

# (Hybrid) Baryons in the Flux–Tube Model

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**Abstract.** We construct baryons and hybrid baryons in the non-relativistic flux–tube model of Isgur and Paton. The motion of the flux–tube with the three quark positions fixed, except for centre of mass corrections, is discussed. It is shown that the problem can to an excellent approximation be reduced to the independent motion of a junction and strings.

Hybrids are bound states where there is an explicit excitation in the gluon field of QCD. Particularly, “hybrids baryons” may be viewed as quark–quark–quark–glue composites.

The experimental interest in hybrid baryons centers around the excited baryon resonance ( $< 2.2$  GeV) program at TJNAF, mostly in Hall B. Hybrid baryon production is expected. If hybrid baryons obey similar decay selection rules to hybrid mesons, they may be distinguishable based on their strong decays.

Hybrid baryons have only been constructed in the MIT bag model [1]. We are motivated to build a model consistent<sup>2</sup> with predictions from QCD lattice gauge theory, i.e. the Isgur–Paton non-relativistic flux–tube model [2]. This model is motivated from the strong coupling limit of the hamiltonian lattice gauge theory formulation of QCD (HLGT).

This talk will deal with fixed quark positions relative to each other. However, we shall allow the quarks with fixed relative positions to move in order to work in the centre of mass frame. This is called the “quasi-adiabatic” approximation.

The model is motivated from the strong coupling limit of HLGT, where

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<sup>2)</sup> The hybrid meson interquark potential is consistent with that evaluated from lattice gauge theory [2].

there are “flux–lines” which play the role of glue. In the spirit of the adiabatic approximation, where quarks do not respond to the influence of glue, we neglect operators which make quarks move. The remaining operator taking you away from the strong coupling limit is the plaquette operator. This induces motion of the “flux–line” between the quarks perpendicular to its rest position. We model the flux–line by “beads”, all with the same mass, which is fixed from the energy in the linear flux–line. The beads are separated along their rest positions by a finite lattice spacing, and are allowed to move perpendicular to their rest position. The beads are attracted to each other by a linear potential, vibrating in various string modes.

There is a second essential ingredient from HLG. Three flux–lines can come together at a point called the “junction”. A plaquette operator cannot move the junction in the lowest order of perturbation theory so that the junction is taken to have a different (higher) mass associated with it than the other beads.

The final picture of a (hybrid) baryon is that of three quarks, each connected via a line of beads to the junction in a Mercedes Benz configuration. The three quarks define a plane. The “equilibrium configuration” is the lowest energy configuration: the junction is located such that there are angles of  $120^\circ$  between each of the “triads” that connect each of the quarks to the junction, and the beads all lie on the triads. The junction and beads then vibrate with respect to the equilibrium configuration. There are two important motions which are expected to have physical significance: (1) the motion of the junction perpendicular and within the plane relative to the junction rest position, called the “junction motion”; (2) the motion of the beads in the two directions perpendicular to the line connecting the quark to the junction, called the “string motion”. If the angles between two of the quarks suspended at the third quark is larger than  $120^\circ$  the equilibrium configuration is not the Mercedes Benz configuration. This issue is not considered further here.

We now make the small oscillation approximation, where the beads and junction move near to the equilibrium configuration. We make sure that we work in the centre of mass system, and therefore make the “quasi–adiabatic” approximation. The hamiltonian is written in terms of the junction and string motion coordinates.

We have demonstrated that the hamiltonian can be separated into a part  $H_J$  which corresponds to the motion of the junction in the potential one would use if there were no beads in the problem, with an effective junction mass related to its own mass and the mass of the beads. Another part  $H_S$  is the independent motion of three strings with respect to a fixed junction, with an effective bead mass that is related to the bead and junction masses. There is also an “interaction term”  $H_{int}$  where the strings corresponding to different quarks interact with each other and various string modes associated with the same quark interact with each other, and where the junction interacts with the various string modes.

We shall now demonstrate that the interaction term gives a minor contribution. The free parameters in the model (and the values used for the numerical simulation) are the string tension ( $0.18 \text{ GeV}^2$ ), the ratio of the junction and bead masses (1) and the quark masses ( $0.33 \text{ GeV}$ ). We shall perform a simulation where there is one bead between each quark and the junction and the quarks form an equilateral triangle with the lengths of the triads equal to a typical value of  $2.5 \text{ GeV}^{-1}$ .

First we solve the exact problem numerically. The frequencies parallel and perpendicular to the plane are (in  $\text{GeV}$ )

Parallel	<b>0.607</b>	<b>0.607</b>	0.924	1.08	1.08
Perpendicular	<b>0.828</b>		0.924	0.924	1.37

where the bold faced frequencies are clearly lower than the others. In fact, if we set  $H_{int} = 0$  then we again obtain the same number of frequencies, and here quote the results for the lowest lying frequencies corresponding to the junction motion (in  $\text{GeV}$ ):

Parallel	<b>0.614</b>	<b>0.614</b>
Perpendicular	<b>0.869</b>	

We hence conclude that the lowest frequencies are the ones corresponding to the junction motion, and that we can neglect  $H_{int}$  safely for the lowest frequency. In retrospect, the reason why  $H_{int}$  can be neglected is because we chose the physically appropriate coordinates for the problem: the junction and string motions.

To compare the frequencies of the full hamiltonian and the junction frequencies of the hamiltonian with  $H_{int} = 0$ , we calculate the deviation  $\epsilon$  of them from one another.  $\epsilon_- = 1\%$  and  $\epsilon_+ = 1\%$  for the low and high parallel frequencies respectively (in this case the frequencies are equal).  $\epsilon = 5\%$  perpendicular to the plane. For the lowest frequency, it is hence sufficient to work with a hamiltonian of the form  $H_J + H_S$  from now on, and there are three types of hybrid baryons: the one corresponding to junction motion perpendicular to the plane which is always the heaviest; and two corresponding to motion parallel to the plane. For a generic quark configuration, one of the parallel frequencies is always below or equal to the other, so that they are not usually degenerate. The flux-tube model thus contains three low lying hybrid baryons, corresponding to vibrations along three perpendicular axes, but each with a different excitation frequency above the baryon.

To access the error in the hamiltonian with  $H_{int} = 0$  more fully, we vary parameters around the central values above, one at a time; with the quark mass up to the charm quark mass of  $1.5 \text{ GeV}$ , the ratio of the junction to the bead mass up to 10, and with triads with lengths from  $0.5 - 5 \text{ GeV}^{-1}$

(the triad lengths not being equal in general). We found that that  $\epsilon_- \lesssim 5\%$  and  $\epsilon_+ \lesssim 6\%$  for parallel frequencies and  $\epsilon \lesssim 40\%$  for the perpendicular frequency. The error introduced by neglecting  $H_{int}$  is therefore rather minimal for the lowest frequency. Hence, to a good approximation, the dynamics of the lowest frequency can be simplified to junction and string motion which are independent of one another.

Significant progress has been made towards building a realistic flux-tube model of (hybrid) baryons. We have constructed the full multibead hamiltonian in the quasi-adiabatic approximation and in the small oscillations approximation. We demonstrated that the junction bead decouples from the other beads to a high degree of accuracy.

## REFERENCES

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