$$\left.\frac{d\sigma_{str}}{d\Omega_\pi}\right|_{spin=\frac{1}{2}} = \left.\frac{d\sigma_{str}}{d\Omega_\pi}\right|_{spin=0} \cdot (1+\xi)$$

$$\xi = \left(\frac{L}{K}\right)^2 \cdot \frac{1}{A^2}; \quad L \lesssim K$$

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Photoproduction of Neutral Pions from Complex Nuclei*

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The photoproduction of neutral pions from complex nuclei is expressed in terms of the production amplitudes from single nucleons, as well as certain properties of the nuclear ground state. Methods are developed for the evaluation of the nuclear matrix elements. Correlations between nucleons lead to suppression of the incoherent cross section for small momentum transfers. The coherent nuclear production is strongly peaked at an angle $\theta \approx 2/kR$. Pions can also be produced by the coupling of the incident photon with the nuclear Coulomb field. The cross section for this process is peaked at an angle $\theta \approx m_\pi^2/2k^2$. Final-state interactions of the produced pion are included by means of the Fernbach-Serber-Taylor model. This leads to attenuation of the nuclear production and to a change in shape of the Coulomb production. The theoretical predictions are compared with experimental measurements at 250 and 900 MeV.

... have since yielded a value of (2.1±0.4) theorem. The projection operators D_λ already transform like scalars to that we need to consider the scalar parts of $t_2{}^\dagger t_1$ only. Recalling that $t_n = K + \mathbf{L}\cdot\boldsymbol{\sigma}_n + M\tau_{3n} + \tau_{3n}\mathbf{L}\cdot\boldsymbol{\sigma}_n$, one can without much trouble decompose $t_2{}^\dagger t_1$ into parts which transform like irreducible tensors of rank 0, 1, and 2, both in ordinary space and in isospace. The scalar part is given by

$$|K|^2+\tfrac{1}{3}|L|^2(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)+\tfrac{1}{3}|M|^2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)+\tfrac{1}{9}|N|^2 \times(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\,. \quad (4.9)$$

By using (4.1), (4.2), and (4.9), one can now easily write down the values of (4.8) corresponding to the four different λ:

$$\begin{aligned} t_{ss}{}^2 &= |K|^2+\tfrac{1}{3}|L|^2-\ \ |M|^2-\tfrac{1}{3}|N|^2,\\ t_{sa}{}^2 &= |K|^2-\ \ |L|^2+\tfrac{1}{3}|M|^2-\tfrac{1}{3}|N|^2,\\ t_{as}{}^2 &= |K|^2+\tfrac{1}{3}|L|^2+\tfrac{1}{3}|M|^2+\tfrac{1}{9}|N|^2,\\ t_{aa}{}^2 &= |K|^2-\ \ |L|^2-\ \ |M|^2+\ \ |N|^2. \end{aligned} \quad (4.10)$$

The total two-particle matrix element which appeared in Eq. (3.11), is finally given by

$$\langle 0|R_{12}t_2{}^\dagger t_1|0\rangle = \textstyle\sum_\lambda t_\lambda{}^2\langle 0|R_{12}D_\lambda|0\rangle\,. \quad (4.11)$$

In Eq. (3.10) we also encountered a one-particle matrix element $\langle 0|t_1{}^\dagger t_1|0\rangle$. The scalar part of $t_1{}^\dagger t_1$ is given by

$$|K|^2+|L|^2+|M|^2+|N|^2\,.$$

This is the only part which has a nonvanishing expectation value in the closed-shell states so that we can write

$$\langle 0|t_1{}^\dagger t_1|0\rangle = |K|^2+|L|^2+|M|^2+|N|^2. \quad (4.12)$$

By combining the spin-isospin matrix elements (4.10) with the spatial matrix elements (5.7) calculated in the next section, we shall obtain a simple relationship between the one-particle and two-particle contributions to the cross section. In the limit of vanishing momentum transfer p, the radial operator R_{12} becomes unity and this simple relation can be expressed as

$$\begin{aligned} A(A-1)\langle 0|t_2{}^\dagger t_1|0\rangle &= A(A-1)\textstyle\sum_\lambda t_\lambda{}^2\langle 0|D_\lambda|0\rangle\\ &= A^2|K|^2-A\{|K|^2+|L|^2+|M|^2+|N|^2\}\\ &= A^2|K|^2-A\langle 0|t_1{}^\dagger t_1|0\rangle\,. \end{aligned} \quad (4.13)$$

For nuclei which do not satisfy the restrictive demands which we imposed, the separation (4.7) of spatial and spin-isospin matrix elements is not generally valid. In the limiting case where the radial operator becomes unity, however, a relation analogous to (4.13) can be derived for a general nucleus in terms of the partition quantum numbers T, S, Y (also denoted by P, P', P'') introduced by Wigner[15] to characterize the symmetry properties of the nucleus. In the cases of interest $2T$ can be identified with the eigenvalue of the operator $(-\sum_i\tau_3{}^i)$, usually called the neutron excess $I=A-2Z$. The more general relation is

$$A(A-1)\langle 0|t_2{}^\dagger t_1|0\rangle = |AK-IM|^2+\mathfrak{M}_2+\mathfrak{M}_1 - A\langle 0|t_1{}^\dagger t_1|0\rangle\,, \quad (4.14)$$

where

$$A\langle 0|t_1{}^\dagger t_1|0\rangle = A\{|K|^2+|L|^2+|M|^2+|N|^2\} - 2I\,\Re(K^*M+\mathbf{L}^*\cdot\mathbf{N}), \quad (4.15)$$

$$\mathfrak{M}_1 = 2S\{|\mathbf{L}-\mathbf{N}|^2-|L_z-N_z|^2\}+4(S-Y)\times(\mathbf{L}^*\cdot\mathbf{N}-L_z{}^*N_z), \quad (4.16)$$

$$\mathfrak{M}_2 = |2SL_z-2YN_z|^2\,. \quad (4.17)$$

Here $\Re(z)$ denotes the real part of z. When the operators $(\sum_i\sigma_z{}^i)$ and $(-\sum_i\sigma_z{}^i\tau_3{}^i)$ are diagonal, they may usually be identified with $2S$ and $2Y$, respectively. The quantities $\mathfrak{M}_1$ and $\mathfrak{M}_2$ are strongly structure-dependent. To a good approximation we can assume that they vanish for even-even nuclei while for odd-A nuclei $\mathfrak{M}_1+\mathfrak{M}_2=|\mathbf{L}\pm\mathbf{N}|^2$ depending on whether Z or N is odd. Unless we specify otherwise, the discussion will henceforth be limited to the closed-shell nuclei for which (4.13) holds.

V. NUCLEAR FORM FACTORS

For the calculation of the spatial matrix elements it is convenient to introduce two-particle densities

$$\rho_\lambda(\mathbf{x}_1,\mathbf{x}_2) = \textstyle\sum_s \displaystyle\int \Psi^*(1\cdots A)D_\lambda\Psi(1\cdots A)d^3x_3\cdots d^3x_A\,,$$

$$\Psi(1\cdots A) = \langle \mathbf{x}_1\cdots\mathbf{x}_A|0\rangle\,, \quad (5.1)$$

where $\sum_s$ indicates that the inner product with respect to all spin-isospin variables must be taken. With this definition

$$\langle 0|e^{i\mathbf{p}\cdot(\mathbf{x}_1-\mathbf{x}_2)}D_\lambda|0\rangle = \int \rho_\lambda(\mathbf{x}_1,\mathbf{x}_2)e^{i\mathbf{p}\cdot(\mathbf{x}_1-\mathbf{x}_2)}d^3x_1d^3x_2\,. \quad (5.2)$$

If we once again confine our attention to the closed-shell nuclei described in Sec. IV, the two-particle densities can be expressed in terms of the "mixed density"

$$d(\mathbf{x}_1,\mathbf{x}_2) = \sum_{j=1}^{A}\varphi_j(\mathbf{x}_1)\varphi_j{}^*(\mathbf{x}_2)\,. \quad (5.3)$$

Each single-nucleon spatial wave function appears in the sum four times. The single-nucleon density is given by

$$\rho(\mathbf{x}) = A^{-1}d(\mathbf{x},\mathbf{x})\,. \quad (5.4)$$

We also define a correlation function

$$h(\mathbf{x}_1,\mathbf{x}_2) = -\frac{1}{4}\frac{d(\mathbf{x}_1,\mathbf{x}_2)d(\mathbf{x}_2,\mathbf{x}_1)}{d(\mathbf{x}_1,\mathbf{x}_1)d(\mathbf{x}_2,\mathbf{x}_2)}\,, \quad (5.5)$$

which is a manifestation of the way in which the position

[15] E. P. Wigner, Phys. Rev. 51, 106 (1937).