Full Coupled Channel Approach to S=-2 s-shell Hypernuclar Systems

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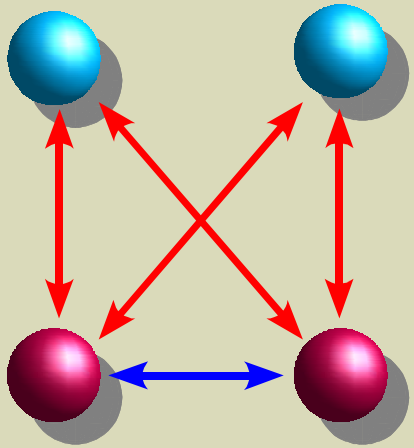
Introduction:

Strangeness S=−1, −2 light hypernuclei

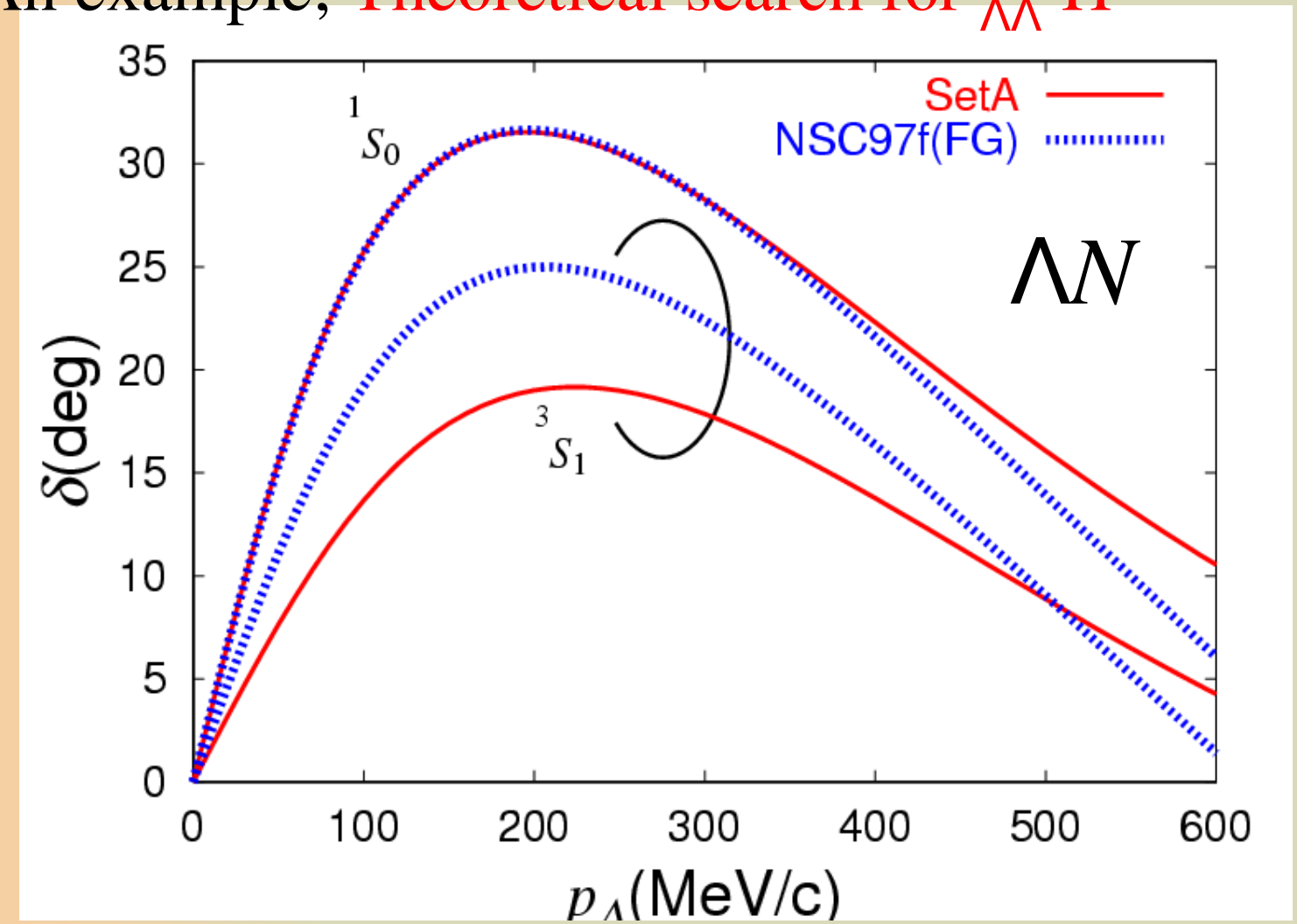
- S=−1 hypernuclei ➜ ΛN interaction
- S=−2 hypernuclei ➜ $\Lambda\Lambda$ interaction
  - ➥ multi-strangeness systems
- A key issue of S=−2 study: The total binding energies of the S=−2 hypernuclei strongly depend on the strength of the ΛN interaction than the strength of the $\Lambda\Lambda$ interaction.
  - For example, ${}^{\Lambda\Lambda}{}^{4}\mathrm{H} \sim pn\Lambda\Lambda$
  - Number of ΛN pairs: 4
  - Number of $\Lambda\Lambda$ pairs: 1



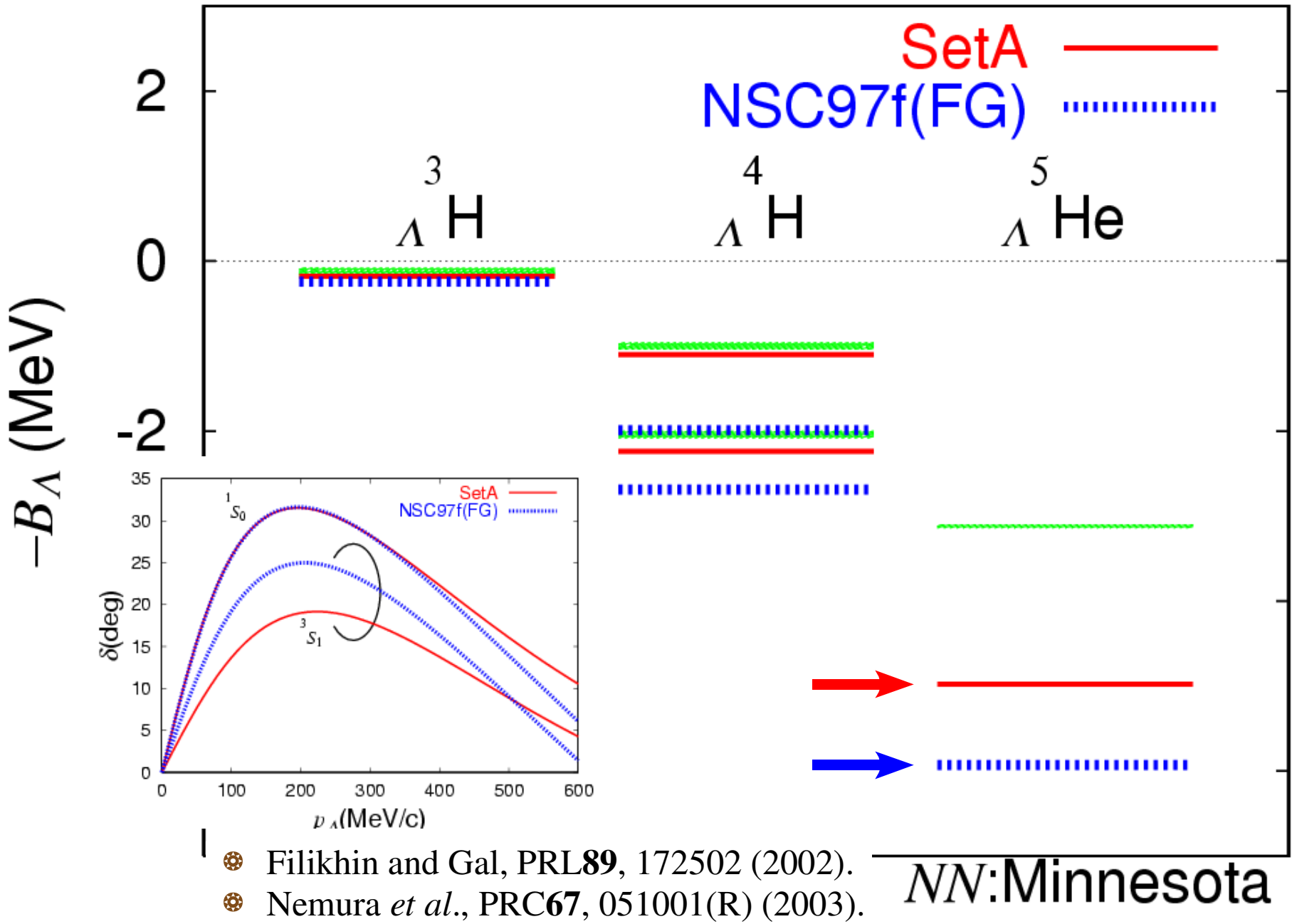
Introduction:

Determination of σ·σ term of the ΛN interaction is crucial to study the S=−2 systems.

- An example; Theoretical search for ${}^{\Lambda\Lambda}{}^{4}\mathrm{H}$



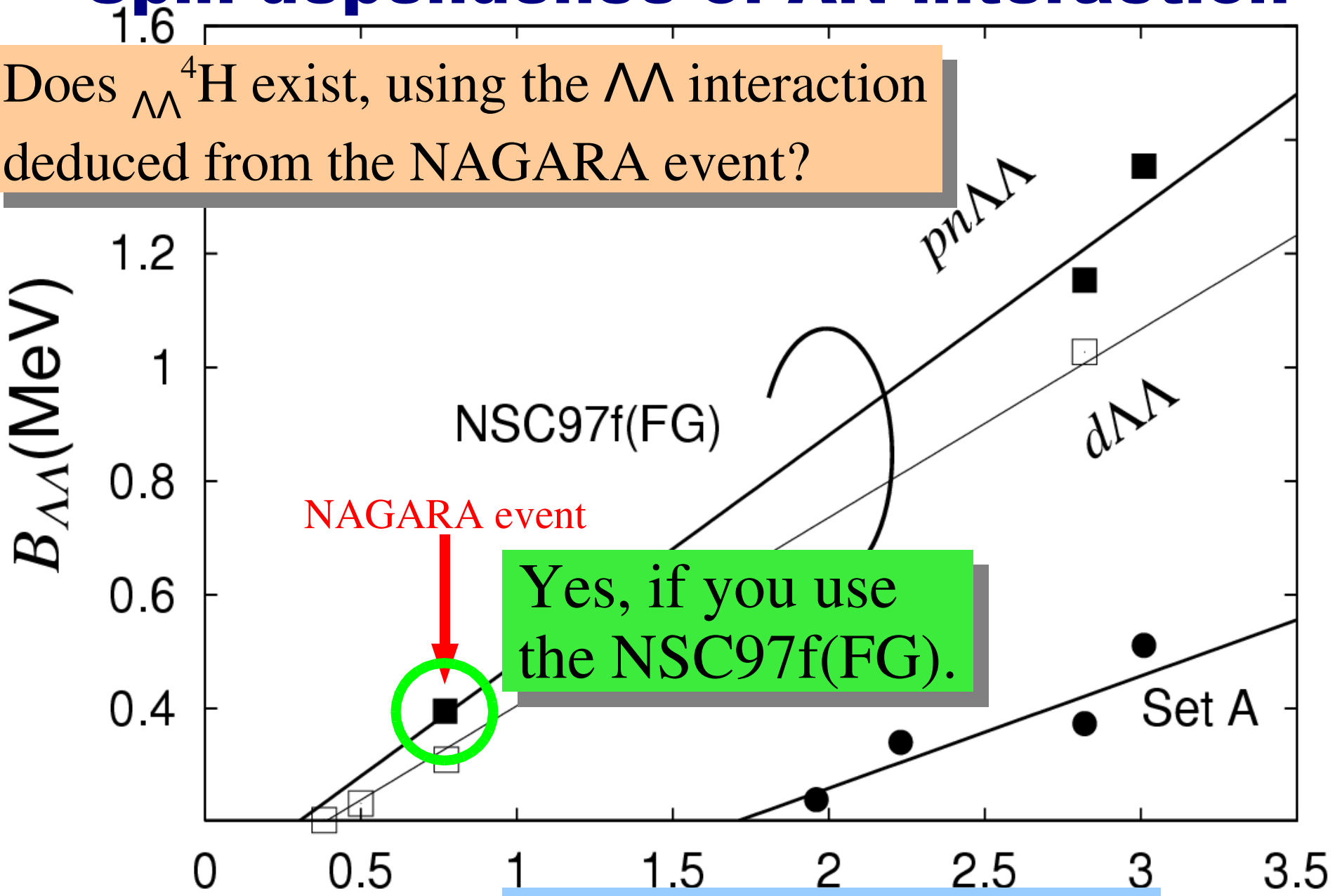
Spin dependence of ΛN interaction



- Filikhin and Gal, PRL**89**, 172502 (2002).
- Nemura *et al*., PRC**67**, 051001(R) (2003).

Spin dependence of ΛN interaction

Does ${}^{4}_{\Lambda\Lambda}\mathrm{H}$ exist, using the ΛΛ interaction deduced from the NAGARA event?



Yes, if you use the NSC97f(FG).

No, if you use the Set A.

- Filikhin and Gal, PRL**89**, 172502
- Nemura *et al.*, PRC**67**, 051001(R)

Spin dependence of ΛN interaction

- Does ${}^{\Lambda\Lambda}{}^{4}\mathrm{H}$ exist, using the ΛΛ interaction deduced from the NAGARA event?
  - Yes, if you use the NSC97f(FG).
  - No, if you use the Set A.
- The problem is
  what the *ΛN* interaction we should use.

Spin dependence of ΛN interaction:

Determination of σ·σ term of the Λ*N* interaction is crucial to study the *S*=−2 systems.

- Is the Λ*N* interaction acceptable, which reproduces the B_Λ values of *A*=3 and 4 hypernuclei? ➜ No!

- There is a long standing problem on *s*-shell Λ hypernuclei: anomalously small binding of ${}^{5}_{\Lambda}\mathrm{He}$.

Anomalously small binding of ${}^{5}_{\Lambda}$He

- Why is the ${}^{5}_{\Lambda}$He anomaly important for the study of S=−2 hypernuclei?
  - The algebraic structure of σ·σ part of ΛN potential for S=−2 hypernuclei is same as that for ${}^{5}_{\Lambda}$He.

$$H=\sum_{i=1}^{A}\frac{\boldsymbol{p}_i^{\,2}}{2m_i}-T_{CM}+\sum_{i<j}^{N}V(N_i N_j)$$

$$+\sum_{i=1}^{N}\sum_{j=1}^{Y}V(N_i \Lambda_j)+V(\Lambda_1 \Lambda_2), \qquad (A=N+Y)$$

Spin-isospin factor

$$\left\langle \sum_{i=1}^{N}\sum_{j=1}^{Y}V(N_i \Lambda_j)\right\rangle,\ \left\langle V(\Lambda_1 \Lambda_2)\right\rangle$$

Anomalously small binding of ${}^{5}_{\Lambda}$He

- Why is the ${}^{5}_{\Lambda}$He anomaly important for the study of S=−2 hypernuclei?
  - The algebraic structure of σ·σ part of ΛN potential for S=−2 hypernuclei is same as that for ${}^{5}_{\Lambda}$He.

S=−1 hypernuclei

$$\left\langle \sum_{i=1}^{N} V(N_i \Lambda) \right\rangle$$

${}^{3}_{\Lambda}$H	$\frac{1}{2}\bar{v}_t + \frac{3}{2}\bar{v}_s$
${}^{4}_{\Lambda}$H , ${}^{4}_{\Lambda}$He	$\frac{3}{2}\bar{v}_t + \frac{3}{2}\bar{v}_s$
${}^{4}_{\Lambda}$H* , ${}^{4}_{\Lambda}$He*	$\frac{5}{2}\bar{v}_t + \frac{1}{2}\bar{v}_s$
${}^{5}_{\Lambda}$He	$3\bar{v}_t + \bar{v}_s$ ⇇ 3:1

S=−2 hypernuclei

$$\left\langle \sum_{i=1}^{N}\sum_{j=1}^{Y} V(N_i \Lambda_j) \right\rangle \quad \left\langle V(\Lambda_1 \Lambda_2) \right\rangle$$

${}^{4}_{\Lambda\Lambda}$H	$3\bar{v}_t + \bar{v}_s$	$\bar{u}_s$
${}^{5}_{\Lambda\Lambda}$H , ${}^{5}_{\Lambda\Lambda}$He	$\frac{9}{2}\bar{v}_t + \frac{3}{2}\bar{v}_s$	$\bar{u}_s$
${}^{6}_{\Lambda\Lambda}$He	$6\bar{v}_t + 2\bar{v}_s$	$\bar{u}_s$

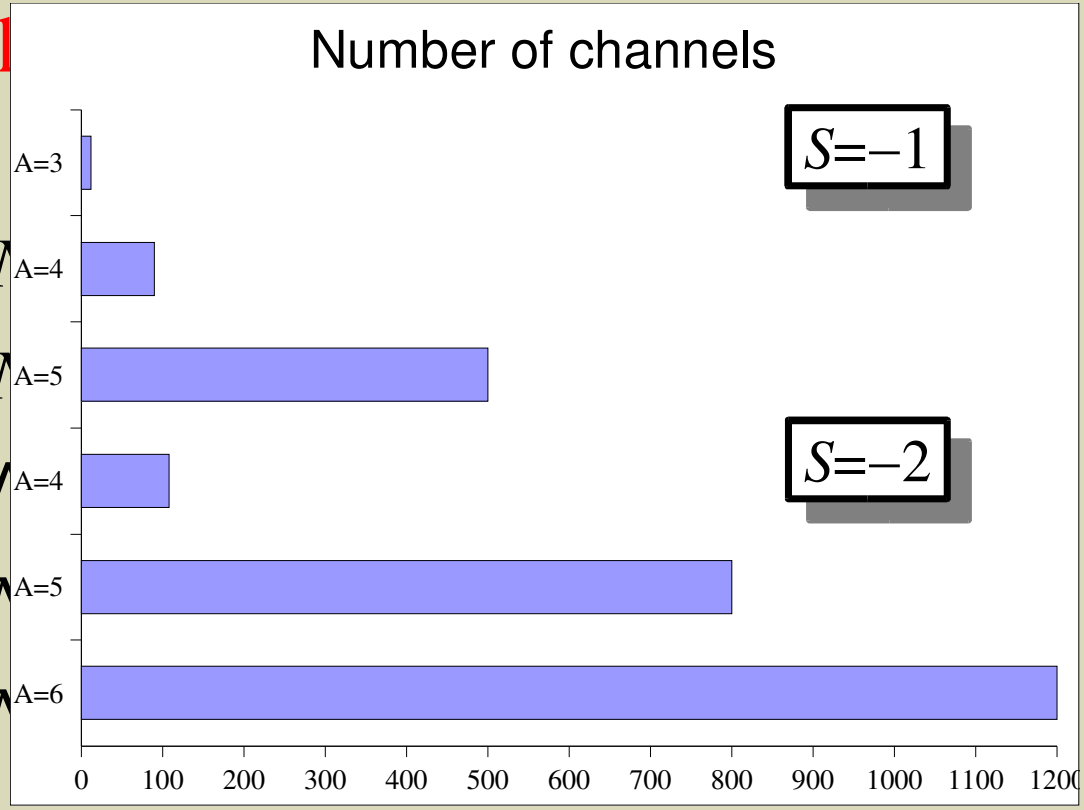
⇈ 3:1 ⇈

The purpose of this work

- Systematic study for the compete set of *s*-shell Λ hyper-nuclei with the strangeness $S=-1, -2$.
  - To resolve the ${}^{5}_{\Lambda}\mathrm{He}$ anomaly, using D2' potential.
  - To perform a systematic study for $A=4$-6, $S=-2$ hypernuclear systems, using the *YN* potential of the $A=5$ anomaly-free.
- ${}^{3}_{\Lambda}\mathrm{H}$ ~ $NN\Lambda + NN\Sigma$
- ${}^{4}_{\Lambda}\mathrm{H}$, ${}^{4}_{\Lambda}\mathrm{He}$ ~ $NNN\Lambda + NNN\Sigma$
- ${}^{5}_{\Lambda}\mathrm{He}$ ~ $NNNN\Lambda + NNNN\Sigma$
- ${}^{4}_{\Lambda\Lambda}\mathrm{H}$ ~ $NN\Lambda\Lambda + NN\Lambda\Sigma + NNN\Xi + NN\Sigma\Sigma$
- ${}^{5}_{\Lambda\Lambda}\mathrm{H}$, ${}^{5}_{\Lambda\Lambda}\mathrm{He}$ ~ $NNN\Lambda\Lambda + NNN\Lambda\Sigma + NNNN\Xi + NNN\Sigma\Sigma$
- ${}^{6}_{\Lambda\Lambda}\mathrm{He}$ ~ $NNNN\Lambda\Lambda + NNNN\Lambda\Sigma + NNNNN\Xi + NNNN\Sigma\Sigma$

The purpose of this work

- Systematic study for the compete set of *s*-shell Λ hyper-nuclei with the strangeness S=−1, −2.
  - To resolve the ${}^{5}_{\Lambda}$He anomaly, using D2' potential.
  - To perform a systematic study for A=4-6, S=−2 hypernuclear systems, using the *YN* potential
- ${}^{3}_{\Lambda}$H ~ $NN\Lambda + NN\Sigma$
- ${}^{4}_{\Lambda}$H, ${}^{4}_{\Lambda}$He ~ $NNN\Lambda + NN$
- ${}^{5}_{\Lambda}$He ~ $NNNN\Lambda + N$
- ${}^{4}_{\Lambda\Lambda}$H ~ $NN\Lambda\Lambda + NN$
- ${}^{5}_{\Lambda\Lambda}$H, ${}^{5}_{\Lambda\Lambda}$He ~ $NNN\Lambda\Lambda + N$
- ${}^{6}_{\Lambda\Lambda}$He ~ $NNNN\Lambda\Lambda +$



NN, YN and YY potentials

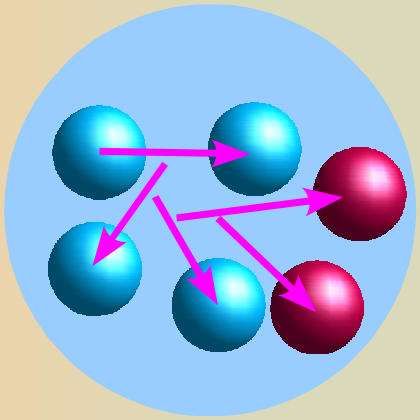
- *NN* interaction: Minnesota potential
  - The *NN* interaction reproduces the low energy *NN* scattering data, and also reproduces reasonably well both the binding energies and sizes of ^{2}H, ^{3}H, ^{3}He, and ^{4}He.
- *YN* interaction: D2' potential
  - The *YN* interaction reproduces the experimental B_{Λ} of *A*=3-5 hypernuclei; Free from the $^{5}_{\Lambda}$He anomaly.
- *YY* interaction: Nijmegen model D simulated (r_c=0.56 fm)
  - Full coupled channel;

	$^1S_0$	$^3S_1$
I=0	ΛΛ-*N*Ξ-ΣΣ	*N*Ξ
I=1	*N*Ξ-ΛΣ	*N*Ξ-ΛΣ-ΣΣ
I=2	ΣΣ	

Ab initio calculation with stochastic variational method

- The variational trial function must be flexible enough to incorporate both
  - Explicit Σ degrees of freedom and
  - Higher orbital angular momenta.
- $\Psi = \Sigma_i c_i \Phi_{JMTM_T}(x; \mathbf{A}_i, u_i)$
- $\Phi_{JMIM_I}(x, \mathbf{A}_i, u_i)$

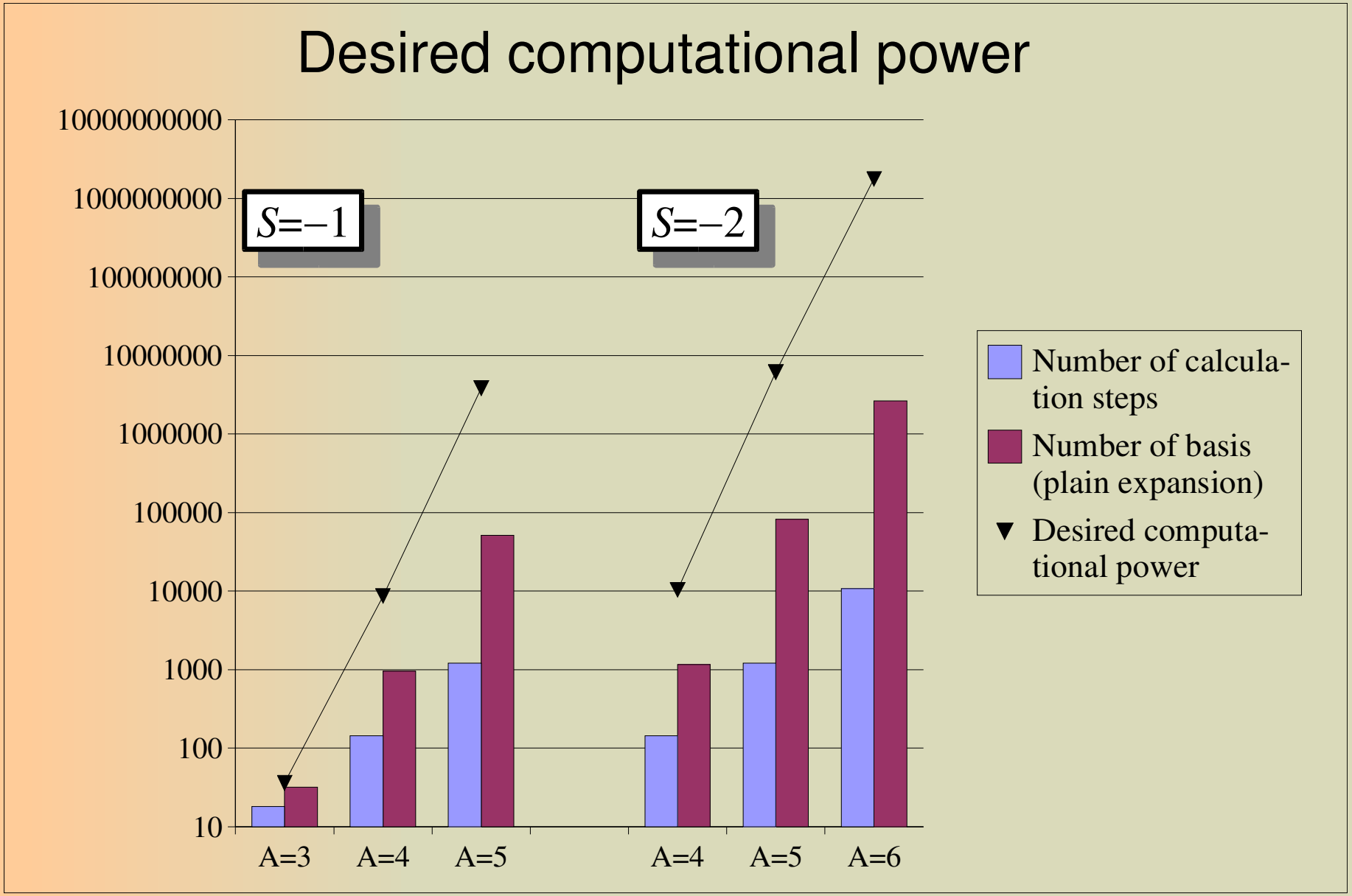
  $= \mathcal{A}\{G(x; \mathbf{A}_i)[\theta_{(kl)_i}(x; u_i) \times \chi_{s_i}]_{JM} \eta_{IM_I}\}$



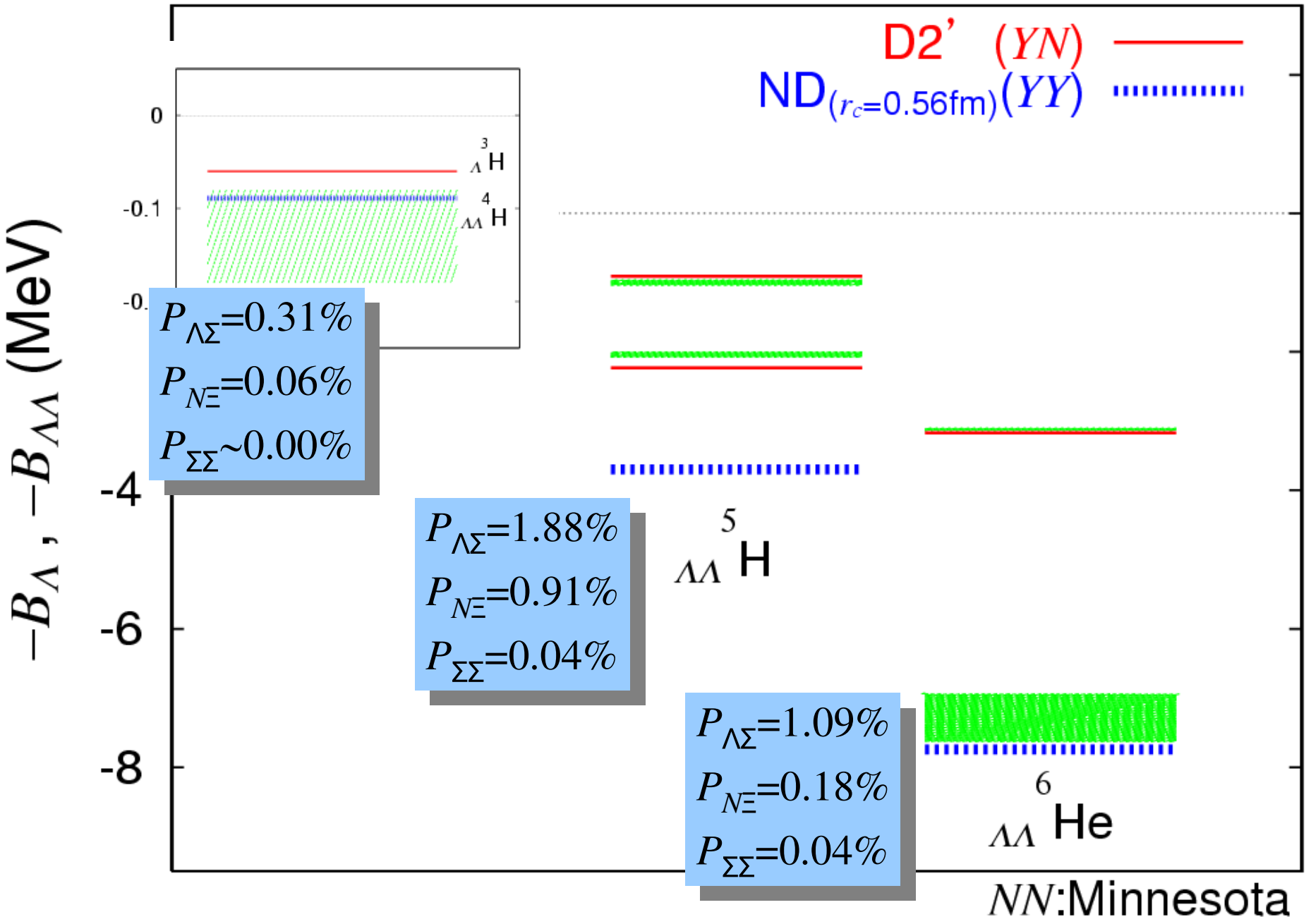
Complete six-body treatment

Ab initio calculation with SVM

- SVM is capable of handling the massive calculation.



Results


D2' (YN)
ND(rc=0.56fm)(YY)
-B_Λ , -B_ΛΛ (MeV)
0
-0.1
-0.
-4
-6
-8
3 Λ H
4 ΛΛ H
5 ΛΛ H
6 ΛΛ He
P_ΛΣ=0.31%
P_NΞ=0.06%
P_ΣΣ~0.00%
P_ΛΣ=1.88%
P_NΞ=0.91%
P_ΣΣ=0.04%
P_ΛΣ=1.09%
P_NΞ=0.18%
P_ΣΣ=0.04%
NN:Minnesota

Summary

- A key issue of S=–2 study: The binding enegy strongly depends on the spin-dependent part of YN potential.
- What is the YN potential we should utilize for S=–2 study (e.g., ${}^{4}_{\Lambda\Lambda}\mathrm{H}$)?
  - The YN potential to reproduce $B_{\Lambda}({}^{3}_{\Lambda}\mathrm{H})$? ➜ No!
  - The YN potential to reproduce the spin-doublet of $B_{\Lambda}({}^{4}_{\Lambda}\mathrm{H})$-$B_{\Lambda}({}^{4}_{\Lambda}\mathrm{H}^{*})$ and/or $B_{\Lambda}({}^{4}_{\Lambda}\mathrm{He})$-$B_{\Lambda}({}^{4}_{\Lambda}\mathrm{He}^{*})$ ➜ No!
  - The YN potential has to resolve the ${}^{5}_{\Lambda}\mathrm{He}$ anomaly, because the algebraic structure of $\sigma\cdot\sigma$ part of ΛN potential for S=–2 hypernuclei is same as that for ${}^{5}_{\Lambda}\mathrm{He}$.
- Explicit Σ degree of freedom has to be taken into account so as to resolve the ${}^{5}_{\Lambda}\mathrm{He}$ anomaly.

Summary

- We have made an *ab initio* calculation of full coupled channel scheme for $S=-2$ *s*-shell hypernuclei.
  - ${}^{\Lambda\Lambda}_{\ }{}^{4}\mathrm{H}$ $\sim NN\Lambda\Lambda + NN\Lambda\Sigma + NN N\Xi + NN\Sigma\Sigma$
  - ${}^{\Lambda\Lambda}_{\ }{}^{5}\mathrm{H}$, ${}^{\Lambda\Lambda}_{\ }{}^{5}\mathrm{He}$ $\sim NNN\Lambda\Lambda + NNN\Lambda\Sigma + NNN N\Xi + NNN\Sigma\Sigma$
  - ${}^{\Lambda\Lambda}_{\ }{}^{6}\mathrm{He}$ $\sim NNNN\Lambda\Lambda + NNNN\Lambda\Sigma + NNNN N\Xi + NNNN\Sigma\Sigma$
- Nijmegen model D simulated *YY* interaction was used.
  - r_c=0.56271 fm for ${}^1S_0$ and r_c=0.44915 fm for ${}^3S_1$
    (These are same as the parameters for *NN* potential)
  - Full coupled channel;

	${}^1S_0$	${}^3S_1$
$I=0$	$\Lambda\Lambda$-$N\Xi$-$\Sigma\Sigma$	$N\Xi$
$I=1$	$N\Xi$-$\Lambda\Sigma$	$N\Xi$-$\Lambda\Sigma$-$\Sigma\Sigma$
$I=2$	$\Sigma\Sigma$	

Summary

- We have made an *ab initio* calculation of full coupled channel scheme for $S=-2$ s-shell hypernuclei.
  - ${}^{\Lambda\Lambda}{}^{4}\mathrm{H}$ $\sim NN\Lambda\Lambda + NN\Lambda\Sigma + NNN\Xi + NN\Sigma\Sigma$
  - ${}^{\Lambda\Lambda}{}^{5}\mathrm{H}$, ${}^{\Lambda\Lambda}{}^{5}\mathrm{He}$ $\sim NNN\Lambda\Lambda + NNN\Lambda\Sigma + NNNN\Xi + NNN\Sigma\Sigma$
  - ${}^{\Lambda\Lambda}{}^{6}\mathrm{He}$ $\sim NNNN\Lambda\Lambda + NNNN\Lambda\Sigma + NNNNN\Xi + NNNN\Sigma\Sigma$
- Nijmegen model D simulated YY potential (r_c=0.56 fm),
- D2' YN interaction, and
- Minnesota NN potentials were used.
  - We found that

    ${}^{\Lambda\Lambda}{}^{4}\mathrm{H}$, ${}^{\Lambda\Lambda}{}^{5}\mathrm{H}$, and ${}^{\Lambda\Lambda}{}^{6}\mathrm{He}$ have a bound state.

In the future study

- Structural aspects of light hypernuclear systems
  - Rearrangement effect on S=–2 systems,
    due to tensor ΛN-ΣN coupling.