (1951).

[Sh89]   J. R. Shepard, E. Rost, and J. A. McNeil, *Phys. Rev.* **C40**, 2320 (1989).

[We67]   S. Weinberg, *Phys. Rev. Lett.* **18**, 188 (1967); *Phys. Rev.* **166**, 1568 (1968); *Physica* **A96**, 327 (1979).

[We87]   K. Wehrberger and F. Beck, *Phys. Rev.* **C35**, 298 (1987).

[We88]   K. Wehrberger and F. Beck, *Phys. Rev.* **C37**, 1148 (1988).

[Zh91a]   J.-K. Zhang and D. S. Onley, *Nucl. Phys.* **A526**, 245 (1991).

3. Pion Dynamics and Chiral Symmetry

[Bi82]   B. L. Birbrair, V. N. Fomenko, and L. N. Savushkin, *J. Phys.* **G8**, 1517 (1982).

[Bo83]   J. Boguta, *Phys. Lett.* **120B**, 34 (1983); **128B**, 19 (1983).

[Ca96]   G. Carter, P. J. Ellis, and S. Rudaz, *Nucl. Phys.* **A603**, 367 (1996).

[Co69]   S. Coleman, J. Wess, and B. Zumino, *Phys. Rev.* **177**, 2239 (1969).

[De92a]   F. de Jong and R. Malfliet, *Phys. Rev.* **C46**, 2567 (1992).

[Do92]   J. F. Donoghue, E. Golowich, and B. R. Holstein, *Dynamics of the Standard Model* (Cambridge Univ. Press, 1992).

[Du77]   J. W. Durso, M. Saarela, G. E. Brown, and A. D. Jackson, *Nucl. Phys.* **A278**, 445 (1977).

[Du80]   J. W. Durso, A. D. Jackson, and B. J. Verwest, *Nucl. Phys.* **A345**, 471 (1980).

[Fo95a]   V. N. Fomenko, P. Ring, and L. N. Savushkin, *Phys. of Atomic Nuclei* **58**, 2156 (1995).

[Fu93] R. J. Furnstahl and B. D. Serot, *Phys. Rev.* **C47**, 2338 (1993).

[Fu93a] R. J. Furnstahl and B. D. Serot, *Phys. Lett.* **B316**, 12 (1993).

[He94a] E. K. Heide, S. Rudaz, and P. J. Ellis, *Nucl. Phys.* **A571**, 713 (1994).

[He96] T. R. Hemmert and B. R. Holstein, "Systematic $1/M$ expansion for spin-3/2 particles in baryon chiral perturbation theory", e-print hep-ph/9606456.

[Ho82] C. J. Horowitz and B. D. Serot, *Phys. Lett.* **108B**, 377 (1982).

[Ja75] A. D. Jackson, D. O. Riska, and B. Verwest, *Nucl. Phys.* **A249**, 397 (1975).

[Ke74] A. K. Kerman and L. D. Miller, in *Second High-Energy Heavy Ion Summer Study*, Lawrence Berkeley Laboratory report LBL–3675 (1974).

[Ki94] H.-C. Kim, J. W. Durso, and K. Holinde, *Phys. Rev.* **C49**, 2355 (1994).

[Ko93a] P. Ko and S. Rudaz, *Phys. Rev.* **D50**, 6877 (1994).

[Li89] W. Lin and B. D. Serot, *Phys. Lett.* **B233**, 23 (1989).

[Li90] W. Lin and B. D. Serot, *Nucl. Phys.* **A512**, 637 (1990).

[Li91] W. Lin and B. D. Serot, *Nucl. Phys.* **A524**, 601 (1991).

[Sa69] J. J. Sakurai, *Currents and Mesons* (Univ. of Chicago Press, 1969).

[Sa85] S. Sarkar and S. K. Chowdhury, *Phys. Lett.* **153B**, 358 (1985).

[Ta96] H.-B. Tang and P. J. Ellis, *Phys. Lett.* **B387**, 9 (1996).

[We93] K. Wehrberger, *Phys. Rep.* **225**, 273 (1993).

4. Few-Nucleon Systems

[Am92] A. Amorim and J. A. Tjon, *Phys. Rev. Lett.* **68**, 772 (1992).

[An83] M. R. Anastasio, L. S. Celenza, W. S. Pong, and C. M. Shakin, *Phys. Rep.* **100**, 327 (1983).

[Br84] R. Brockmann and R. Machleidt, *Phys. Lett.* **149B**, 283 (1984); *Phys. Rev.* **C42**, 1965 (1990).

[Br87] G. E. Brown, W. Weise, G. Baym, and J. Speth, *Comments Nucl. Part. Phys.* **17**, 39 (1987).

[Br90a] R. Brockmann and R. Machleidt, *Phys. Rev.* **C42**, 1965 (1990).

[Co91a] T. D. Cohen, R. J. Furnstahl, and D. K. Griegel, *Phys. Rev. Lett.* **67**, 961 (1991).

[Co92a] T. D. Cohen, R. J. Furnstahl, and D. K. Griegel, *Phys. Rev.* **C45**, 1881 (1992).

[Co94a]   F. Coester and D. O. Riska, *Ann. Phys. (N.Y.)* **234**, 141 (1994).

[Co95a]   S. A. Coon, M. T. Peña, and D. O. Riska, *Phys. Rev.* **C52**, 2925 (1995).

[Co96]   T. D. Cohen, J. L. Friar, G. A. Miller, and U. van Kolck, *Phys. Rev.* **C53**, 2661 (1996).

[De91]   F. de Jong and R. Malfliet, *Phys. Rev.* **C44**, 998 (1991).

[De93]   N. K. Devine and S. J. Wallace, *Phys. Rev.* **C48**, R973 (1993).

[Fo95]   J. L. Forest, V. R. Pandharipande, and J. L. Friar, *Phys. Rev.* **C52**, 568 (1995).

[Fu92a]   R. J. Furnstahl, D. K. Griegel, and T. D. Cohen, *Phys. Rev.* **C46**, 1507 (1992).

[Gr87]   F. Gross and D. O. Riska, *Phys. Rev.* **C36**, 1928 (1987).

[Gr90]   F. Gross, J. W. Van Orden, and K. Holinde, *Phys. Rev.* **C41**, R1909 (1990); **C45**, 2094 (1992).

[Ho84a]   C. J. Horowitz and B. D. Serot, *Phys. Lett.* **137B**, 287 (1984).

[Ho87]   C. J. Horowitz and B. D. Serot, *Nucl. Phys.* **A464**, 613 (1987); **A473**, 760 (E) (1987).

[Ho94b]   C. J. Horowitz, H. O. Meyer, and D. K. Griegel, *Phys. Rev.* **C49**, 1337 (1994).

[Hu90]   E. Hummel and J. A. Tjon, *Phys. Rev.* **C42**, 423 (1990).

[Hu93]   H. Huber, F. Weber, and M. K. Weigel, *Phys. Lett.* **B317**, 485 (1993); *Phys. Rev.* **C51**, 1790 (1995).

[Ja90]   M. Jaminon and C. Mahaux, *Phys. Rev.* **C41**, 697 (1990).

[Ji93a]   X. Jin, T. D. Cohen, R. J. Furnstahl, and D. K. Griegel, *Phys. Rev.* **C47**, 2882 (1993).

[Ji94a]   X. Jin, M. Nielsen, T. D. Cohen, R. J. Furnstahl, and D. K. Griegel, *Phys. Rev.* **C49**, 464 (1994).

[Ko66]   D. S. Koltun and A. Reitan, *Phys. Rev.* **141**, 1413 (1966).

[Le89a]   Y. J. Lee, C. Nuppenau, and A. D. MacKellar, *Nucl. Phys.* **A504**, 447 (1989); *Phys. Lett.* **B233**, 263 (1989).

[Le92]   T.-S. H. Lee and D. O. Riska, *Phys. Rev. Lett.* **70**, 2237 (1992).

[Ma94e]   R. Machleidt and G. Q. Li, *Phys. Rep.* **242**, 5 (1994).

[Me90]   H. O. Meyer, M. A. Ross, R. E. Pollock, A. Berdoz, F. Dohrmann, J. E. Goodwin, M. G. Minty, H. Nann, P. V. Pancella, S. F. Pate, B. v. Przewoski, T. Rinckel, and F. Sperisen, *Phys. Rev. Lett.* **65**, 2846 (1990).

[Me92]   H. O. Meyer, C. Horowitz, H. Nann, P. V. Pancella, S. F. Pate, R. E. Pollock, B. von Przewoski, T. Rinckel, M. A. Ross, and F. Sperisen, *Nucl. Phys.* **A539**, 633 (1992).

[Ne85]   J. W. Negele, *Comments Nucl. Part. Phys.* **14**, 303 (1985).

[Or92]   C. Ordóñez and U. van Kolck, *Phys. Lett.* **B291**, 459 (1992).

[Or96]   C. Ordóñez, L. Ray, and U. van Kolck, *Phys. Rev.* **C53**, 2086 (1996).

[Po88]   P. Poschenrieder and M. K. Weigel, *Phys. Rev.* **C38**, 471 (1988); *Phys. Lett.* **B200**, 231 (1988).

[Sc69]   M. E. Schillaci, R. R. Silbar, and J. E. Young, *Phys. Rev.* **179**, 1539 (1969).

[Va93]   U. van Kolck, Ph.D. Thesis, Univ. of Texas, 1993.

[Va95]   J. W. Van Orden, N. K. Devine, and F. Gross, *Phys. Rev. Lett.* **75**, 4369 (1995).

5. Effective Field Theory

[Ba88]   M. Bando, T. Kugo, and K. Yamawaki, *Phys. Rep.* **164**, 217 (1988).

[Ba94b]   R. D. Ball and R. S. Thorne, *Ann. Phys. (N.Y.)* **236**, 117 (1994).

[Be95]   V. Bernard, N. Kaiser, U. G. Meissner, *Int. J. Mod. Phys.* **E4**, 193 (1995).

[Bi95]   M. C. Birse, *Phys. Rev.* **C51**, R1083 (1995).

[Ca69]   C. G. Callan, Jr., S. Coleman, J. Wess, and B. Zumino, *Phys. Rev.* **177**, 2247 (1969).

[Co87]   T. D. Cohen, M. K. Banerjee, and C.-Y. Ren, *Phys. Rev.* **C36**, 1653 (1987).

[Do89]   J. F. Donoghue, C. Ramirez, and G. Valencia, *Phys. Rev.* **D39**, 1947 (1989).

[Ec89]   G. Ecker, J. Gasser, A. Pich, and E. De Rafael, *Nucl. Phys.* **B321**, 311 (1989).

[Ec89a]   G. Ecker, J. Gasser, H. Leutwyler, and E. De Rafael, *Phys. Lett.* **B223**, 425 (1989).

[Fr82]   J. Friedrich and N. Voegler, *Nucl. Phys.* **A373**, 192 (1982).

[Fr96]   J. L. Friar, D. G. Madland, and B. W. Lynn, *Phys. Rev.* **C53**, 3085 (1996).

[Fu88]   R. J. Furnstahl and C. J. Horowitz, *Nucl. Phys.* **A485**, 632 (1988).

[Fu95]   R. J. Furnstahl, H.-B. Tang, and B. D. Serot, *Phys. Rev.* **C52**, 1368 (1995).

[Fu96a]   R. J. Furnstahl, B. D. Serot and H.-B. Tang, "A chiral effective lagrangian for nuclei", e-print nucl-th/9608035, *Nucl. Phys. A*, in press.

[Fu96b]   R. J. Furnstahl, B. D. Serot and H.-B. Tang, "Vacuum nucleon loops and naturalness", e-print nucl-th/9611046.

[Ge91]   H. Georgi, *Nucl. Phys.* **B363**, 339 (1991).

[Ge93a]   H. Georgi, *Phys. Lett.* **B298**, 187 (1993).

[Ge94]   H. Georgi, *Annu. Rev. Nuc. Part. Sci.* **43**, 209 (1993).

[Ka95a]   D. B. Kaplan, "Effective field theories", lectures at the Seventh Summer School in Nuclear Physics, Seattle, WA (1995), UW/PT 95–05.

[Ka96]   D. B. Kaplan, M. J. Savage, and M. B. Wise, *Nucl. Phys.* **B478**, 629 (1996).

[Le89]   G. P. Lepage, in *From Actions to Answers* (TASI–89), ed. T. DeGrand and D. Toussaint (World Scientific, 1989), p. 483.

[Le94]   H. Leutwyler, *Ann. Phys. (N.Y.)* **235**, 165 (1994).

[Li90a]   K. Lim, Ph.D. Thesis, Indiana Univ., 1990.

[Ly93]   B. W. Lynn, *Nucl. Phys.* **B402**, 281 (1993).

[Ma84a]   A. Manohar and H. Georgi, *Nucl. Phys.* **B234**, 189 (1984).

[Me88]   Ulf-G. Meissner, *Phys. Rep.* **161**, 213 (1988).

[Me96]   Ulf-G. Meissner, "The role of massive states in chiral perturbation theory", e-print hep-ph/9610459.

[Ni92]   B. A. Nikolaus, T. Hoch, and D. G. Madland, *Phys. Rev.* **C46**, 1757 (1992).

[Pe68]   R. D. Peccei, *Phys. Rev.* **176**, 1812 (1968); **181**, 1902 (1969).

[Pe87]   R. J. Perry, *Phys. Lett.* **B199**, 489 (1987).

[Po92]   J. Polchinski, in *Recent Directions in Particle Theory: From Superstrings and Black Holes to the Standard Model* (TASI–92), ed. J. Harvey and J. Polchinski (World Scientific, 1993), p. 235.

[Re86]   P.-G. Reinhard, M. Rufa, J. Maruhn, W. Greiner, and J. Friedrich, *Z. Phys.* **A323**, 13 (1986).

[Wa95a]   S. J. Wallace, F. Gross, and J. A. Tjon, *Phys. Rev. Lett.* **74**, 228 (1995).

[We90]   K. Wehrberger, R. Wittman, and B. D. Serot, *Phys. Rev.* **C42**, 2680 (1990).

6. Relativistic Mean-Field Theory

[Bo84]   A. Bouyssy, S. Marcos, and P. van Thieu, *Nucl. Phys.* **A422**, 541 (1984).

[Fi89]   J. Fink, V. Blum, P.-G. Reinhard, J. A. Maruhn, and W. Greiner, *Phys. Lett.* **B218**, 277 (1989).

[Fu89c]   R. J. Furnstahl and C. E. Price, *Phys. Rev.* **C40**, 1398 (1989).

[Fu91a]   R. J. Furnstahl and C. E. Price, *Phys. Rev.* **C44**, 895 (1991).

[Ga90]   Y. K. Gambhir, P. Ring, and A. Thimet, *Ann. Phys. (N.Y.)* **198**, 132 (1990).

[Re89]   P.-G. Reinhard, *Rep. Prog. Phys.* **52**, 439 (1989).

[Ru88]   M. Rufa, P.-G. Reinhard, J. A. Maruhn, W. Greiner, and M. R. Strayer, *Phys. Rev.* **C38**, 390 (1988).

[Va92]   N. Van Giai and L. N. Savushkin, *Sov. J. Part. Nucl.* **23**, 373 (1992).

6.A. Density Functional Theory

[Ce92b]   M. Centelles, X. Viñas, M. Barranco, S. Marcos, and R. J. Lombard, *Nucl. Phys.* **A537**, 486 (1992).

[Ce93a]   M. Centelles, X. Viñas, M. Barranco, and P. Schuck, *Ann. Phys. (N.Y.)* **221**, 165 (1993).

[Ce93b]   M. Centelles, X. Viñas, M. Barranco, N. Ohtsuka, A. Faessler, D. T. Khoa, and M. Müther, *Phys. Rev.* **C47**, 1091 (1993).

[Ce94]   M. Centelles, X. Viñas, and P. Shuck, *Nucl. Phys.* **A567**, 611 (1994).

[Ha94]   S. Haddad and M. K. Weigel, *Nucl. Phys.* **A578**, 471 (1994).

[Ko65]   W. Kohn and L. J. Sham, *Phys. Rev.* **A140**, 1133 (1965).

[Mu92]   H. Müller and R. M. Dreizler, *Z. Phys.* **A341**, 417 (1992); *Nucl. Phys.* **A563**, 649 (1993).

[Mu96]   H. Müller and B. D. Serot, *Nucl. Phys.* **A606**, 508 (1996).

[Sc95]   R. N. Schmid, E. Engel, and R. M. Dreizler, *Phys. Rev.* **C52**, 164 (1995).

[Sc95a]   R. N. Schmid, E. Engel, and R. M. Dreizler, *Phys. Rev.* **C52**, 2804 (1995).

[Sp92]   C. Speicher, R. M. Dreizler, and E. Engel, *Ann. Phys. (NY)* **213**, 312 (1992).

[Sp93]   C. Speicher, E. Engel, and R. M. Dreizler, *Nucl. Phys.* **A562**, 569 (1993).

[Vo92b]   D. Von-Eiff, S. Haddad, and M. K. Weigel, *Phys. Rev.* **C46**, 230 (1992).

[Vo94a]   D. Von-Eiff, W. Stocker, and M. K. Weigel, *Phys. Rev.* **C50**, 1436 (1994).

[Vo94b]   D. Von-Eiff, J. M. Pearson, W. Stocker, and M. K. Weigel, *Phys. Lett.* **B324**, 279 (1994); *Phys. Rev.* **C50**, 831 (1994).

6.B. Nuclear Structure

[Af96]   A. V. Afanasjev, J. König, and P. Ring, *Nucl. Phys.* **A608**, 107 (1996).

[Bo93a]   H. F. Boersma, *Phys. Rev.* **C48**, 472 (1993).

[Bo94]   H. F. Boersma and R. Malfliet, *Phys. Rev.* **C49**, 233 (1994).

[Br92]   R. Brockmann and H. Toki, *Phys. Rev. Lett.* **68**, 3408 (1992).

[En93a]   T. Engeland, M. Hjorth-Jensen, A. Holt, and E. Osnes, *Phys. Rev.* **C48**, R535 (1993).

[Fu95b]   C. Fuchs, H. Lenske, and H. H. Wolter, *Phys. Rev.* **C52**, 3043 (1995).

[Ga94a]   Y. K. Gambhir, *Nucl. Phys.* **A570**, 101c (1994).

[Gm92]   S. Gmuca, *Z. Phys.* **A342**, 387 (1992).

[Gm92a]   S. Gmuca, *Nucl. Phys.* **A547**, 447 (1992).

[Gr92]   C. Greiner and P.-G. Reinhard, *Z. Phys.* **A342**, 379 (1992).

[Ho94a]   A. Holt, T. Engeland, M. Hjorth-Jensen, and E. Osnes, *Nucl. Phys.* **A570**, 137c (1994).

[La95]   G. A. Lalazissis and M. M. Sharma, *Nucl. Phys.* **A586**, 201 (1995).

[La96]   G. A. Lalazissis, M. M. Sharma, and P. Ring, *Nucl. Phys.* **A597**, 35 (1996).

[La96a]   G. A. Lalazissis, M. M. Sharma, P. Ring, and Y. K. Gambhir, *Nucl. Phys.* **A608**, 202 (1996).

[Le95]   H. Lenske and C. Fuchs, *Phys. Lett.* **B345**, 355 (1995).

[Ma92a]   J. P. Maharana, Y. K. Gambhir, J. A. Sheikh, and P. Ring, *Phys. Rev.* **C46**, R1163 (1992).

[Ma92b]   S. Marcos, N. Van Gai, and L. N. Savushkin, *Nucl. Phys.* **A549**, 143 (1992).

[Ma96a]   J. P. Maharana, L. S. Warrier, and Y. K. Gambhir, *Ann. Phys. (N.Y.)* **250**, 237 (1996).

[Mi92]   E. Migli, T. L. Ainsworth, and J. Speth, *Phys. Lett.* **B288**, 1 (1992).

[Ne95]   Y. Nedjadi and J. R. Rook, *Nucl. Phys.* **A585**, 641 (1995).

[Pa91]   S. K. Patra and C. R. Praharaj, *Phys. Lett.* **B273**, 13 (1991).

[Pa93]   S. K. Patra and P. K. Panda, *Phys. Rev.* **C47**, 1514 (1993).

[Pa93a]   S. K. Patra, *Nucl. Phys.* **A559**, 173 (1993).

[Pa93b]   S. K. Patra and C. R. Praharaj, *Nucl. Phys.* **A565**, 442 (1993).

[Pa93c]   S. K. Patra, *Phys. Rev.* **C48**, 1449 (1993).

[Pa94a]   S. K. Patra, S. Yoshida, N. Takigawa, and C. R. Praharaj, *Phys. Rev.* **C50**, 1924 (1994).

[Pa95]   S. K. Patra, S. Yoshida, N. Takigawa, C. R. Praharaj, and A. K. Rath, *Phys. Rev.* **C51**, 2248 (1995).

[Pe94]   J. M. Pearson and M. Farine, *Phys. Rev.* **C50**, 185 (1994).

[Pr94]   M. Prakash, P. J. Ellis, E. K. Heide, and S. Rudaz, *Nucl. Phys.* **A575**, 583 (1994).

[Re91]   P.-G. Reinhard in *Computational Nuclear Physics*, vol. I, ed. K. Langanke, J. A. Maruhn, and S. E. Koonin (Springer-Verlag, 1991), p. 28.

[Ru95]   K. Rutz, J. A. Maruhn, P.-G. Reinhard, and W. Greiner, *Nucl. Phys.* **A590**, 680 (1995).

[Sh92]   M. M. Sharma and P. Ring, *Phys. Rev.* **C45**, 2514 (1992).

[Sh92a]   M. M. Sharma and P. Ring, *Phys. Rev.* **C46**, 1715 (1992).

[Sh93]   M. M. Sharma, M. A. Nagarajan, and P. Ring, *Phys. Lett.* **B312**, 377 (1993).

[Sh93a]   M. M. Sharma, G. A. Lalazissis, and P. Ring, *Phys. Lett.* **B317**, 9 (1993).

[Sh93b]   J. A. Sheikh, J. P. Maharana, and Y. K. Gambhir, *Phys. Rev.* **C48**, 192 (1993).

[Sh93c]   J. A. Sheikh, *Phys. Rev.* **C48**, 476 (1993).

[Sh94]   M. M. Sharma, M. A. Nagarajan, and P. Ring, *Ann. Phys. (N.Y.)* **231**, 110 (1994).

[Su93]   K. Sumiyoshi, D. Hirata, H. Toki, and H. Sagawa, *Nucl. Phys.* **A552**, 437 (1993).

[Su94]   Y. Sugahara and H. Toki, *Nucl. Phys.* **A579**, 557 (1994).

[Ta92]   I. Tanihata, D. Hirata, T. Kobayashi, S. Shimoura, K. Sugimoto, and H. Toki, *Phys. Lett.* **B289**, 261 (1992).

[Ta93]   K. Tanaka, W. Bentz, and A. Arima, *Nucl. Phys.* **A555**, 151 (1993).

[Ue92]   H. Uechi, *Nucl. Phys.* **A541**, 397 (1992).

[Vo92a]   D. Von-Eiff and M. K. Weigel, *Phys. Rev.* **C46**, 1797 (1992).

[Wa94]   L. S. Warrier and Y. K. Gambhir, *Phys. Rev.* **C49**, 871 (1994).

[Yo94]   S. Yoshida, S. K. Patra, N. Takigawa, and C. R. Praharaj, *Phys. Rev.* **C50**, 1398 (1994).

[Za92]   L. Zamick, D. C. Zheng, and H. Müther, *Phys. Rev.* **C45**, 2763 (1992).

[Zh91]   J.-K. Zhang and D. S. Onley, *Phys. Rev.* **C44**, 1915 (1991).

[Zh92]   D. C. Zheng, L. Zamick, and H. Müther, *Phys. Rev.* **C45**, 275 (1992).

[Zh93]   J.-K. Zhang, Y. Jin, and D. S. Onley, *Phys. Rev.* **C48**, 2607 (1993).

[Zh94]   Z. Y. Zhu, W. Q. Shen, Y. H. Cai, and Y. G. Ma, *Phys. Lett.* **B328**, 1 (1994).

6.C. Electroweak Interactions in Nuclei

[Al93]   W. M. Alberico, M. B. Barbaro, A. De Pace, T. W. Donnelly, and A. Molinari, *Nucl. Phys.* **A563**, 605 (1993).

[An96]   S. Ananyan, Ph.D. Thesis, College of William and Mary, 1996, to be published.

[Ba93]   G. Barenboim, A. O. Gattone, and E. D. Izquierdo, *Phys. Rev.* **C48**, 2537 (1993).

[Ba94]   M. B. Barbaro, A. De Pace, T. W. Donnelly, and A. Molinari, *Nucl. Phys.* **A569**, 701 (1994).

[Bo93]   S. Boffi, C. Giusti, and F. D. Pacati, *Phys. Rep.* **226**, 1 (1993).

[Bo94b]   I. Bobeldijk *et al.*, *Phys. Rev. Lett.* **73**, 2684 (1994).

[De92]   M. J. Dekker, P. J. Brussaard, and J. A. Tjon, *Phys. Lett.* **B289**, 255 (1992).

[Ei94]   E. Eich, F. Beck, and A. Richter, *Nucl. Phys.* **A572**, 57 (1994).

[En93]   J. Engel, E. Kolbe, K. Langanke, and P. Vogel, *Phys. Rev.* **D48**, 3048 (1993).

[Fe92]   H. W. Fearing and M. S. Welsh, *Phys. Rev.* **C46**, 2077 (1992).

[Fe94]   T. C. Ferrée and D. S. Koltun, *Phys. Rev.* **C49**, 1961 (1994).

[Fr94]   M. R. Frank, *Phys. Rev.* **C49**, 555 (1994).

[Ga92]   A. O. Gattone, E. D. Izquierdo, and M. Chiapparini, *Phys. Rev.* **C46**, 788 (1992).

[Ga94]   S. Gardner and J. Piekarewicz, *Phys. Rev.* **C50**, 2822 (1994).

[Gi95]   A. Gil, M. Kleinmann, H. Müther, and E. Oset, *Nucl. Phys.* **A584**, 621 (1995).

[He94]   M. Hedayati-Poor and H. S. Sherif, *Phys. Lett.* **B326**, 9 (1994).

[Ho93]   C. J. Horowitz, *Phys. Rev.* **C47**, 826 (1993).

[Ho93a]   C. J. Horowitz and J. Piekarewicz, *Phys. Rev.* **C47**, 2924 (1993).

[Ho94]   C. J. Horowitz and O. Yilmaz, *Phys. Rev.* **C49**, 3042 (1994).

[Iz94]   E. D. Izquierdo and A. O. Gattone, *Phys. Rev.* **C49**, 2005 (1994).

[Ji92]   Y. Jin, D. S. Onley, and L. E. Wright, *Phys. Rev.* **C45**, 1333 (1992).

[Ji93]   Y. Jin, J. K. Zhang, D. S. Onley, and L. E. Wright, *Phys. Rev.* **C47**, 2024 (1993).

[Jo94]   J. I. Johansson and H. S. Sherif, *Nucl. Phys.* **A575**, 477 (1994).

[Jo95]   J. Jourdan, *Phys. Lett.* **B353**, 189 (1995).

[Ki95]   H.-C. Kim, J. Piekarewicz, and C. J. Horowitz, *Phys. Rev.* **C51**, 2739 (1995).

[Ki96]   K. S. Kim, Ph.D. Thesis, Ohio Univ., 1996; K. S. Kim and L. E. Wright, submitted to *Physical Review C.*

[Ko95]   D. S. Koltun and T. C. Ferrée, *Phys. Rev.* **C52**, 901 (1995).

[Kr92]   G. Krein, D. P. Menezes, and M. Nielsen, *Phys. Lett.* **B294**, 7 (1992).

[Li95a]   G. Q. Li and C. M. Ko, *Nucl. Phys.* **A582**, 731 (1995).

[Li95b]   L.-G. Liu, Q. Zhou, and T. Lai, *Phys. Rev.* **C51**, R2302 (1995).

[Lo92]   G. M. Lotz and H. S. Sherif, *Nucl. Phys.* **A537**, 285 (1992).

[Pa94]   T.-S. Park, I. S. Towner, and K. Kubodera, *Nucl. Phys.* **A579**, 381 (1994).

[Pi92]   J. Piekarewicz and R. A. Rego, *Phys. Rev.* **C45**, 1654 (1992).

[Pi95]   J. Piekarewicz and J. R. Shepard, *Phys. Rev.* **C51**, 806 (1995).

[Po92a]   S. J. Pollock, E. N. Fortson, and L. Wilets, *Phys. Rev.* **C46**, 2587 (1992).

[Pr96]   G. Prézeau, The College of William and Mary, *to be published.*

[Ri94]   A. S. Rinat and M. F. Taragin, *Nucl. Phys.* **A571**, 733 (1994).

[To92]   I. S. Towner, *Nucl. Phys.* **A542**, 631 (1992).

[To95]   I. S. Towner and J. C. Hardy, in *Symmetries and Fundamental Interactions in Nuclei*, ed. W. C. Haxton and E. M. Henley (World Scientific, 1995), p. 183.

[Va96]   V. Van der Sluys, J. Ryckebusch, and M. Waroquiur, *Phys. Rev.* **C54**, 1322 (1996).

[Wa94a]   E. K. Warburton and I. S. Towner, *Phys. Rep.* **242**, 103 (1994).

6.D. Strangeness in Nuclei

[Ca92]   J. C. Caillon and J. Labarsouque, *Phys. Rev.* **C45**, 2503 (1992).

[Co91]   J. Cohen and H. J. Weber, *Phys. Rev.* **C44**, 1181 (1991).

[Co92]   J. Cohen and J. V. Noble, *Phys. Rev.* **C46**, 801 (1992).

[Co93]   J. Cohen, *Phys. Rev.* **C48**, 1346 (1993).

[Co94]   E. D. Cooper, B. K. Jennings, and J. Mareš, *Nucl. Phys.* **A580**, 419 (1994).

[Co95]   E. D. Cooper, B. K. Jennings, and J. Mareš, *Nucl. Phys.* **A585**, 157c (1995).

[Do93]   C. B. Dover and A. Gal, *Nucl. Phys.* **A560**, 559 (1993).

[Do95]   C. B. Dover, H. Feshbach, and A. Gal, *Phys. Rev.* **C51**, 541 (1995).

[Ga95]   A. Gal and C. B. Dover, *Nucl. Phys.* **A585**, 1c (1995).

[Gl93]   N. K. Glendenning, D. Von-Eiff, M. Haft, H. Lenske, and M. K. Weigel, *Phys. Rev.* **C48**, 889 (1993).

[Je90]   B. K. Jennings, *Phys. Lett.* **B246**, 325 (1990).

[Ji94]   X. Jin and R. J. Furnstahl, *Phys. Rev.* **C49**, 1190 (1994).

[Ji95]   X. Jin and M. Nielsen, *Phys. Rev.* **C51**, 347 (1995).

[Ko95a] C. M. Kormanyos and R. J. Peterson, *Nucl. Phys.* **A585**, 113c (1995).

[Ko95b] C. M. Kormanyos, R. J. Peterson, J. R. Shepard, J. E. Wise, S. Bart, R. E. Chrien, L. Lee, B. L. Clausen, J. Piekarewicz, M. B. Barakat, E. V. Hungerford, R. A. Michael, K. H. Hicks, and T. Kishimoto, *Phys. Rev.* **C51**, 669 (1995).

[La92]   A. Lang, W. Cassing, U. Mosel, and K. Weber, *Nucl. Phys.* **A541**, 507 (1992).

[Li95]   G. Q. Li and C. M. Ko, *Phys. Lett.* **B349**, 405 (1995).

[Lo94]   R. J. Lombard, S. Marcos, and J. Mareš, *Phys. Rev.* **C50**, 2900 (1994).

[Lo95]   R. J. Lombard, S. Marcos, and J. Mareš, *Phys. Rev.* **C51**, 1784 (1995).

[Ma93a] J. Mareš and J. Žofka, *Z. Phys.* **A345**, 47 (1993).

[Ma94]   J. Mareš, and B. K. Jennings, *Phys. Rev.* **C49**, 2472 (1994).

[Ma95]   J. Mareš, and B. K. Jennings, *Nucl. Phys.* **A585**, 347c (1995).

[Ma95a] J. Mareš, E. Friedman, A. Gal, and B. K. Jennings, *Nucl. Phys.* **A594**, 311 (1995).

[Ru92]   V. N. Russkikh and Yu. B. Ivanov, *Nucl. Phys.* **A543**, 751 (1992).

[Sc92]   J. Schaffner, C. Greiner, and H. Stöcker, *Phys. Rev.* **C46**, 322 (1992).

[Sc93]   J. Schaffner, C. B. Dover, A. Gal, C. Greiner, and H. Stöcker, *Phys. Rev. Lett.* **71**, 1328 (1993).

7. Matter Under Extreme Conditions

7.A. Relativistic Transport Theory

[Ay94]   S. Ayik, Y. B. Ivanov, V. N. Russkikh, and W. Nörenberg, *Nucl. Phys.* **A578**, 640 (1994).

[Bl88a]   B. Blättel, V. Koch, W. Cassing, and U. Mosel, *Phys. Rev.* **C38**, 1767 (1988).

[Bl89]   B. Blättel, V. Koch, K. Weber, W. Cassing, and U. Mosel, *Nucl. Phys.* **A495**, 381c (1989).

[Bl91]   B. Blättel, V. Koch, A. Lang, K. Weber, W. Cassing, and U. Mosel, *Phys. Rev.* **C43**, 2728 (1991).

[Bu93]   B. W. Bush and J. R. Nix, *Ann. Phys. (N.Y.)* **277**, 97 (1993).

[Bu93a]   B. W. Bush and J. R. Nix, *Nucl. Phys.* **A560**, 586 (1993).

[Bu95]   B. W. Bush and J. R. Nix, *Nucl. Phys.* **A583**, 705 (1995).

[El92]   H. Elsenhans, L. Sehn, A. Faessler, H. Müther, N. Ohtsuka, and H. H. Wolter, *Nucl. Phys.* **A536**, 750 (1992).

[Fa93]   X. S. Fang, C. M. Ko, and Y. M. Zheng, *Nucl. Phys.* **A556**, 499 (1993).

[Fa93a]   X. S. Fang, C. M. Ko, G. E. Brown, and V. Koch, *Phys. Rev.* **C47**, 1678 (1993).

[Fa94]   X. S. Fang, C. M. Ko, G. Q. Li, and Y. M. Zheng, *Phys. Rev.* **C49**, R608 (1994).

[Fu92]   C. Fuchs, L. Sehn, and H. H. Wolter, *Nucl. Phys.* **A545**, 151c (1992).

[Fu95a]   C. Fuchs, H. H. Wolter, *Nucl. Phys.* **A589**, 732 (1995).

[Ha93a]   C. Hartnack, H. Kruse, and H. Stöcker, in *Computational Nuclear Physics*, vol. II, ed. K. Langanke, J. A. Maruhn, and S. E. Koonin (Springer-Verlag, 1993), p. 128.

[Ha93b]   R. Hakim and L. Mornas, *Phys. Rev.* **C47**, 2846 (1993).

[Iv94]   Yu. B. Ivanov and V. N. Russkikh, *Nucl. Phys.* **A580**, 614 (1994).

[Ju92]   Chr. Jung, W. Cassing, and U. Mosel, *Nucl. Phys.* **A549**, 577 (1992).

[Ko87]   C. M. Ko, Q. Li, and R. Wang, *Phys. Rev. Lett.* **59**, 1084 (1987).

[Ko88]   C. M. Ko and Q. Li, *Phys. Rev.* **C37**, 2270 (1988).

[Ko90]   V. Koch, B. Blättel, W. Cassing, and U. Mosel, *Phys. Lett.* **B241**, 174 (1990).

[Ko91]   V. Koch, B. Blättel, W. Cassing, and U. Mosel, *Nucl. Phys.* **A532**, 715 (1991).

[La90]   A. Lang, B. Blättel, V. Koch, W. Cassing, and U. Mosel, *Phys. Lett.* **B245**, 147 (1990).

[Li89a] Q. Li, J. Q. Wu, and C. M. Ko, *Phys. Rev.* **C39**, 849 (1989).

[Li93]   G. Q. Li and R. Machleidt, *Phys. Rev.* **C48**, 1702 (1993).

[Li94]   G. Q. Li, C. M. Ko, X. S. Fang, and Y. M. Zheng, *Phys. Rev.* **C49**, 1139 (1994).

[Ma92]   T. Maruyama, B. Blättel, W. Cassing, A. Lang, U. Mosel, and K. Weber, *Phys. Lett.* **B297**, 228 (1992).

[Ma93]   R. Malfliet, *Nucl. Phys.* **A553**, 763c (1993).

[Ma94a] T. Maruyama, W. Cassing, U. Mosel, S. Teis, and K. Weber, *Nucl. Phys.* **A573**, 653 (1994).

[Ma94b] G. Mao, Z. Li, Y. Zhuo, and Y. Han, *Phys. Rev.* **C49**, 3137 (1994).

[Me93]   D. P. Menezes, F. S. Navarra, M. Nielsen, and U. Ornik, *Phys. Rev.* **C47**, 2635 (1993).

[Mo94]   L. Mornas, *Nucl. Phys.* **A573**, 554 (1994).

[Mo95]   L. Mornas and U. Ornik, *Nucl. Phys.* **A587**, 828 (1995).

[Mr94]   S. Mrówczyński and U. Heinz, *Ann. Phys. (N.Y.)* **229**, 1 (1994).

[Ni93]   M. Nielsen, C. Providência, and J. da Providência, *Phys. Rev.* **C47**, 200 (1993).

[Po95]   J. Pochodzalla *et al.*, *Phys. Rev. Lett.* **75**, 1040 (1995).

[Ru94]   V. N. Russkikh, Yu. B. Ivanov, Yu. E. Pokrovsky, and P. A. Henning, *Nucl. Phys.* **A572**, 749 (1994).

[Sc94]   M. Schönhofen, M. Cubero, B. L. Friman, W. Nörenberg, and Gy. Wolf, *Nucl. Phys.* **A572**, 112 (1994).

[St86]   H. Stöcker and W. Greiner, *Phys. Rep.* **137**, 277 (1986).

[Te93]   S. Teis, W. Cassing, T. Maruyama, and U. Mosel, *Phys. Lett.* **B319**, 47 (1993).

[Te94]   S. Teis, W. Cassing, T. Maruyama, and U. Mosel, *Phys. Rev.* **C50**, 388 (1994).

[Wa89]   J. D. Walecka, *Fundamentals of Statistical Mechanics: Manuscript and Notes of Felix Bloch, Prepared by John Dirk Walecka* (Stanford Univ. Press, 1989).

[We92] K. Weber, B. Blättel, W. Cassing, H.-C. Dönges, V. Koch, A. Lang, and U. Mosel, *Nucl. Phys.* **A539**, 713 (1992).

[We93a] K. Weber, B. Blättel, W. Cassing, H.-C. Dönges, A. Lang, T. Maruyama, and U. Mosel, *Nucl. Phys.* **A552**, 571 (1993).

[Zh92a] W-M. Zhang and L. Wilets, *Phys. Rev.* **C45**, 1900 (1992).

7.B. Extrapolation and Connections to QCD

[Ad93] C. Adami and G. E. Brown, *Phys. Rep.* **234**, 1 (1993).

[Ag94] R. Aguirre, O. Civitarese, and A. L. De Paoli, *Nucl. Phys.* **A579**, 573 (1994).

[Ba94a] G. Bao, L. Engvik, M. Hjorth-Jensen, E. Osnes, and E. Østgaard, *Nucl. Phys.* **A575**, 707 (1994).

[Ce92a] L. S. Celenza, A. Pantziris, C. M. Shakin, and W.-D. Sun, *Phys. Rev.* **C45**, 2015 (1992).

[Ce93] L. S. Celenza, C. M. Shakin, W.-D. Sun, and X. Zhu, *Phys. Rev.* **C48**, 159 (1993).

[De95] A. Delfino, C. T. Coelho, and M. Malheiro, *Phys. Lett.* **B345**, 361 (1995); *Phys. Rev.* **C51**, 2188 (1995).

[Dr95] A. Drago, M. Fiolhais, and U. Tambini, *Nucl. Phys.* **A588**, 801 (1995).

[Du95] A. Dumitru, U. Katscher, J. A. Maruhn, H. Stöcker, W. Greiner, and D. H. Rischke, *Phys. Rev.* **C51**, 2166 (1995).

[El91] J. Ellis, J. I. Kapusta, and K. A. Olive, *Phys. Lett.* **B273**, 123 (1991); *Nucl. Phys.* **B348**, 345 (1991).

[El95] P. J. Ellis, R. Knorren, and M. Prakash, *Phys. Lett.* **B349**, 11 (1995).

[Fu91b] R. J. Furnstahl and B. D. Serot, *Phys. Rev. C* **44**, 2141 (1991).

[Ga94b] S. Gao, R.-K. Su, and K. N. Yu, *Phys. Rev.* **C49**, 40 (1994).

[Ga94c] E. Gallego, J. Diaz Alonso, and A. Pérez, *Nucl. Phys.* **A578**, 542 (1994).

[Ga95a] S. Gao, Y.-J. Zhang, and R.-K. Su, *Nucl. Phys.* **A593**, 362 (1995).

[Gl92] N. K. Glendenning, *Phys. Rev.* **D46**, 1274 (1992).

[Gl92a] N. K. Glendenning, F. Weber, and S. A. Moszkowski, *Phys. Rev.* **C45**, 844 (1992).

[Ha92] Y. Han, Q. Shen, and Y. Zhuo, *Phys. Rev.* **C46**, 2396 (1992).

[Ha94a]   S. Haddad and M. K. Weigel, *Phys. Rev.* **C49**, 3228 (1994).

[Ha94b]   Y. Han, Q. Shen, Y. Zhuo, and T. Geng, *Nucl. Phys.* **A569**, 732 (1994).

[He93]   G. Hejc, H. Baier, and W. Bentz, *Phys. Lett.* **B317**, 257 (1993).

[He95]   G. Hejc, W. Bentz, and H. Baier, *Nucl. Phys.* **A582**, 401 (1995).

[Hu94]   H. Huber, F. Weber, and M. K. Weigel, *Phys. Rev.* **C50**, R1287 (1994).

[Ja92]   M. Jaminon, R. Mendez Galain, G. Ripka, and P. Stassart, *Nucl. Phys.* **A537**, 418 (1992).

[Ja93a]   M. Jaminon and G. Ripka, *Nucl. Phys.* **A564**, 505 (1993).

[Ka95]   J. I. Kapusta, A. P. Vischer, and R. Venugopalan, *Phys. Rev.* **C51**, 901 (1995).

[Ko95c]   C. M. Ko and G. Q. Li, *Nucl. Phys.* **A583**, 591 (1995).

[Ku91]   T. Kunihiro, *Phys. Lett.* **B271**, 395 (1991).

[Li92]   G. Q. Li, R. Machleidt, and R. Brockmann, *Phys. Rev.* **C45**, 2782 (1992).

[Li95c]   G. Q. Li and C. M. Ko, *Phys. Lett.* **B351**, 37 (1995).

[Lo92a]   I. Lovas, L. Molnár, K. Sailer, and W. Greiner, *Phys. Rev.* **C45**, 1693 (1992).

[Lo94a]   I. Lovas, L. Molnár, K. Sailer, and W. Greiner, *Phys. Lett.* **B328**, 168 (1994).

[Ma92c]   F. Matera and A. Dellafiore, *Phys. Lett.* **B296**, 1 (1992).

[Ma93b]   B.-Q. Ma, Q.-R. Zhang, D. H. Rischke, and W. Greiner, *Phys. Lett.* **B315**, 29 (1993).

[Ma94d]   T. Maruyama, H. Fujii, T. Muto, and T. Tatsumi, *Phys. Lett.* **B337**, 19 (1994).

[Mo94a]   A. Moncada, F. G. Scholtz, and F. J. W. Hahne, *Phys. Rev.* **C50**, 1412 (1994).

[Mu95]   H. Müller and B. D. Serot, *Phys. Rev.* **C52**, 2072 (1995).

[Mu96a]   H. Müller, "The deconfinement phase transition in asymmetric matter", preprint IU/NTC 96–13, *Nucl. Phys. A*, in press.

[Qi93]   Z.-X. Qian, H.-Q. Song, and R.-K. Su, *Phys. Rev.* **C48**, 154 (1993).

[Ri95]   D. H. Rischke, *Nucl. Phys.* **A583**, 663 (1995).

[Ro92]   A. Rosenhauer, E. F. Staubo, L. P. Csernai, T. Øvergård, and E. Østgaard, *Nucl. Phys.* **A540**, 630 (1992).

[Ra93]   M. Rashdan, *Phys. Rev.* **C48**, 1323 (1993).

[Sa94]   K. Saito and A. W. Thomas, *Nucl. Phys.* **A574**, 659 (1994).

[Sa94a]   S. Sarkar and B. Malakar, *Phys. Rev.* **C50**, 757 (1994).

[Sa94b]   K. Saito and A. W. Thomas, *Phys. Lett.* **B327**, 9 (1994).

[Sa95]   K. Saito and A. W. Thomas, *Phys. Rev.* **C51**, 2757 (1995).

[Sc94a]   J. Schaffner, A. Gal, I. N. Mishustin, H. Stöcker, and W. Greiner, *Phys. Lett.* **B334**, 268 (1994).

[So93]   H. Q. Song, Z. X. Qian, and R. K. Su, *Phys. Rev.* **C47**, 2001 (1993).

[So94]   H. Q. Song, Z. X. Qian, and R. K. Su, *Phys. Rev.* **C49**, 2924 (1994).

[Su92]   K. Sumiyoshi, H. Toki, and R. Brockmann, *Phys. Lett.* **B276**, 393 (1992).

[Su92a]   R.-K. Su and Z.-X. Qian, *Phys. Rev.* **C46**, 1528 (1992).

[Su95]   K. Sumiyoshi, H. Kuwabara, and H. Toki, *Nucl. Phys.* **A581**, 725 (1995).

[Su95a]   R.-K. Su, T. Long, and Y.-J. Zhang, *Phys. Rev.* **C51**, 1265 (1995).

[Ta92a]   K. Tanaka and W. Bentz, *Nucl. Phys.* **A540**, 383 (1992).

[Vo92]   D. Von-Eiff and M. K. Weigel, *Phys. Rev.* **C46**, 1288 (1992).

[We94]   W. Weise, *Nucl. Phys.* **A574**, 347c (1994).

[Zi90]   J. Zimanyi and S. A. Moszkowski, *Phys. Rev.* **C42**, 1416 (1990).

8. Loops in QHD

[Al92]   M. P. Allendes and B. D. Serot, *Phys. Rev.* **C45**, 2975 (1992).

[Al95]   M. P. Allendes, Ph.D. Thesis, Indiana Univ., 1995.

[Ao95]   Z. Aouissat, R. Rapp, G. Chanfray, P. Schuck, and J. Wambach, *Nucl. Phys.* **A581**, 471 (1995).

[Be92]   W. Bentz, A. Arima, and H. Baier, *Nucl. Phys.* **A541**, 333 (1992).

[Be93]   P. Bernardos, V. N. Fomenko, N. Van Giai, M. L. Quelle, S. Marcos, R. Niembro, and L. N. Savushkin, *Phys. Rev.* **C48**, 2665 (1993).

[Be93a]   C. Bedau and F. Beck, *Nucl. Phys.* **A560**, 518 (1993).

[Be96]   P. Bernardos, V. N. Fomenko, M. L. Quelle, S. Marcos, R. Niembro, and L. N. Savushkin, *J. Phys.* **G22**, 361 (1996).

[Bi83]   A. F. Bielajew, *Nucl. Phys.* **A404**, 428 (1983).

[Bi84]   A. F. Bielajew and B. D. Serot, *Ann. Phys. (N.Y.)* **156**, 215 (1984).

[Bo87]   A. Bouyssy, J.-F. Mathiot, N. Van Giai, and S. Marcos, *Phys. Rev.* **C36**, 380 (1987).

[Bo94a]   H. F. Boersma and R. Malfliet, *Phys. Rev.* **C49**, 1495 (1994).

[Ca95]   J. C. Caillon and J. Labarsouque, *Phys. Lett.* **B352**, 193 (1995).

[Ce92]   L. S. Celenza, A. Pantziris, C. M. Shakin, and W.-D. Sun, *Phys. Rev.* **C46**, 571 (1992).

[Do96]   J. F. Donoghue, Lectures at the Int. School on Effective Field Theory, Almunecar, Spain, 1995, e-print hep-ph/9607351.

[Du93]   J. W. Durso, H.-C. Kim, and J. Wambach, *Phys. Lett.* **B298**, 267 (1993).

[Fe93]   T. C. Ferrée, C. E. Price, and J. R. Shepard, *Phys. Rev.* **C47**, 573 (1993).

[Fr92]   R. Friedrich, K. Wehrberger, and F. Beck, *Phys. Rev.* **C46**, 188 (1992).

[Fr93]   M. R. Frank, *Phys. Rev.* **C47**, 1811 (1993).

[Fr93a]   R. Fritz, H. Müther, and R. Machleidt, *Phys. Rev. Lett.* **71**, 46 (1993).

[Fr94a]   R. Fritz and H. Müther, *Phys. Rev.* **C49**, 633 (1994).

[He91]   P. A. Henning, *Phys. Lett.* **B272**, 186 (1991).

[He92a]   T. Herbert, K. Wehrberger, and F. Beck, *Nucl. Phys.* **A541**, 699 (1992).

[Ja93]   M. Jaminon, R. Méndez-Galain, and P. Stassart, *Nucl. Phys.* **A556**, 509 (1993).

[Je94]   H.-C. Jean, J. Piekarewicz, and A. G. Williams, *Phys. Rev.* **C49**, 1981 (1994).

[Ka92]   M. Katô, W. Bentz, and K. Tanaka, *Phys. Rev.* **C45**, 2445 (1992).

[Ko93]   C. L. Korpa and R. Malfliet, *Phys. Lett.* **B315**, 209 (1993).

[Kr93]   G. Krein, M. Nielsen, R. D. Puff, and L. Wilets, *Phys. Rev.* **C47**, 2485 (1993).

[Ma94c]   Z. Ma, H. Shi, and B. Chen, *Phys. Rev.* **C50**, 3170 (1994).

[Mi91]   J. Milana, *Phys. Rev.* **C44**, 527 (1991).

[Na91]   M. Nakano and A. Hasegawa, *Phys. Rev.* **C43**, 618 (1991).

[Na94]   M. Nakano, A. Hasegawa, H. Kouno, and K. Koide, *Phys. Rev.* **C49**, 3061 (1994).

[Na94a]   M. Nakano, T. Mitsumori, M. Muraki, K. Koide, H. Kouno, and A. Hasegawa, *Phys. Rev.* **C49**, 3076 (1994).

[Pa92]   P. K. Panda, S. P. Misra, and R. Sahu, *Phys. Rev.* **C 45**, 2079 (1992).

[Pi93]   J. Piekarewicz and A. G. Williams, *Phys. Rev.* **C47**, R2462 (1993).

[Pi93a]   J. Piekarewicz, *Phys. Rev.* **C48**, 1555 (1993).

[Pr92]   M. Prakash, P. J. Ellis, and J. I. Kapusta, *Phys. Rev.* **C45**, 2518 (1992).

[Ru95a]   J. J. Rusnak and R. J. Furnstahl, *Z. Phys.* **A352**, 345 (1995).

[Se95]   B. D. Serot and H.-B. Tang, *Phys. Rev.* **C51**, 969 (1995).

[Ta91]   K. Tanaka, W. Bentz, A. Arima, and F. Beck, *Nucl. Phys.* **A528**, 676 (1991).

[Ta93a]   H.-B. Tang, Ph.D. Thesis, Indiana Univ., 1993.

[Vo94]   D. Von-Eiff, S. Haddad, and M. K. Weigel, *Phys. Rev.* **C50**, 1244 (1994).

[Zh94a]   J.-K. Zhang and D. S. Onley, *Phys. Rev.* **C49**, 762 (1994).